GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 6-Dec-04 COURSE: ECE-2025

NAME: ______________________________ GT # ____________________

LAST, FIRST (e.g. gtg123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning CLEARLY to receive partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	

PROBLEM fall-04-F.1:

In each of the following cases, ***simplify the expression as much as possible.***

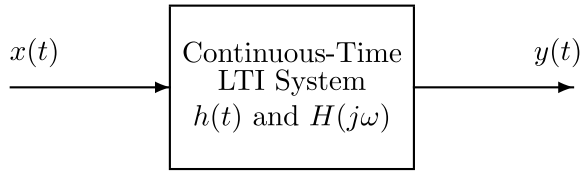
(a) $\cos(3\pi/4t - \pi/2)\delta(t-3) =$ ____________________

(b) $[(t+1)u(t+1)] * \delta(t-2) =$ ____________________

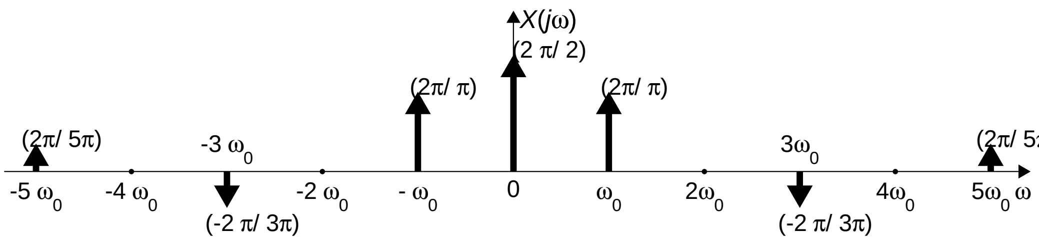
(c) $\int_0^2 e^{-10\tau}\delta(\tau-3)d\tau =$ ____________________

(d) $x[n] = 5\cos(0.3\pi n + \pi/2) + 5\sqrt{2}\cos(0.3\pi n - \pi/4) =$ ____________________

PROBLEM fall-04-F.2:



The periodic input to the above LTI system has the Fourier transform $X(j\omega)$ drawn below:



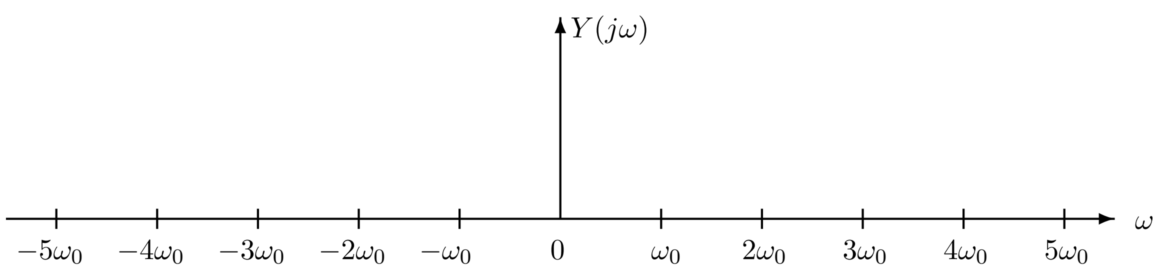
where the dark arrows denote impulses.

(a) If the frequency response of the filter is given by

$$H(j\omega) = \begin{cases} e^{-2j\omega} & 5\omega_0/2 < |\omega| < 7\omega_0/2 \\ 0 & \text{otherwise} \end{cases}$$

determine $y(t)$. Your answer should be written as a real time function, i.e., there should be no j's in your answer.

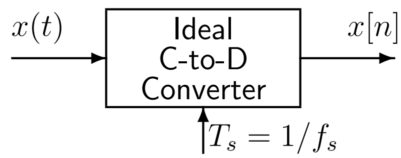
(b) If $y(t) = x(t-1)$ sketch $Y(j\omega)$, the spectrum of $y(t)$, on the axes below.



PROBLEM fall-04-F.3:

The individual parts of this problem are independent.

