GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
FINAL EXAM

DATE: 11/13-Dec-13 COURSE: ECE-2026-B/A

NAME: ______________________________ GT#: ______________
LAST, FIRST ex: gtaburDEll

Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01: Mon - (Juang)	L02: Wed - (Bloch)	L03: Mon - (Casinovi)
L04: Wed - (Bloch)	L05: Tues - (Bhatti)	L06: Thurs - (Coyle)
L07: Tues - (Bhatti)	L08: Thurs - (Coyle)	L09: Tues - (AlRegib)
L10: Thurs - (Ma)	L11: Tues - (Causey)	L12: Thurs - (Ma)
L13: Tues - (Causey)		L14: Thurs - (AlRegib)

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **Explanations and justifications are required to receive full credit**. You will be given partial credit in case of wrong answers, provided that you give sufficient correct reasoning.
- You must **write your answer in the boxes provided on the exam paper itself**. **Only answers in these boxes will be graded as the final solution**. If space is needed for scratch work, use the backs of previous pages.
- ***RELAX, BREATHE, AND TAKE YOUR TIME***

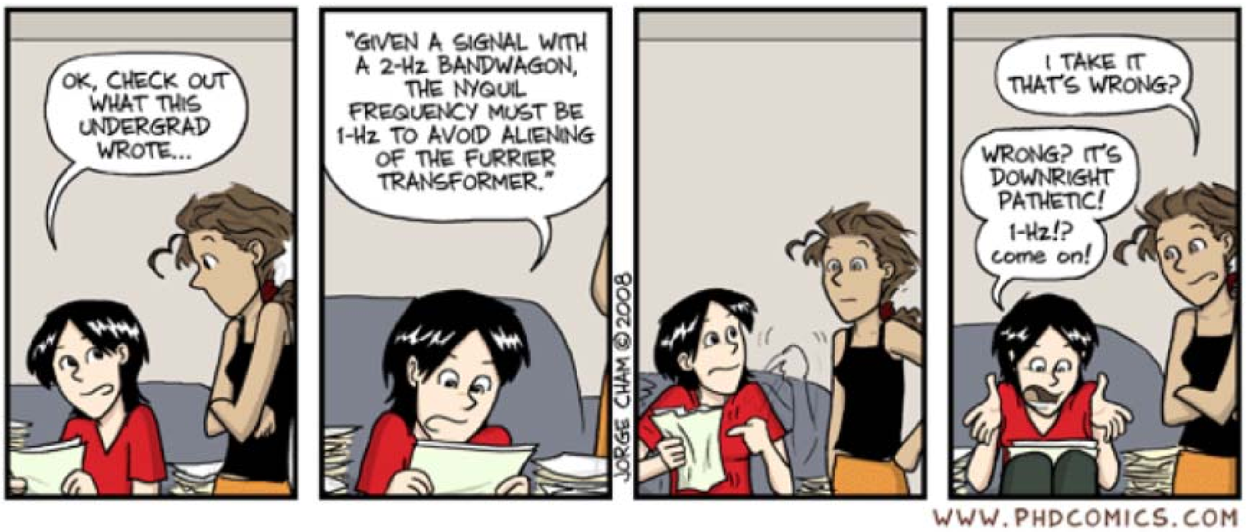
Problem	*Value*	*Score*
1	10	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
Total	170	

Discrete-time Fourier Transform (DTFT)	
Sequence	DTFT
$x[n]$	$X(e^{j\hat{\omega}})$
$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
$x[n]e^{j\hat{\omega}_c n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_c)})$
$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
$x[n]=a^n u[n], \quad \lvert a\rvert<1$	$X(e^{j\hat{\omega}})=\dfrac{1}{1-ae^{-j\hat{\omega}}}$
$x[n]=\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$X(e^{j\hat{\omega}})=\begin{cases}1, & \lvert\hat{\omega}\rvert\le\hat{\omega}_b\\ 0, & \hat{\omega}_b<\lvert\hat{\omega}\rvert\le\pi\end{cases}$
x[n]=u[n]-u[n-L]	$X(e^{j\hat{\omega}})=\dfrac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega}(L-1)/2}$

Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	*z-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$
General cosine	$r^n\cos(\hat{\omega}_0 n)u[n]$	$\dfrac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	z*-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay	$x[n-n_d]$	$z^{-n_d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

PROBLEM fa-13-Final.1:
During the semester, you have learned a new vocabulary pertaining to the processing of signals. Therefore, the following comic strip about a GTA grading papers (produced by a 1997 ME graduate from Georgia Tech) should now qualify as humor:



(a) What is the correct answer that the GTA is looking for? (i.e., ***explain why 1-Hz is wrong***).

