GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Fall 2012
Final Exam

December 12, 2012

NAME: ____________ (FIRST) ______________ (LAST) GT username: ______________ (e.g., gtxyz123)

Circle your recitation section in the chart below (otherwise you lose 3 points!):

	Mon	Tue	Wed	Thu
9:30 – 11am				L06 (Fekri)
12 – 11:30pm		L07 (Al-Regib)		L08 (Fekri)
1:30 – 3pm		L09 (Al-Regib)		L10 (Rozell)
3 – 4:30pm	L01 (Juang)	L11 (Davenport)	L02 (Zajic)	L12 (Rozell)
4:30 – 6pm	L03 (Baxley)	L13 (Davenport)	L04 (Zajic)	
6 – 7:30pm	L05 (Baxley)			

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” × 11”) of hand-written notes permitted. Calculators are permitted.
- JUSTIFY your reasoning CLEARLY to receive full credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
No/Wrong Rec	–3	
Total		

POTENTIALLY USEFUL TABLES

Discrete-time Fourier Transform (DTFT)	
Sequence	DTFT
$x[n]$	$X(e^{j\hat{\omega}})$
$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
$x[n]e^{j\hat{\omega}_c n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_c)})$
$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
$\frac{\sin(\hat{\omega}_b n)}{\pi n}$	$X(e^{j\hat{\omega}})=\begin{cases}1, & \lvert\hat{\omega}\rvert\le\hat{\omega}_b \\ 0, & \hat{\omega}_b<\lvert\hat{\omega}\rvert\le\pi\end{cases}$

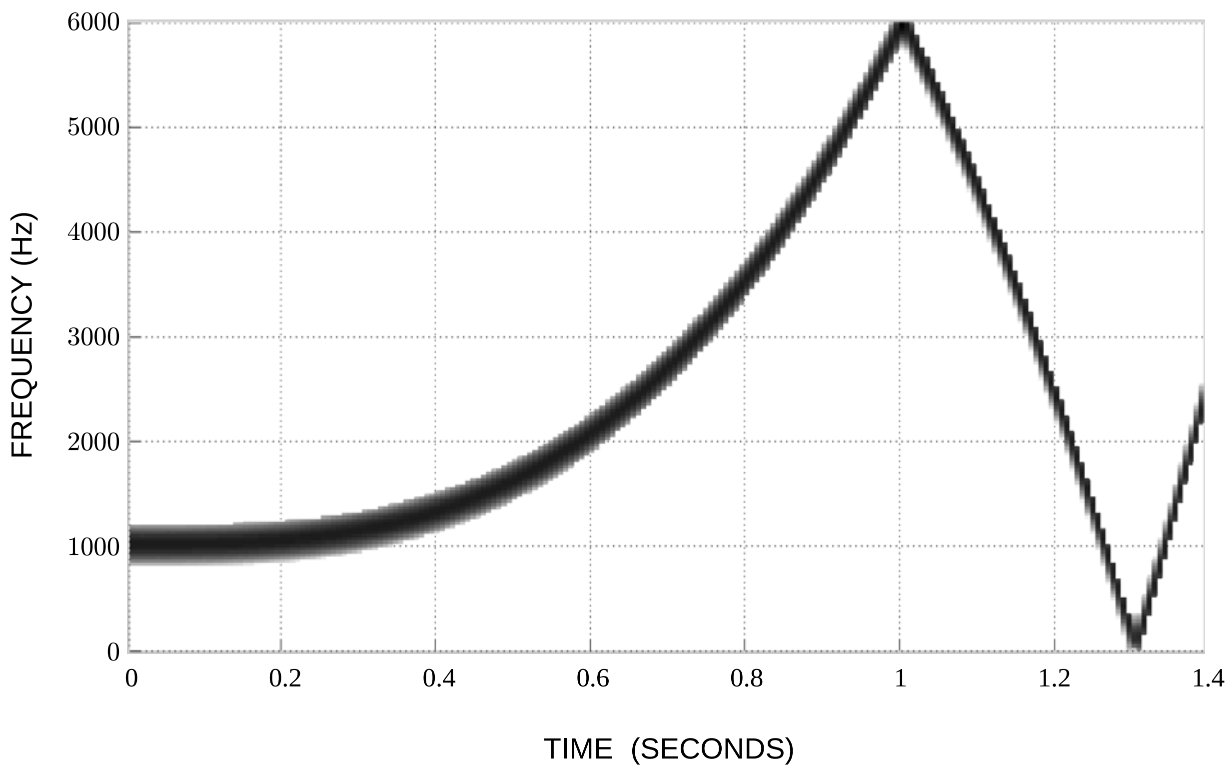
Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	z*-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\frac{1}{1-az^{-1}}$
General cosine	$r^n\cos(\hat{\omega}_0 n)u[n]$	$\frac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	z*-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay	$x[n-n_d]$	$z^{-n_d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

PROB. F12-Final.1. Consider the eight lines of MATLAB code shown below:

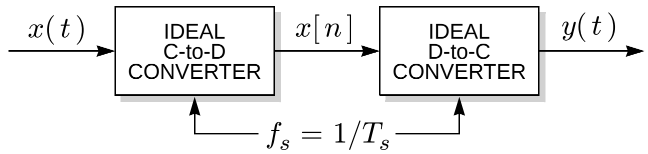
```
dur   = [          ];
fsamp = [          ];
ALPHA = [          ];
BETA  = [          ];

tt = 0:(1/fsamp):dur;
psi = ALPHA*pi*(tt.^4) + BETA*pi*tt;
xx = real(exp(1i*psi));
spectrogram(xx, ... );   % see footnote1
```

Find numerical values for the unspecified parameters `dur`, `fsamp`, `ALPHA`, and `BETA` so that running the above code produces the following spectrogram:



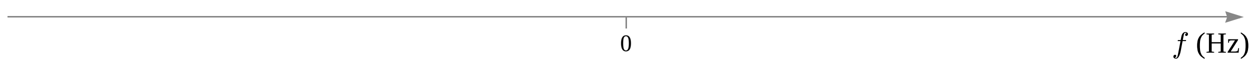
1. To avoid confusion, the remaining arguments of `spectrogram` are not shown. They are not relevant. If you are curious, however, the complete command is `spectrogram(xx,100,20,400,fsamp,'yaxis')`.

PROB. F12-Final.2. Consider the following ideal sampling and reconstruction system:



where the input is: $x(t) = \cos(2\pi t + 0.9\pi) - \cos(4\pi t - 0.1\pi) + \cos(12\pi t + 0.1\pi)$.