(a) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the sampling rate is 8000 samples/sec, then determine two possible positive values of the input frequency of $x(t)$ that are less than 8000Hz.:



ANS 1: =

ANS 2: =

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;
xx = (3/pi) * cos(pi*1.25*nn + pi/3);
soundsc(xx,fsamp)
```

Although the sinusoid was not written to have a frequency of 2100 Hz, it is possible to play out the vector `xx` so that it sounds like a 2100 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 2100 Hz tone. Write your answer as an integer.

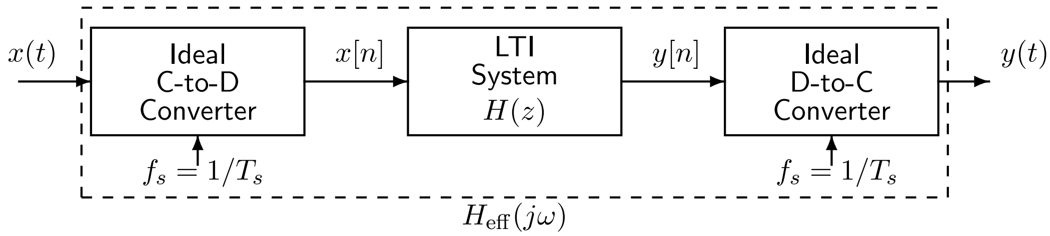
`fsamp =` Hz

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by:
$x(t) = \Re e\{e^{j4000\pi t} + e^{-j3000\pi t}\}$.

ANS = samples/sec.

PROBLEM fall-04-F.4:

Consider the following system for discrete-time filtering of a continuous-time signal:

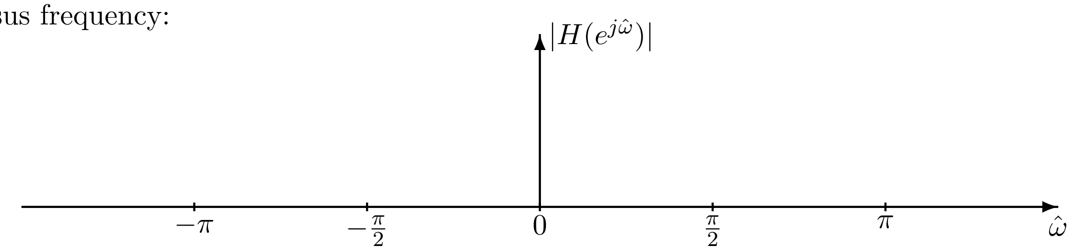


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,0,1,0,1],xx)
```

where `xx` is an array of samples of $x[n]$ and `yy` holds samples of $y[n]$.

(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



(b) Assume that the input signal $x(t)$ is a sum of cosines:

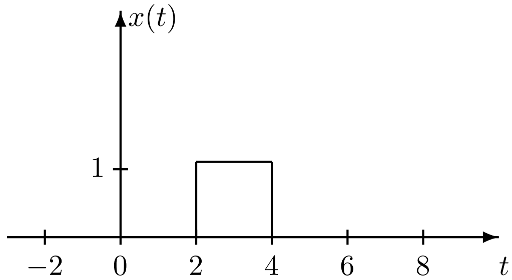
$$x(t) = 3\cos(100\pi t - \pi/4) + 2\cos(400\pi t + \pi/3)$$

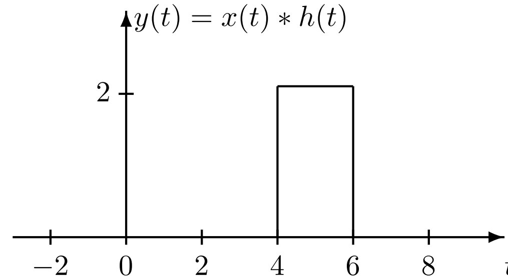
For this input signal, determine the output signal $y(t)$ when the sampling rate is $f_s =$ **300 samples/sec.** Your answer should be expressed as a sum of cosines.

PROBLEM fall-04-F.5:

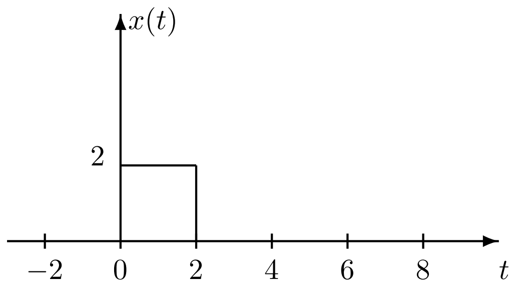
The parts of this problem are completely independent.