If the bandwidth is 2-Hz, Nyquist requires 2x that so the correct answer is 4-Hz

(b) In the second comic pane, ***four*** technical terms (consisting of six words total) from DSP are morphed into other words (i.e., the terms are close to what they should be but humorously wrong). For example, "*Nyquil Frequency*" **should be** "*Nyquist Rate*" Identify the remaining ***three*** terms (consisting of four words total), and give the correct DSP technical word for each.

Bandwagon -> Bandlimited or Bandwidth

Aliening -> Aliasing

Furrier Transformer -> Fourier Transform

PROBLEM fa-13-Final.2:

Each question is independent of the others. **Show the steps** you use to find the solution to each question.

(a) Simplify $x(t) = \Re e\{(2-2j)e^{j30\pi t}\}$ to the form $x(t) = A\cos(\omega t + \theta)$

$2-2j \Rightarrow 2\sqrt{2}e^{j\,-\pi/4}$

$x(t) = 2\sqrt{2}\cos(30\pi t - \pi/4)$

(b) Simplify the expression $z = \sum_{k=1}^{20}\{\delta[k-3] + \delta[k-5]\}\, e^{j0.25\pi k}$ to the form $z = re^{j\theta}$

$e^{j0.25\pi\cdot 3} + e^{j0.25\pi\cdot 5}$

$= e^{j3\pi/4} + e^{j5\pi/4} \Rightarrow e^{j3\pi/4} + e^{-j3\pi/4}$

$= 2\Re e\{e^{j3\pi/4}\} = -\sqrt{2} \approx 1.4142$

$r = \sqrt{2} \approx 1.4142$ $\quad\theta = -\pi$

(c) Find the zeros for the following system function: $H(z) = 1 - z^{-6}$

$$z^6 - 1 = 0 \Rightarrow z^6 = 1$$

$$z^6 = e^{j2\pi k} \rightarrow z = e^{j\frac{2\pi}{6}k}, \quad k = 0, \dots, 5$$

zeros = $z = e^{j\pi/3 k}, \; k = 0, \dots, 5$

$$z = 0, e^{j\pi/3}, e^{j\frac{2\pi}{3}}, e^{j\pi}, e^{j\frac{4\pi}{3}}, e^{j5\pi/3}$$

$e^{j\frac{4\pi}{3}} \xrightarrow{\text{or}} e^{-j\frac{2\pi}{3}}$, $e^{j5\pi/3} \xrightarrow{\text{or}} e^{-j\pi/3}$

PROBLEM fa-13-Final.3:

For each part of the below questions, pick a correct frequency **<u>from the list on the right hand side</u>** and enter its letter in the answer box. **Justify your answer** with a brief explanation and/or appropriate work to receive any credit.

Frequency

(a) 8000 Hz
(b) 4000 Hz
(c) 2000 Hz
(d) 1600 Hz
(e) 1200 Hz
(f) 1000 Hz
(g) 800 Hz
(h) 500 Hz
(i) 400 Hz

(a) Determine the Nyquist rate for sampling the signal $x(t)$ defined by:
$x(t) = (A + B\sin(300\pi t)\cos(500\pi t))$.

ANS = 800 Hz (g)

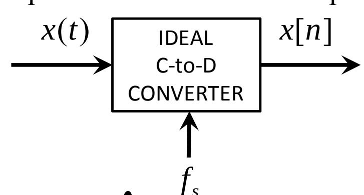
$x(t) = A\cos(500\pi t) + B\sin(300\pi t)\cos(500\pi t)$

modulation adds frequencies

highest freq: $300\pi + 500\pi \Rightarrow 800\pi \rightarrow 2\pi(400)$

$\therefore$ Nyquist rate $\Rightarrow 2\cdot 400 \Rightarrow 800$

(b) If the output from an ideal C/D converter is $x[n] = 12\cos(0.4\pi n)$, and the sampling rate is 1000 samples/sec, then determine one possible value of the input frequency for $x(t) = 12\cos(2\pi f_0)$.