(a) Draw the two-sided line spectrum for $x(t)$ in the space below, taking care to label both the frequency and complex amplitude of each line:



(b) The fundamental frequency of $x(t)$ is $f_0 =$ ______ Hz.

(c) In order for $y(t) = x(t)$, the sampling frequency must satisfy $f_s >$ ______ (samples/sec).

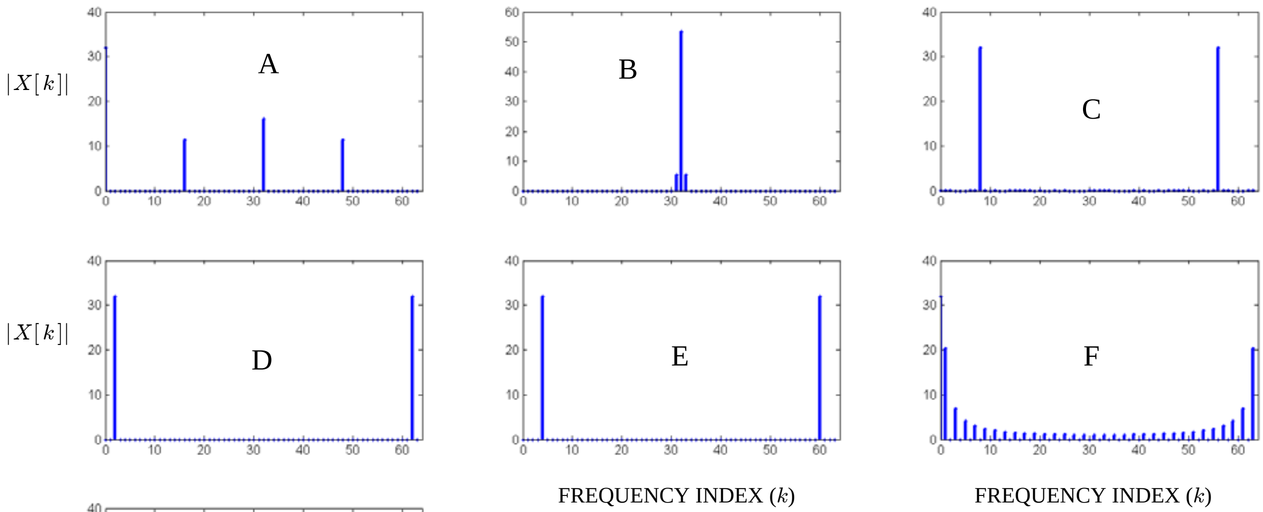
(d) If $y(t) = \cos(2\pi t + 0.9\pi)$, the sampling frequency is $f_s =$ ______ (samples/sec).

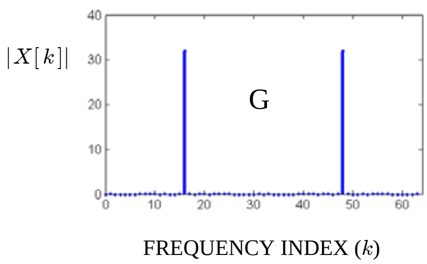
(e) If $y(t) = -\cos(4\pi t - 0.1\pi)$, the sampling frequency is $f_s =$ ______ (samples/sec).

(f) If the sampling frequency is $f_s = 0.2$ sample/sec, the output will be $y(t) =$ ______ .

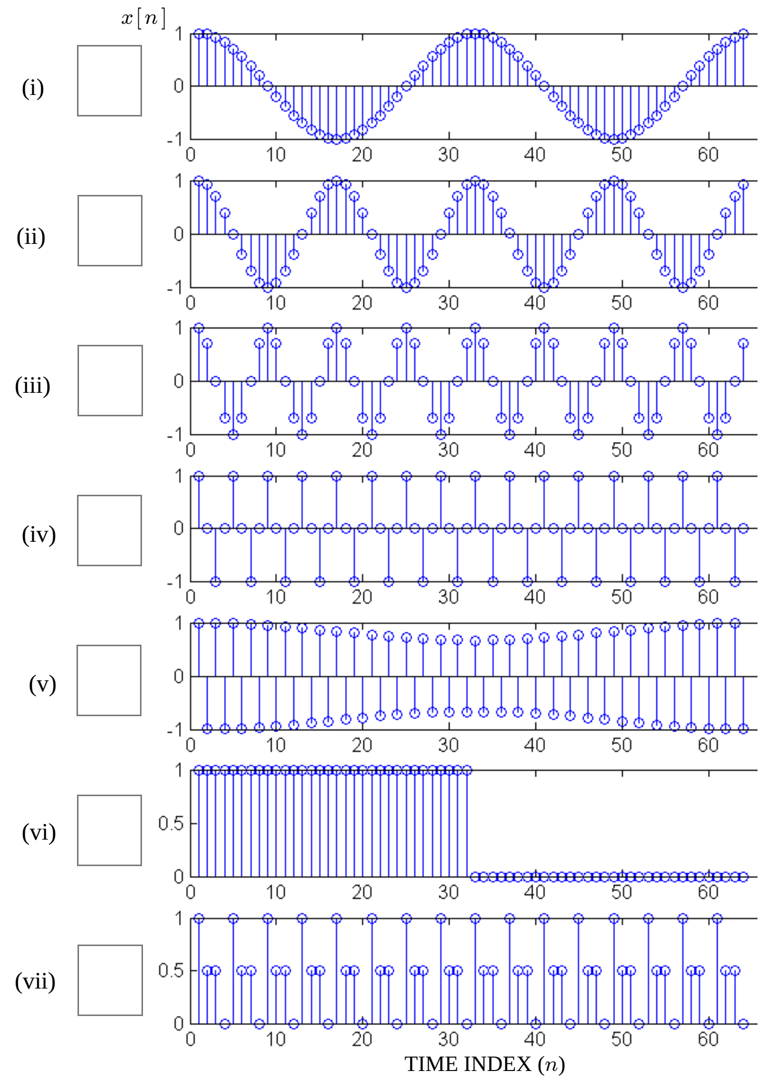
(put in simplest possible form)

PROB. F12-Final.3. Shown below are sketches of the 64-point DFT spectrum for eight different length-64 signal vectors, labelled A through G. (The spectrum is represented by a stem plot of the coefficient magnitude $|X[k]|$ versus the frequency index $k \in \{0,\ 1,\ \ldots\ 63\}$.)