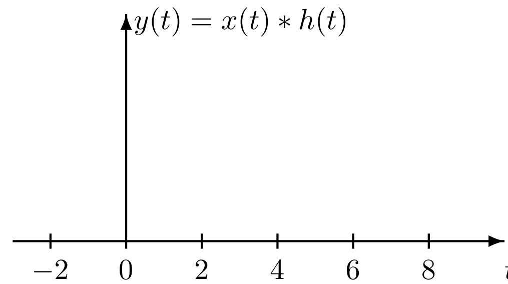
(a) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) =$



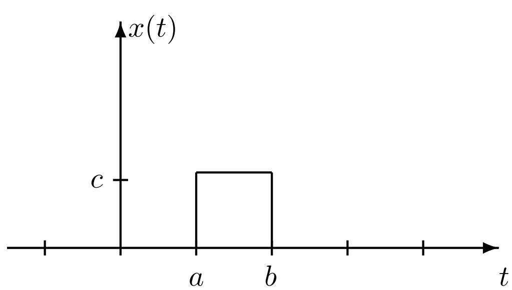


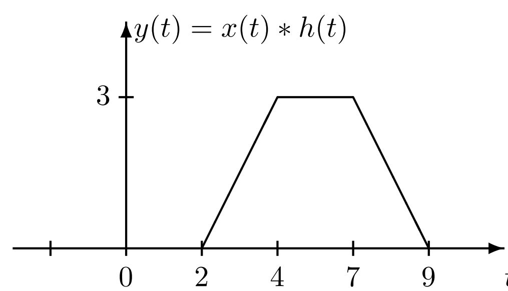
(b) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*





(c) If $h(t) = u(t-1) - u(t-3)$ and $y(t) = x(t) * h(t)$, determine the values of a, b, and c in the graph of $x(t)$ on the left, if $y(t)$ is given by the graph on the right.





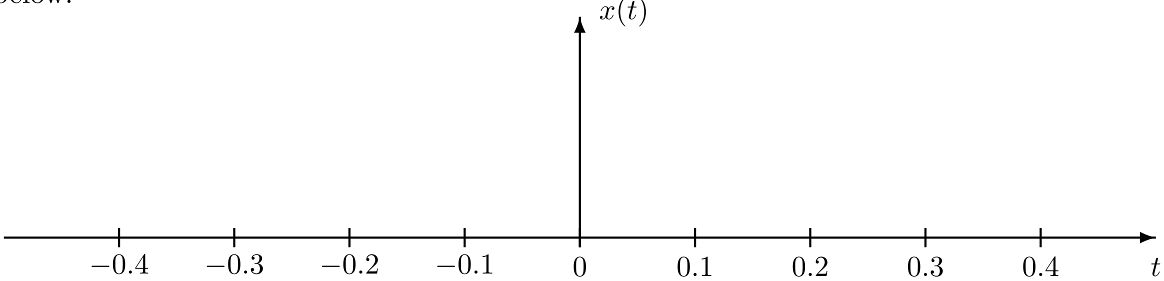
$a =$

$b =$

$c =$

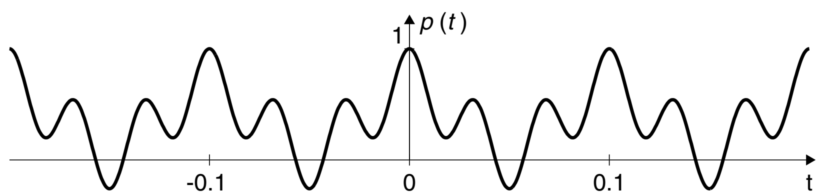
PROBLEM fall-04-F.6:

(a) Consider the signal $x(t) = \dfrac{\sin(10\pi t)}{4\pi t}$. Make a carefully labeled sketch of $x(t)$ in the space below.



(b) Determine the Fourier transform of $y(t) = x(2t - 0.2)$, using $x(t)$ from part (a).