ANS = 1200 (e) OR 800 (g)

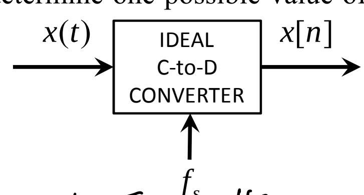
$\hat{w} = \frac{2\pi(f_0)}{f_s} + 2\pi l \rightarrow f_0 = \frac{\hat{w} - 2\pi l}{2\pi} f_s = \frac{\hat{w} f_s}{2\pi} - l f_s$

$l = 0$, $f_0 = \frac{0.4\pi}{2\pi} \cdot 1000 = 200$

$l = -1$, $f_0 = 200 + 1000 = 1200$

$l = 1$, $f_0 = 200 - 1000 = -800$

(c) If the output from an ideal C/D converter is $x[n] = 12\cos(0.5\pi n)$, and the input signal $x(t)$ is defined by: $x(t) = 12\cos(2400\pi t)$then determine one possible value of the sampling frequency (f_s) of the C-to-D converter:



ANS = 1600 (d)

$f_s \Rightarrow$ '+' or '−'

$\hat{w} = \frac{2\pi f}{f_s} + 2\pi l$ $\quad f_s = \frac{2\pi f}{\hat{w} - 2\pi l} = \frac{2400\pi}{0.5\pi - 2\pi l} = \frac{4800}{1 - 4l}$

$l = 0$, ~~$f_s = 4800$~~

$l = 1$, $f_s = -1600$

$l = -1$, ~~$f_s = 960$~~

PROBLEM fa-13-Final.4:
Which of the following signals are periodic? For those that are periodic, find the fundamental period, T_0, and the Fourier Series coefficients a_k for all k.

(a) $x(t) = -10 + \cos\left(300\pi t - \frac{\pi}{3}\right) + \cos\left(25t + \frac{\pi}{6}\right)$

Periodic? **Yes or No** (Circle answer)

If YES: $T_0 =$ $\boxed{}$ seconds

No f_0 for 300π and 25

$a_k =$ N/A

(b) $x(t) = \cos\left(\sqrt{2}\pi t + \frac{\pi}{4}\right) + \cos\left(3\sqrt{2}\pi t - \frac{\pi}{8}\right)$

Periodic? **Yes or No** (Circle answer)

If YES: $T_0 =$ $\boxed{\sqrt{2}}$ seconds

≈ 1.4142

$GCD\left(\frac{\sqrt{2}}{2}, 3\frac{\sqrt{2}}{2}\right) \Rightarrow \sqrt{2}/2 \Rightarrow f_0$

$\omega_0 = \sqrt{2}\pi \quad T_0 = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$a_k =$

$a_1 = \frac{1}{2}e^{j\pi/4} \qquad a_3 = \frac{1}{2}e^{-j\pi/8}$

$a_{-1} = \frac{1}{2}e^{-j\pi/4} \qquad a_{-3} = \frac{1}{2}e^{j\pi/8}$

(c) $x(t) = \sum_{k=-\infty}^{\infty} \frac{k^2-1}{|k|^3} e^{-j100k\pi t}$

Periodic? **Yes** or **No** (Circle answer)

If YES: $T_0 =$ $\boxed{1/50}$ seconds

$= 0.02$

$\omega_0 = 100\pi, \ f_0 = 50$

$a_k =$ $\boxed{\frac{k^2-1}{|k|^3}}$

(d) $x(t) = 10\cos(1500\pi t)\cos\left(300\pi t - \frac{\pi}{7}\right)$

Periodic? **Yes** or **No** (Circle answer)