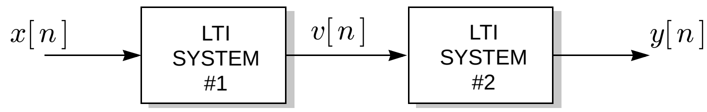




Match one of the seven signals $\{x[n]\}$ below to its corresponding DFT spectrum by writing a letter (A through G) in each answer box:



PROB. F12-Final.4. Consider the cascade connection of two LTI systems, as shown below, where the output of system#1 is the input to system#2, and the overall output is the output of system#2:



System #1 is defined by its impulse response $h_1[n] = 2\delta[n-1] - 2\delta[n-2]$, and system #2 is defined by its difference equation $y[n] = 3v[n-1] + 3v[n-2]$.

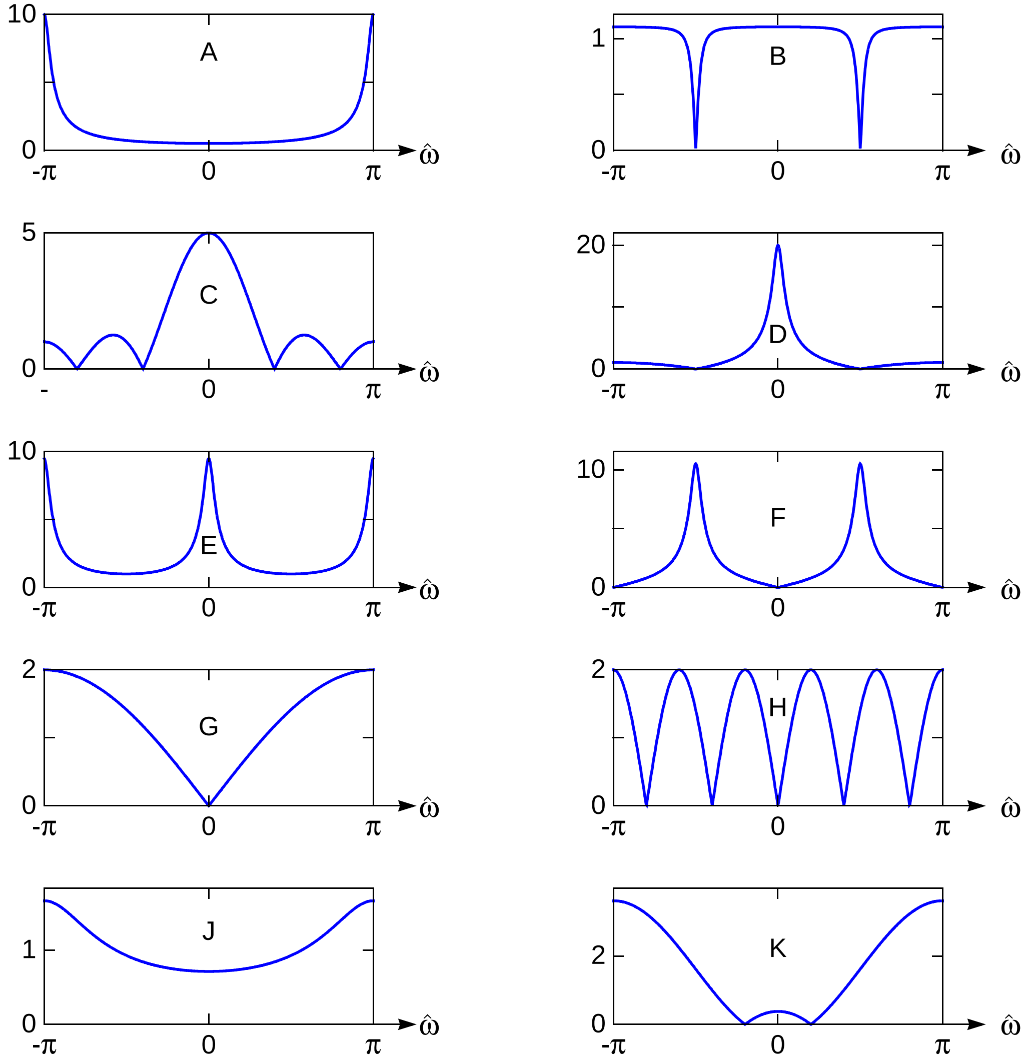
(a) If the input *to system #2* is $v[n] = 0.2$ (a constant), the output is: $y[n] = \boxed{}$.

(b) If the input *to system #2* is $v[n] = 0.2\cos(0.25\pi n)$, the output is: $y[n] = \boxed{}$.

(c) If the input *to system #1* is $x[n] = (-1)^n$, the *overall* output is: $y[n] = \boxed{}$.

(d) If the input *to system #1* is $x[n] = \cos(0.5\pi n)$, the *overall* output is: $y[n] = \boxed{}$.

PROB. F12-Final.5. Shown below are the magnitude responses of ten different LTI systems:



Match each magnitude response above to its corresponding system (defined by either a difference equation, a system function, or a MATLAB statement) by writing a letter from {A, B, C, ... H, J, K} in each answer box:

(i) $y[n] = -0.9y[n-1] + x[n]$

(ii) $H(z) = \dfrac{1}{1-0.9z^{-1}} - \dfrac{1}{1+0.9z^{-1}}$

(iii) $H(z) = 1 - 2\cos(\pi/5)z^{-1} + z^{-2}$

(iv) `yy = conv(xx, ones(1,5));`

(v) $y[n] = 0.9y[n-1] + x[n] + x[n-2]$

(vi) $H(z) = \dfrac{1}{1+0.4z^{-1}}$

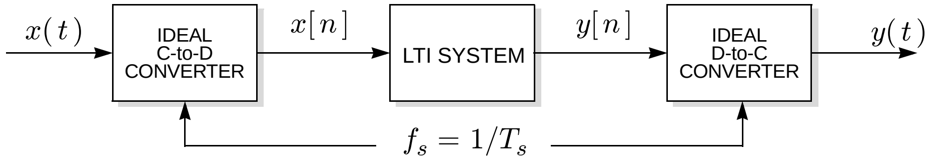
(vii) $H(z) = \dfrac{1-z^{-2}}{1+0.81z^{-2}}$