(c) Now consider the periodic signal $p(t)$ plotted below:

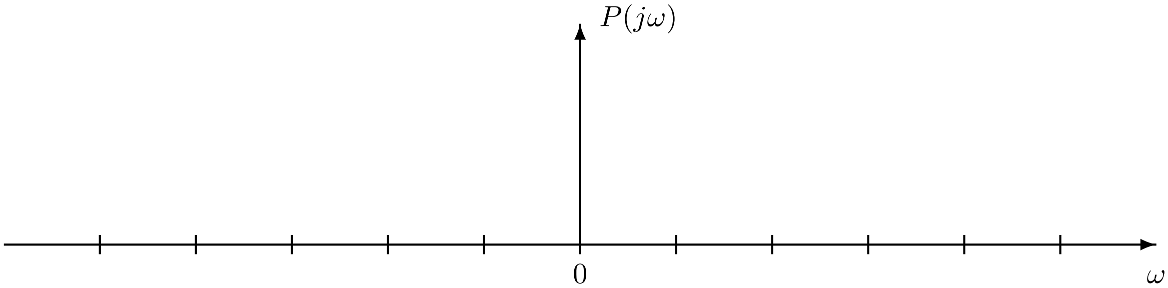


The Fourier series for this input can be simplified to the following form:

$$p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_0 t) + \frac{2}{\pi}\cos(3\omega_0 t)$$

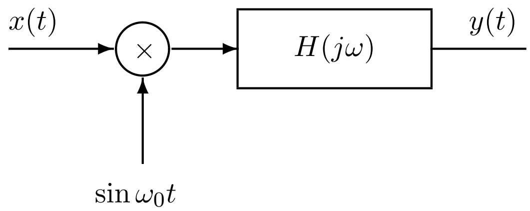
$\omega_0 =$ ______________ rad/sec

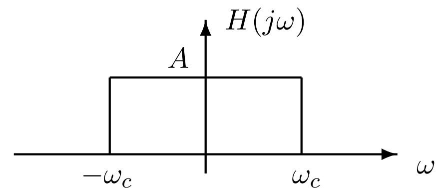
First determine the value of ω_0 and put your result in the box. Then, **either** write an equation for $P(j\omega)$, the Fourier transform of $p(t)$, in the space below, **or** plot it on the axes below. **You must label your plot carefully to receive full credit.**



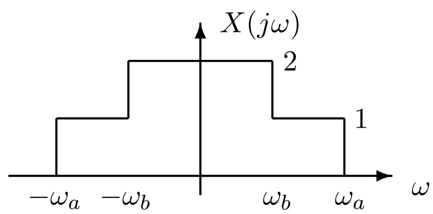
PROBLEM fall-04-F.7:
The two parts of this problem are independent.

(a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine.

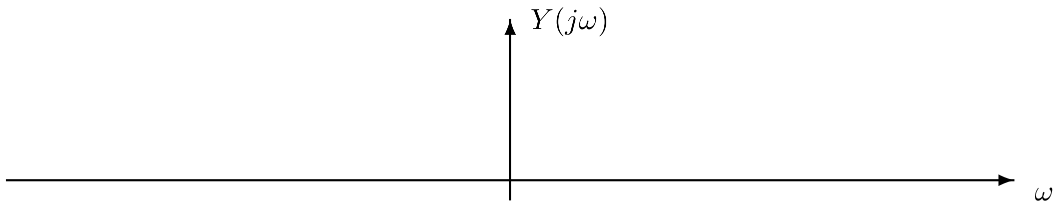




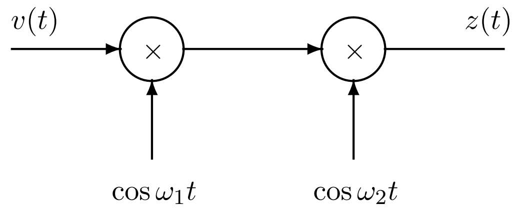
Assume that $x(t)$ has the spectrum, $X(j\omega)$ shown below



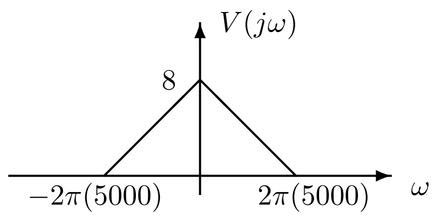
Sketch the spectrum, $Y(j\omega)$ of the output of the scrambler on the axes below if $\omega_a/(2\pi) =$ 5 kHz, $\omega_b/(2\pi) = 2$ kHz, $\omega_c/(2\pi) = 5$ kHz, and $\omega_0/(2\pi) = 5$ kHz. Be sure to LABEL YOUR PLOT.