If YES: $T_0 =$ $\boxed{1/300}$ seconds

$X(t) = 5\left[\cos(1800\pi t - \pi/7) + \cos(1200\pi t + \pi/7)\right]$

900 600

GCD $\Rightarrow 300 \Rightarrow f_0$

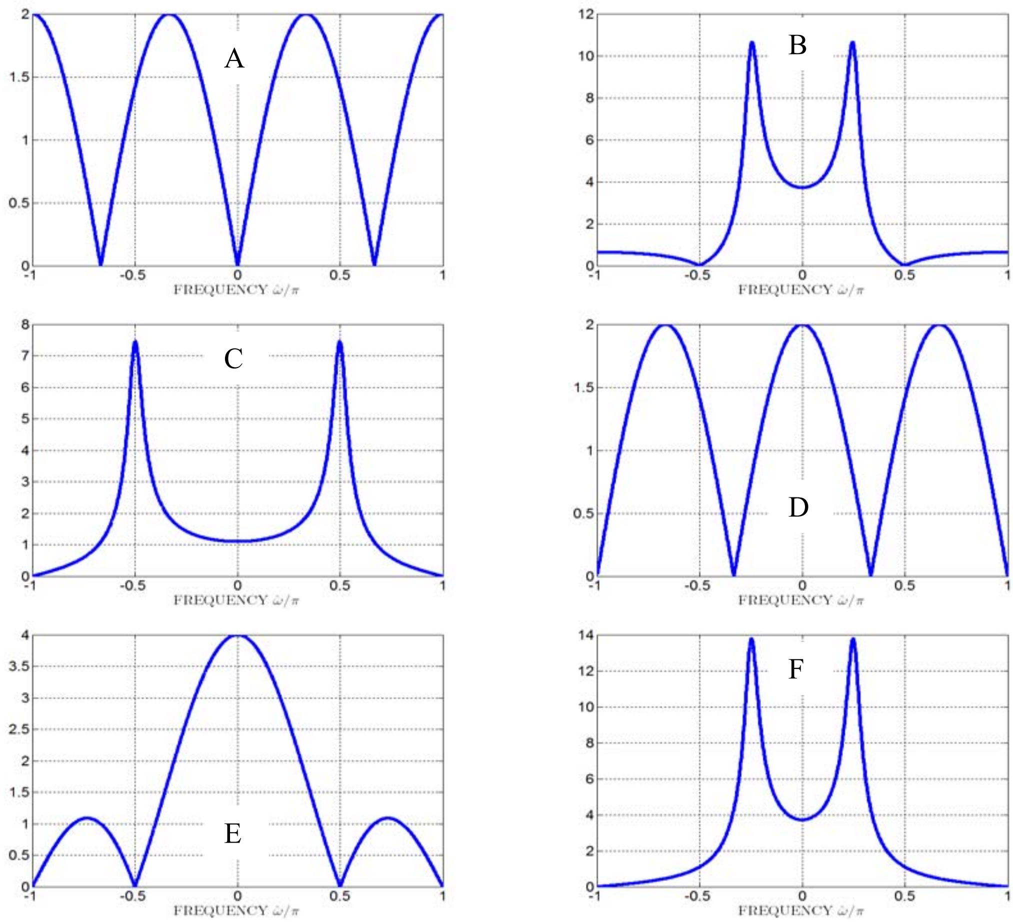
$a_k =$

$a_2 = \frac{5}{2}e^{j\pi/7}$ $a_3 = \frac{5}{2}e^{j-\pi/7}$

$a_{-2} = \frac{5}{2}e^{-j\pi/7}$ $a_{-3} = \frac{5}{2}e^{j\pi/7}$

PROBLEM fa-13-Final.5:

Shown below are the magnitude responses of six different LTI systems

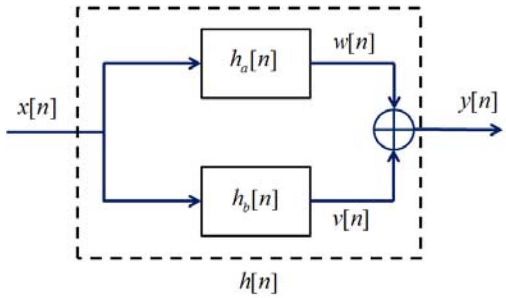


Match the magnitude response above to its corresponding system (defined by either a difference equation, a system function, or a MATLAB statement) by writing a letter from {A,…,F} in the answer box:

F	$y[n] = 1.8\cos\left(\frac{\pi}{4}\right)y[n-1] - 0.81y[n-2] + x[n] + x[n-1]$
C	`yy=filter([1 1],[1 0 0.81],x);`
A	$H(z) = 1 - z^{-3}$
D	$y[n] = x[n] + x[n-3]$
E	`yy=conv(xx,ones(1,4));`
B	$H(z) = \frac{1+z^{-2}}{1-1.8\cos\left(\frac{\pi}{4}\right)z^{-1}+0.81z^{-2}}$

PROBLEM fa-13-Final.6:
Consider the system below where

$$h_a[n] = \frac{1}{A}(u[n] - u[n-A]); \quad h_b[n] = \frac{1}{B} h_a[n-B]$$



The overall impulse response is $h[n] = h_a[n] + h_b[n]$. Assume that A=9 and B=4.

(a) Find the overall frequency response $H(e^{j\hat{\omega}})$

$$H_a(e^{j\hat{w}}) = \frac{\sin(\hat{w}\frac{9}{2})}{9\sin(\hat{w}/2)} e^{-j\hat{w}4}$$

$$H_b = \frac{1}{4} \cdot H_a(e^{j\hat{w}}) \cdot e^{-j4\hat{w}} = \frac{1}{36} \frac{\sin(\hat{w}\frac{9}{2})}{\sin(\hat{w}/2)} e^{-j8\hat{w}}$$

$$H(e^{j\hat{w}}) = H_a(e^{j\hat{w}}) + H_b(e^{j\hat{w}})$$
$$= \left(1 + \frac{1}{4}e^{-j\hat{w}4}\right) H_a(e^{j\hat{w}})$$
$$= \left(1 + \frac{1}{4}e^{-j\hat{w}4}\right)\left(\frac{\sin(\hat{w}\frac{9}{2})}{9\sin(\hat{w}/2)}\right) e^{-j4\hat{w}}$$

$H(e^{j\hat{\omega}}) =$ $\left(1 + \frac{1}{4}e^{-j\hat{w}4}\right) \frac{\sin(\hat{w}\frac{9}{2})}{9\sin(\hat{w}/2)} e^{-j4\hat{w}}$

(b) Find the output $y[n]$ when $x[n] = (-1)^n$ $= \cos(\pi n)$

$$H(e^{j\pi}) = \left(1 + \frac{1}{4}e^{-j4\pi}\right)\frac{\sin(\pi\frac{9}{2})}{9\sin(\pi/2)}e^{-j4\pi}$$

$$= (1 + 1/4)\left(\frac{1}{9}\right) = \left(\frac{5}{4}\right)\left(\frac{1}{9}\right) = \underline{\frac{5}{36}}$$