(viii) $H(z) = \dfrac{1+z^{-2}}{1+0.81z^{-2}}$

(ix) $y[n] = x[n] - x[n-5]$

(x) `yy = conv(xx, [1,-1]);`

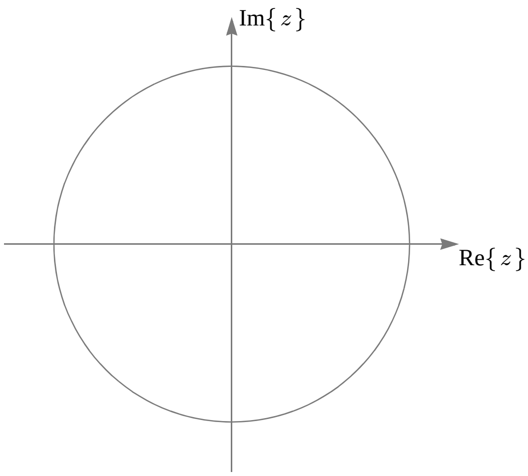
PROB. F12-Final.6. Consider the below system for discrete-time processing of a continuous-time signal:



Assume that the discrete-time LTI system is defined by the difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] - 0.3y[n-1].$$

(a) In this part, assume that $b_1 = -1.4$. Find *all* poles and *all* zeros of the discrete-time LTI system. Give your answer as a pole-zero plot.



(b) The overall system can be used to null one continuous-time sinusoid. The frequency that is nulled is controlled by the value of the filter parameter b_1. If the sampling rate is $f_s = 8000$ Hz, find the value of b_1 so that the overall system nulls out a sinusoid at 60 Hz.

$b_1 = \boxed{}$.

(c) The range of values for b_1 for which the overall system is *not* a nulling filter is $|b_1| > \boxed{}$.

PROB. F12-Final.7. An LTI system has difference equation:

$$y[n] = 1.4x[n] - 0.7y[n-1].$$

(a) This system is [FIR][IIR] (circle one).

(b) The system function is: $H(z) = \boxed{}$.

(c) An equation for the impulse response $h[n]$ that is valid for all $-\infty < n < \infty$ is:

$h[n] = \boxed{}$.

(d) The output in response to $x[n] = \cos(\pi n + \pi/5)$ is:

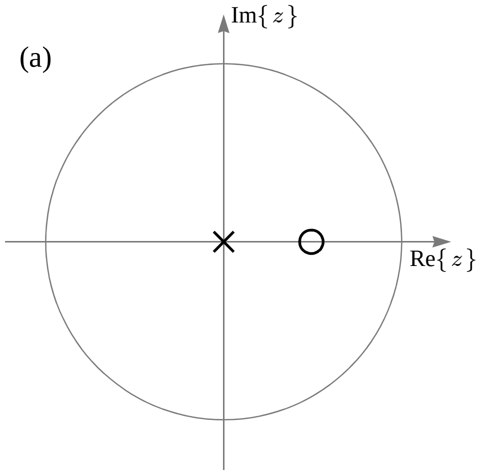
$h[n] = \boxed{}$.

(e) If the *output* of this system is $y[n] = \delta[n]$, then its *input* must be: $x[n] = \boxed{}$.

PROB. F12-Final.8. Shown below are the pole-zero plots for three different LTI systems. For each, write a difference equation for an LTI system that has a matching pole-zero plot. The exact locations of the poles and zeros are not labeled, so you will have to estimate. You don't need to be incredibly precise; you will get full credit if your coefficients are within 15% of correct values.

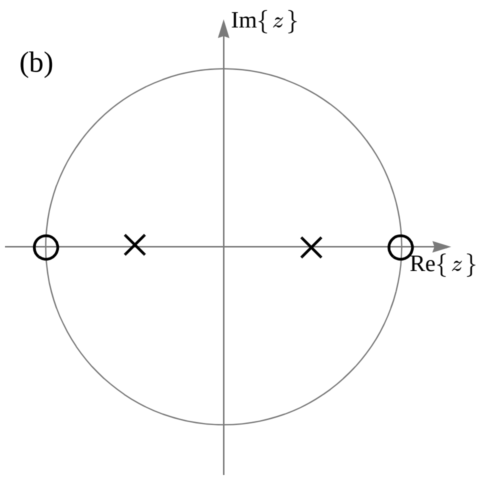
(*Hint:* The coefficients should all be real.)

(a)



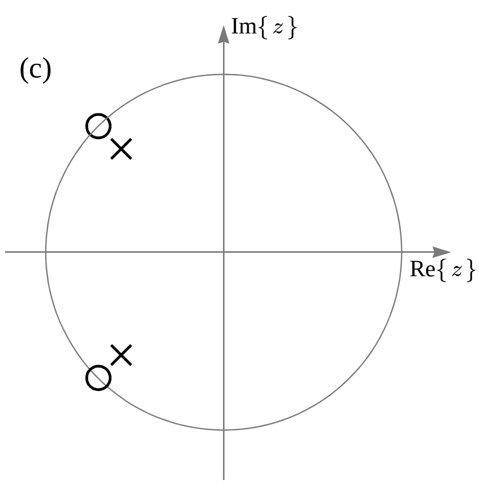
$y[n] =$

(b)



$y[n] =$

(c)



$y[n] =$

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VERSION A

NAME: ANSWER KEY (FIRST) (LAST) GT username: ________ (e.g., gtxyz123)

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4:30 – 6pm	L03 (Baxley)	L13 (Davenport)	L04 (Zajic)	
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4	20	
5	20	
6	20	
7	20	
8	20	
No/Wrong Rec	–3	
Total		

PROB. F12-Final.1. Consider the eight lines of MATLAB code shown below:

```
dur    = 1.4    ;

fsamp  = 12000  ;

ALPHA  = 2500   ;

BETA   = 2000   ;

tt = 0:(1/fsamp):dur;
psi = ALPHA*pi*(tt.^4) + BETA*pi*tt;
xx = real(exp(1i*psi));
spectrogram(xx, ... );    % see footnote1
```

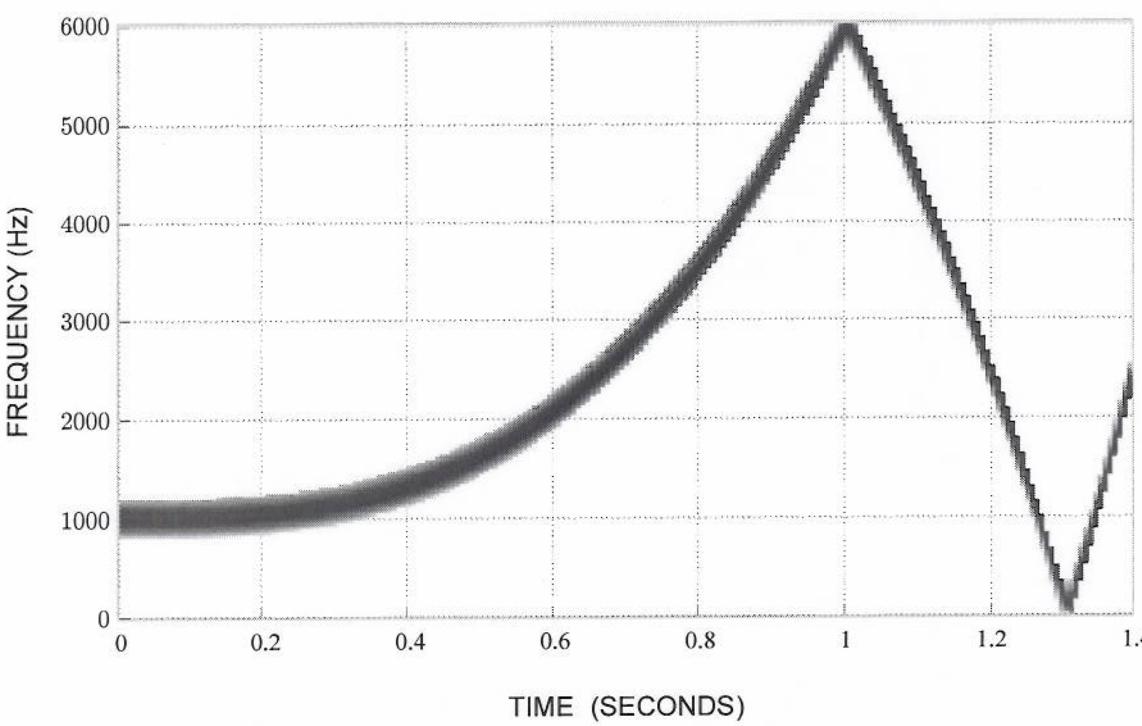
$\psi(t) = \alpha\pi t^4 + \beta\pi t$

$f_i(t) = \frac{\psi'(t)}{2\pi} = 2\alpha t^3 + \beta/2$

@ $t=0$, $f_i(0) = 1000 \Rightarrow \beta = 2000$

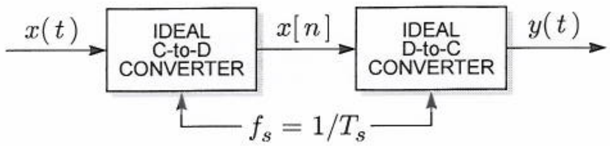
@ $t=1$, $f_i(1) = 6000 \Rightarrow \alpha = 2500$

Find numerical values for the unspecified parameters `dur`, `fsamp`, `ALPHA`, and `BETA` so that running the above code produces the following spectrogram:



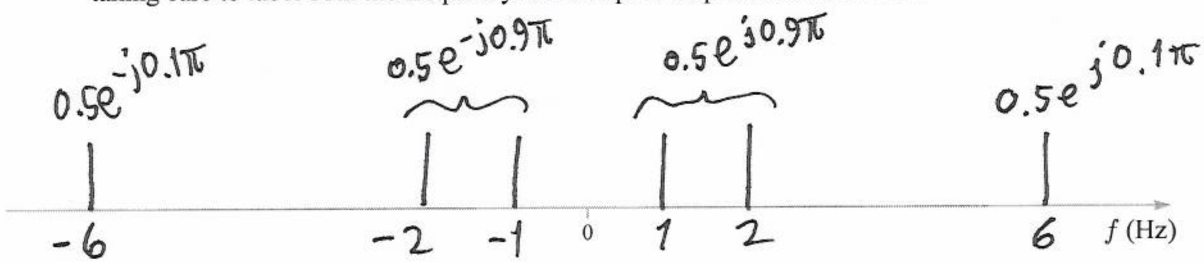
1. To avoid confusion, the remaining arguments of `spectrogram` are not shown. They are not relevant. If you are curious, however, the complete command is `spectrogram(xx,100,20,400,fsamp,'yaxis')`.

PROB. F12-Final.2. Consider the following ideal sampling and reconstruction system:



where the input is: $x(t) = \cos(2\pi t + 0.9\pi) - \cos(4\pi t - 0.1\pi) + \cos(12\pi t + 0.1\pi)$.

(a) Draw the two-sided line spectrum for $x(t)$ in the space below, taking care to label both the frequency and complex amplitude of each line:



(b) The fundamental frequency of $x(t)$ is $f_0 = \boxed{1}$ Hz.

(c) In order for $y(t) = x(t)$, the sampling frequency must satisfy $f_s > \boxed{12}$ (samples/sec).

(d) If $y(t) = \cos(2\pi t + 0.9\pi)$, the sampling frequency is $f_s = \boxed{8}$ (samples/sec).

⇒ highest freq sinusoid aliases to

$\cos\left(\frac{12\pi n}{8} + 0.1\pi\right) = \cos(0.5\pi n - 0.1\pi) \rightarrow \boxed{D/C} \rightarrow \cos(4\pi t - 0.1\pi)$ ⇒ cancels middle sinusoid

(e) If $y(t) = -\cos(4\pi t - 0.1\pi)$, the sampling frequency is $f_s = \boxed{7}$ (samples/sec).