$$y(n) = \frac{1}{12}\cos(\pi n)$$

$y[n] = \frac{5}{36}\cos(\pi n) = \frac{5}{36}(-1)^n$

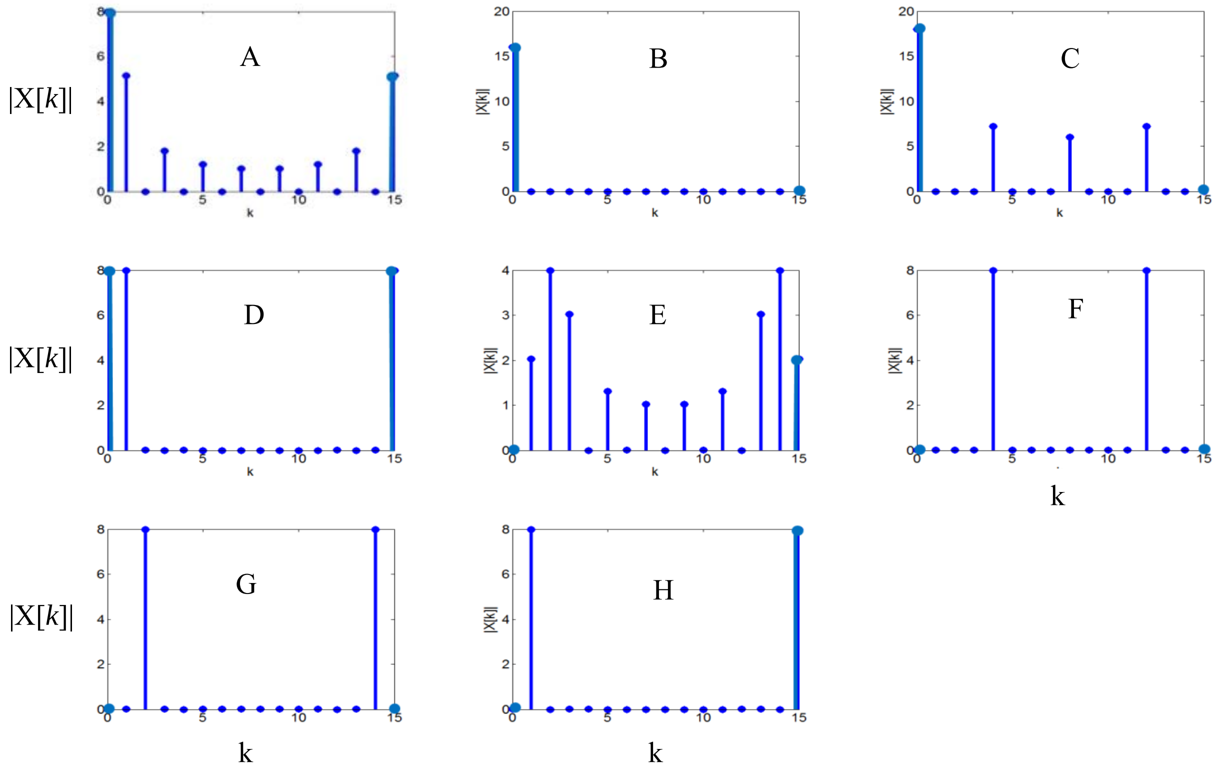
(c) Find the output $y[n]$ when $x[n] = 2$ (i.e., $x[n]$ is a constant)