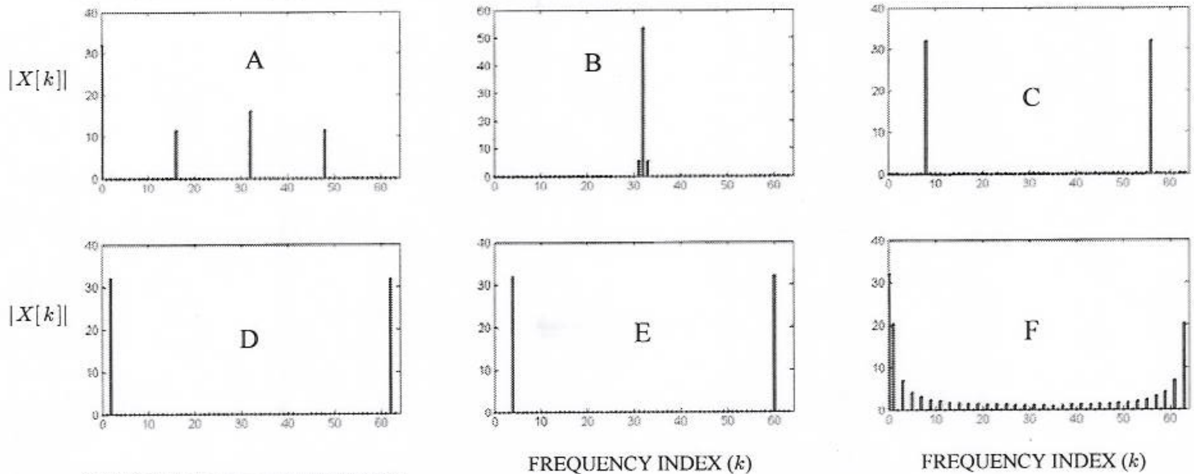
⇒ highest freq sinusoid aliases to

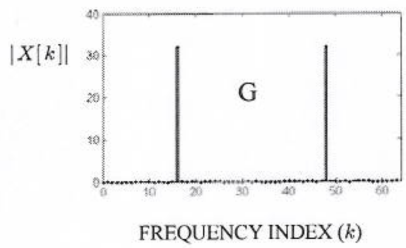
$\cos\left(\frac{12\pi n}{7} + 0.1\pi\right) = \cos\left(\frac{2\pi n}{7} - 0.1\pi\right) \rightarrow \boxed{D/C} \rightarrow \cos(2\pi t - 0.1\pi)$ ⇒ cancels lowest freq sinusoid

(f) If the sampling frequency is $f_s = 0.2$ sample/sec, the output will be $y(t) = \boxed{\cos(0.9\pi) \approx -0.951}$.

(constant) *(put in simplest possible form)*

PROB. F12-Final.3. Shown below are sketches of the 64-point DFT spectrum for eight different length-64 signal vectors, labelled A through G. (The spectrum is represented by a stem plot of the coefficient magnitude $|X[k]|$ versus the frequency index $k \in \{0,\ 1,\ \ldots\ 63\}$.)





Match one of the seven signals $\{x[n]\}$ below to its corresponding DFT spectrum by writing a letter (A through G) in each answer box:

(i) D

(ii) E

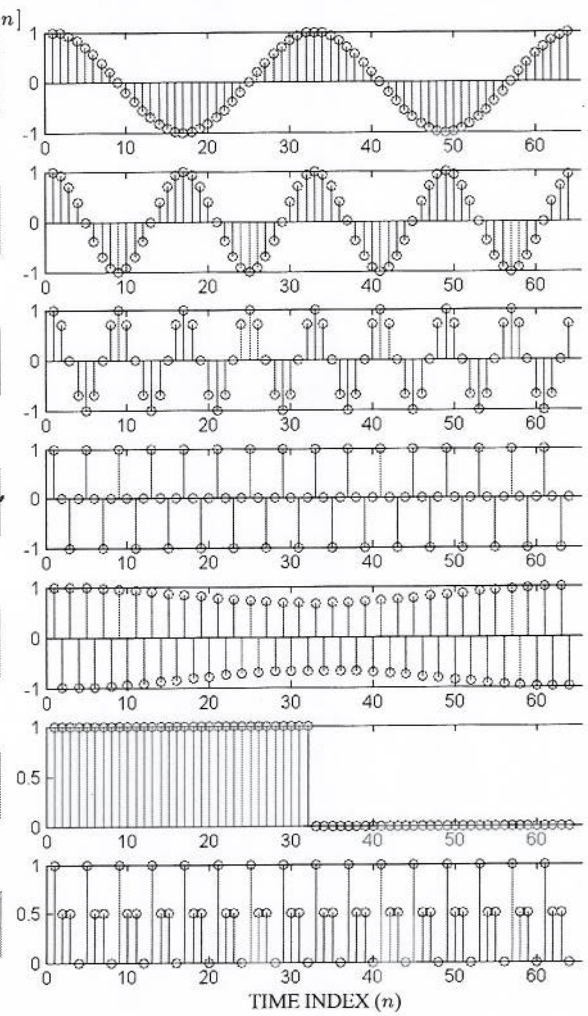
(iii) C

(iv) G

(v) B

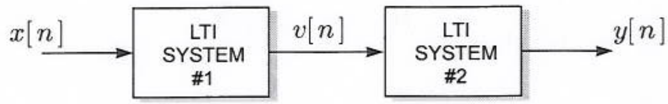
(vi) F

(vii) A



only these have a DC component

PROB. F12-Final.4. Consider the cascade connection of two LTI systems, as shown below, where the output of system#1 is the input to system#2, and the overall output is the output of system#2:



System #1 is defined by its impulse response $h_1[n] = 2\delta[n-1] - 2\delta[n-2]$, and system #2 is defined by its difference equation $y[n] = 3v[n-1] + 3v[n-2]$.

(a) If the input *to system #2* is $v[n] = 0.2$ (a constant), the output is: $y[n] = \boxed{1.2}$.

DC gain $= 3 + 3 = 6$

(b) If the input *to system #2* is $v[n] = 0.2\cos(0.25\pi n)$, the output is: $y[n] = \boxed{1.11\cos\left(\frac{\pi n}{4} - 0.375\pi\right)}$.

$H_2(e^{j\hat{\omega}}) = 3e^{-j\hat{\omega}}(1 + e^{-j\hat{\omega}})$

@ $\hat{\omega} = \frac{\pi}{4} \Rightarrow 3e^{-j\pi/4} + 3e^{-j\pi/2} = 5.54e^{-j\overset{1.18}{\cancel{0.51\pi}}}$

$= 5.54e^{-j0.375\pi}$

(c) If the input *to system #1* is $x[n] = (-1)^n$, the *overall* output is: $y[n] = \boxed{0}$.

@ $\hat{\omega} = \pi$, freq response is

$H_2(e^{j\pi}) = 3e^{-j\pi}(1 + e^{-j\pi}) = 0$

(d) If the input *to system #1* is $x[n] = \cos(0.5\pi n)$, the *overall* output is: $y[n] = \boxed{-12\cos(0.5\pi n)}$.

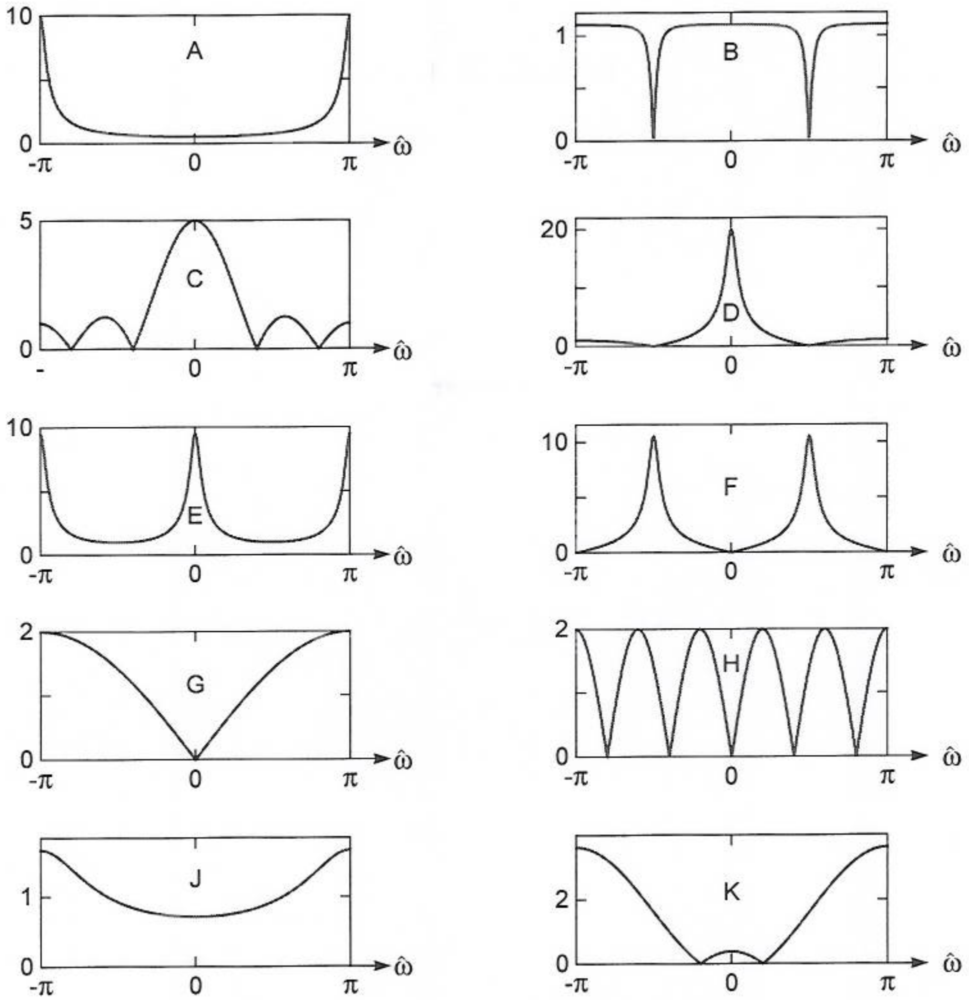
$H(z) = H_1(z)H_2(z) = 6z^{-2}(1 - z^{-2})$

$\Rightarrow$ @ $\hat{\omega} = \frac{\pi}{2}$

$H(j) = 6(-j)^2(1 - (-j)^2)$

$= -6(2) = -12$

PROB. F12-Final.5. Shown below are the magnitude responses of ten different LTI systems:



Match each magnitude response above to its corresponding system (defined by either a difference equation, a system function, or a MATLAB statement) by writing a letter from {A, B, C, ... H, J, K} in each answer box:

$\frac{1}{1+0.9z^{-1}}$ strong HPF

(i) A $y[n] = -0.9y[n-1] + x[n]$

(ii) E $H(z) = \dfrac{1}{1-0.9z^{-1}} - \dfrac{1}{1+0.9z^{-1}} = \dfrac{1.8z^{-1}}{1-.81z^{-2}}$

(iii) K $H(z) = 1 - 2\cos(\pi/5)z^{-1} + z^{-2}$
$= z^{-2}(z-e^{j\pi/5})(z-e^{-j\pi/5})$

(iv) C `yy = conv(xx, ones(1,5));`

(v) D $y[n] = 0.9y[n-1] + x[n] + x[n-2]$

$\dfrac{1+z^{-2}}{1-0.9z^{-1}}$ → zeros @ $\pm\pi/2$

weak HPF

(vi) J $H(z) = \dfrac{1}{1+0.4z^{-1}}$

(vii) F $H(z) = \dfrac{1-z^{-2}}{1+0.81z^{-2}}$ → zeros @ $0, \pm\pi$

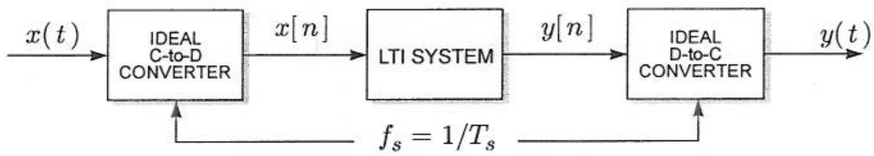
(viii) B $H(z) = \dfrac{1+z^{-2}}{1+0.81z^{-2}}$ → zeros @ $\pm\pi/2$

(ix) H $y[n] = x[n] - x[n-5]$ $1-z^{-5}$

(x) G `yy = conv(xx, [1,-1]);`

$1-z^{-1}$ → zeros @ 0

PROB. F12-Final.6. Consider the below system for discrete-time processing of a continuous-time signal:

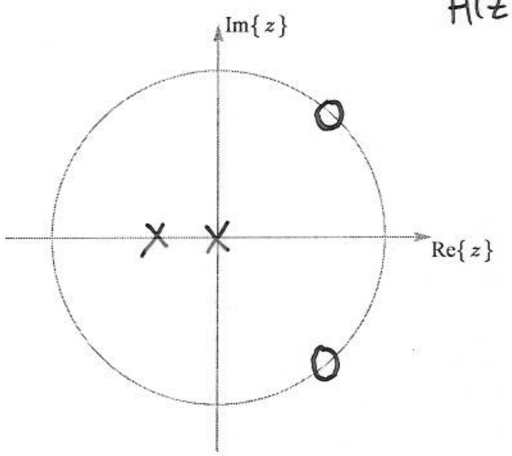


Assume that the discrete-time LTI system is defined by the difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2] - 0.3y[n-1].$$

(a) In this part, assume that $b_1 = -1.4$. Find *all* poles and *all* zeros of the discrete-time LTI system. Give your answer as a pole-zero plot.