$$H(e^{j0}) = (1 + 1/4)\cdot(1) = 5/4$$

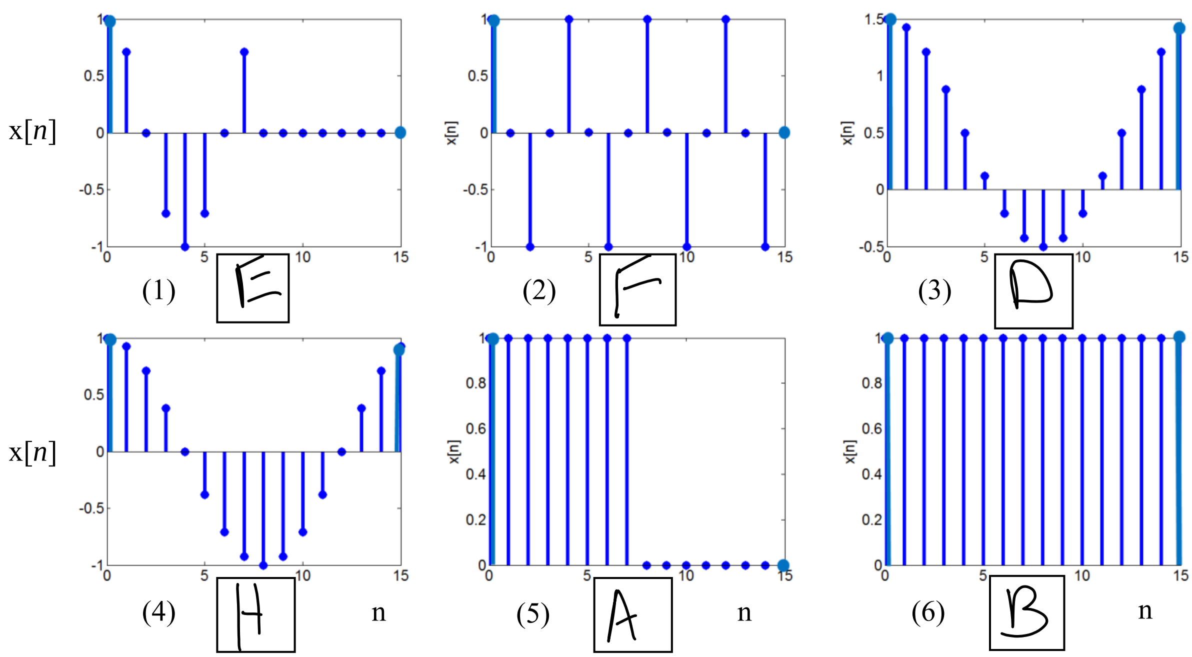
$$X(n) = 2 \cdot 5/4 = 5/2$$

$y[n] = 5/2$

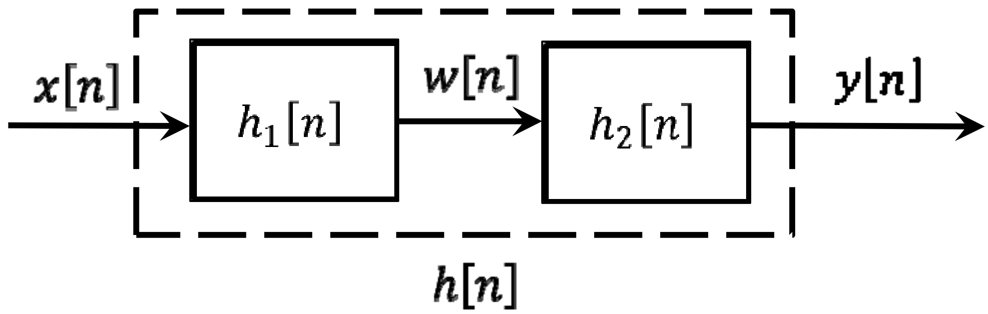
PROBLEM fa-13-Final.7:
Shown below are eight sketches of 16-point DFT spectrums labeled A through H. The spectrum is represented by a stem plot of the coefficient magnitude $|X[k]|$ vs. the frequency index: $k \in \{0,1,\ldots,15\}$.



Match one of the six plots for $x[n]$ shown below with a DFT plot above by putting a letter (A through H) in its box. (NOTE: There are more DFT plots above than stem plots for $x[n]$ shown below. Show relevant work on the back of this page.)



PROBLEM fa-13-Final.8:
Consider the system shown below



where

$$w[n] = .8w[n-1] - .64w[n-2] + x[n] - 0.4x[n-1]$$

(a) Find $H_1(z)$

$H_1(z) = \dfrac{1 - 0.4z^{-1}}{1 - 0.8z^{-1} + 0.64z^{-2}}$

(b) Find the poles and zeros of $H_1(z)$

Zeros: $0.4e^{j0}$, poles: $0.8e^{j\pi/3}$, $0.8e^{-j\pi/3}$

Poles:	Zeros:
$0.8e^{j\pi/3}$ $0.8e^{-j\pi/3}$	$0.4e^{j0}$ 0

(c) Find $h_1[n]$ (HINT: It is NOT necessary to use Partial Fraction Expansion on this problem.)

Use table

$h_1[n] = (0.8^n)\cos\left(\frac{\pi}{3}n\right) u[n]$

(d) Find $H_2(z)$ such that the **overall impulse response** is $h[n] = \delta[n] - \delta[n-1]$

$$H_2(z) = \frac{1}{H_1(z)} \cdot (1 - z^{-1})$$

$H_2(z) = \left(\frac{1 - 0.8z^{-1} + 0.64z^{-2}}{1 - 0.4z^{-1}}\right)(1 - z^{-1})$

PROBLEM fa-13-Final.9:
The following questions are independent of each other.