$$H(z) = \frac{1 + b_1 z^{-1} + z^{-2}}{1 + 0.3z^{-1}} = \frac{z^2 + b_1 z + 1}{z(z+0.3)} = \frac{(z-e^{j\theta})(z-e^{-j\theta})}{z(z+0.3)}$$



where $-2\cos\theta = b_1 = -1.4$

$\Rightarrow \theta = \cos^{-1}(0.7)$

$= 0.253\pi$

$\Rightarrow$ ZEROS @ $e^{\pm j\theta}$

POLES @ $0, -0.3$

(b) The overall system can be used to null one continuous-time sinusoid. The frequency that is nulled is controlled by the value of the filter parameter b_1. If the sampling rate is $f_s = 8000$ Hz, find the value of b_1 so that the overall system nulls out a sinusoid at 60 Hz.

$$\hat{\omega} = \frac{2\pi f_0}{f_s} = \frac{2\pi(60)}{8000} = \frac{3\pi}{200} = 0.015\pi$$

$b_1 = \boxed{-1.998}$.

$$\Rightarrow b_1 = -2\cos\left(\frac{3\pi}{200}\right) = -1.998$$

(c) The range of values for b_1 for which the overall system is *not* a nulling filter is $|b_1| > \boxed{2}$.

$$b_1 = -2\cos\theta$$

PROB. F12-Final.7. An LTI system has difference equation:

$$y[n] = 1.4x[n] - 0.7y[n-1].$$

(a) This system is [FIR] [IIR] (circle one).

(b) The system function is: $H(z) = \boxed{\dfrac{1.4}{1+0.7z^{-1}}}$.

(c) An equation for the impulse response $h[n]$ that is valid for all $-\infty < n < \infty$ is:

$$h[n] = \boxed{1.4(-0.7)^n u[n]}.$$

(d) The output in response to $x[n] = \cos(\pi n + \pi/5)$ is:

@ $z = e^{j\pi}$

$$y[n] = \boxed{\frac{14}{3}\cos\left(\pi n + \frac{\pi}{5}\right)}.$$

$$H(z) = H(-1) = \frac{1.4}{1-0.7} = \frac{1.4}{0.3} = 4.6\bar{6}$$

(e) If the *output* of this system is $y[n] = \delta[n]$, then its *input* must be:

$$x[n] = \boxed{0.714\,\delta[n] + \tfrac{1}{2}\delta[n-1]}.$$

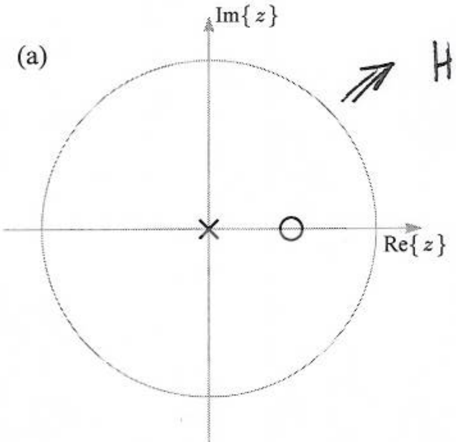
$$1 = Y(z) = X(z)H(z)$$

$$\Rightarrow X(z) = \frac{1}{H(z)} = \frac{1+0.7z^{-1}}{1.4} = 0.714 + 0.5z^{-1}$$

PROB. F12-Final.8. Shown below are the pole-zero plots for three different LTI systems. For each, write a difference equation for an LTI system that has a matching pole-zero plot. The exact locations of the poles and zeros are not labeled, so you will have to estimate. You don't need to be incredibly precise; you will get full credit if your coefficients are within 15% of correct values.

(*Hint:* The coefficients should all be real.)

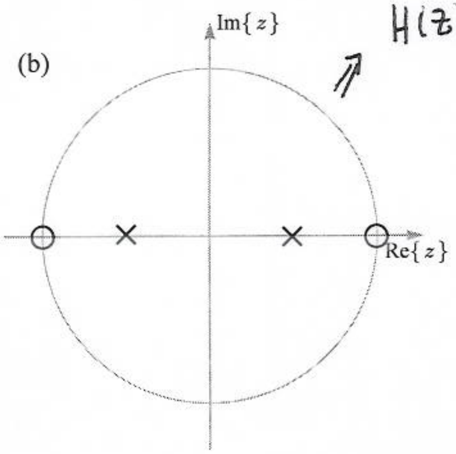
(a)



$$H(z) = \frac{z - 0.5}{z} = 1 - 0.5z^{-1}$$

$y[n] =$ $x[n] - 0.5\,x[n-1]$

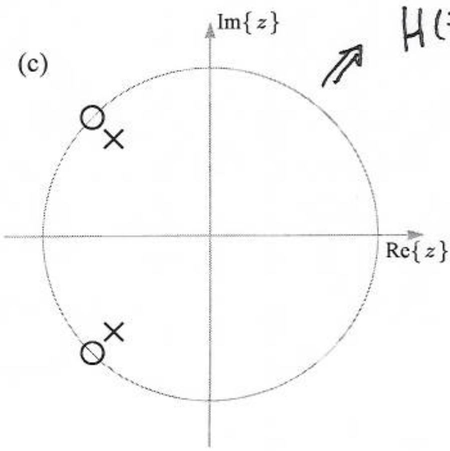
(b)



$$H(z) = \frac{(z+1)(z-1)}{(z+0.5)(z-0.5)} = \frac{z^2 - 1}{z^2 - 0.25} = \frac{1 - z^{-2}}{1 - 0.25z^{-2}}$$

$y[n] =$ $x[n] - x[n-2] + 0.25\,y[n-2]$

(c)



$$H(z) = \frac{(z - e^{j3\pi/4})(z - e^{-j3\pi/4})}{(z - 0.8e^{j3\pi/4})(z - 0.8e^{-j3\pi/4})} = \frac{1 - 2\cos(\frac{3\pi}{4})z^{-1} + z^{-2}}{1 - 1.6\cos(\frac{3\pi}{4})z^{-1} + 0.64z^{-2}}$$

$$= \frac{1 + \sqrt{2}z^{-1} + z^{-2}}{1 + 1.13z^{-1} + 0.64z^{-2}}$$

$y[n] =$ $x[n] + \sqrt{2}\,x[n-1] + x[n-2]$

$- 1.13\,y[n-1] - 0.64\,y[n-2]$