(a) Make a sketch of the spectrogram that would be plotted from the following MATLAB code:

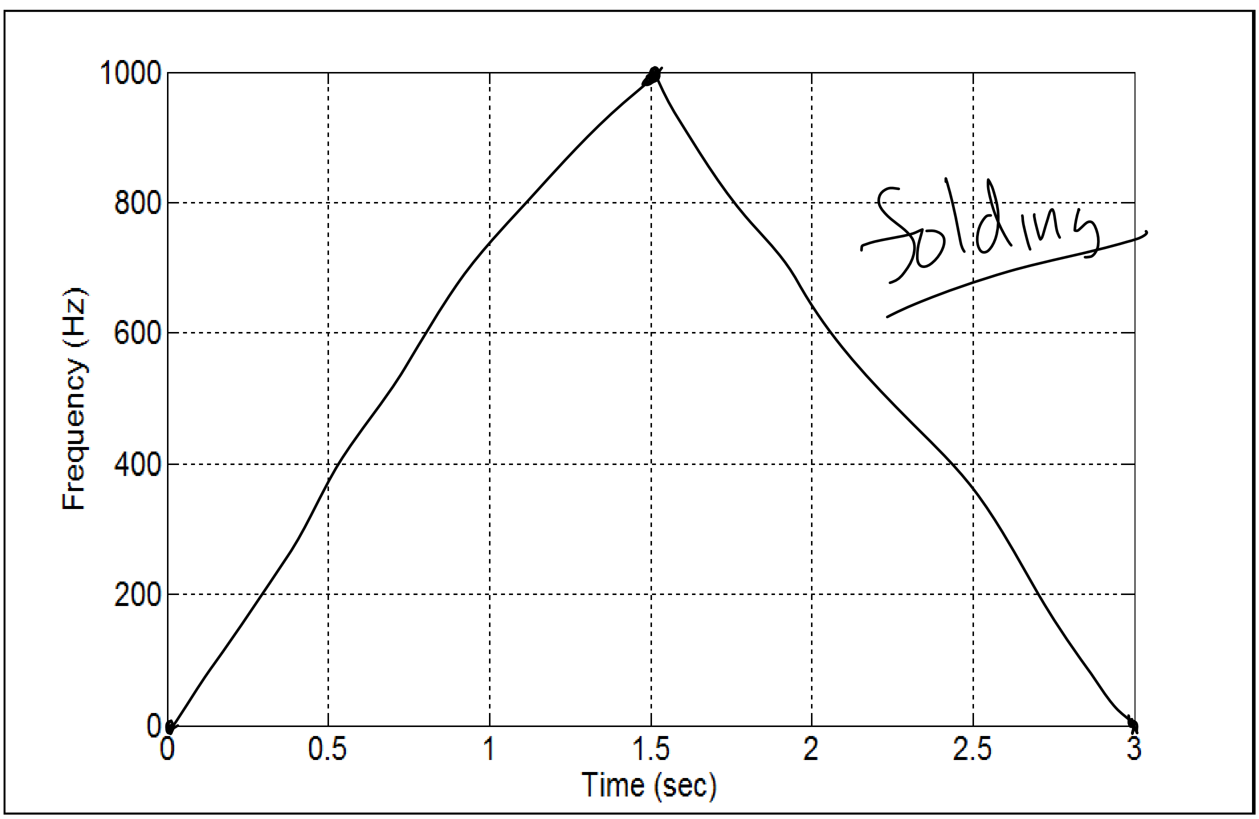
```
fsamp =2000;
t=0:1/fsamp:3;
alpha=(2000*pi)/3;
beta=0;
phi=pi/4;
x=cos(alpha*t.^2+beta.*t+phi);
spectrogram(x, … )
```

(NOTE: Specific parameters of ‘spectrogram’ are irrelevant to your plot and therefore left out)

$$x = \cos\left(\frac{2000\pi}{3}t^2 + \pi/4\right)$$

$$f_i(t) = \frac{1}{2\pi}\,\frac{4000\pi}{3}t = \frac{2000}{3}t$$

$f_s = 2000\text{ Hz} \rightarrow$ fold @ $f_i(t) = 1000\text{ Hz}$
$t = 1.5$



(b) Consider the following lines of MATLAB code

```
xn=[1 ,-1.5];
yn=filter([1,0.9],[1,-1],xn);
```

Is this system stable? YES or NO (circle one)

Explain your answer:

$H(z) = \frac{1+0.9z^{-1}}{1-z^{-1}}$ pole@1

poles need to be inside Unit circle for stability