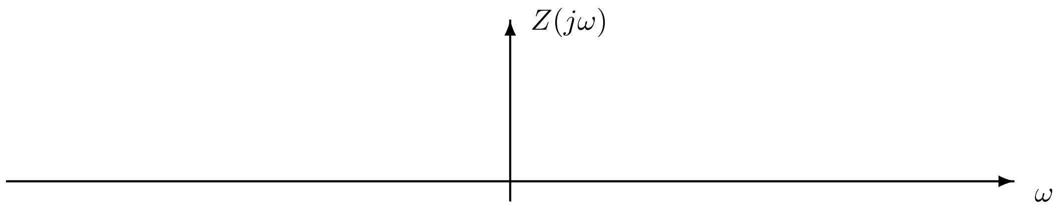


(b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.



Let $f_1 = \omega_1/(2\pi) = 30$ kHz and $f_2 = \omega_2/(2\pi) = 20$ kHz. Sketch the spectrum $Z(j\omega)$ assuming that $V(j\omega)$ has the shape shown in the figure below.





PROBLEM fall-04-F.8:

A discrete-time system is defined by the following system function:

$$H(z) = H(z) = \frac{0.64 + z^{-2}}{1 - 0.64z^{-2}}.$$

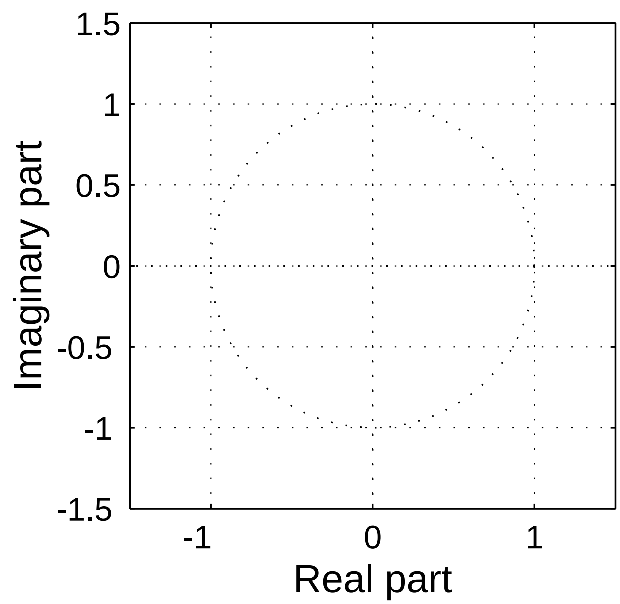
(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

(b) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[                    ];     aa=[                    ];
yy=filter(bb,aa,xx)
```

where `xx` is the input signal to be filtered.

(c) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.



(d) Make a sketch of the magnitude of the frequency response of the system over the range $-\pi < \hat{\omega} \leq \pi$. Indicate where the peaks and valleys are located, and also determine the height of the peaks and the valleys.

PROBLEM fall-04-F.1:

In each of the following cases, *simplify the expression as much as possible.*

(a) $\cos(3\pi/4t - \pi/2)\delta(t-3) =$ $\boxed{\frac{\sqrt{2}}{2}\delta(t-3)}$

$\cos(\frac{\pi}{4} - \frac{\pi}{2})\delta(t-3) = \frac{\sqrt{2}}{2}\delta(t-3)$

(b) $[(t+1)u(t+1)] * \delta(t-2) =$ $\boxed{(t-1)u(t-1)}$

(c) $\int_0^2 e^{-10\tau}\delta(\tau - 3)d\tau =$ $\boxed{\emptyset}$

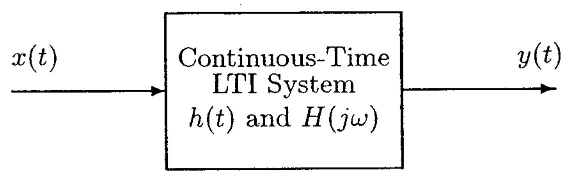
(d) $x[n] = 5\cos(0.3\pi n + \pi/2) + 5\sqrt{2}\cos(0.3\pi n - \pi/4) =$ $\boxed{5\cos(0.3\pi n)}$

$x[n] = \Re e\left[\left(5e^{j\pi/2} + 5\sqrt{2}\,e^{-j\pi/4}\right)e^{j0.3\pi n}\right]$

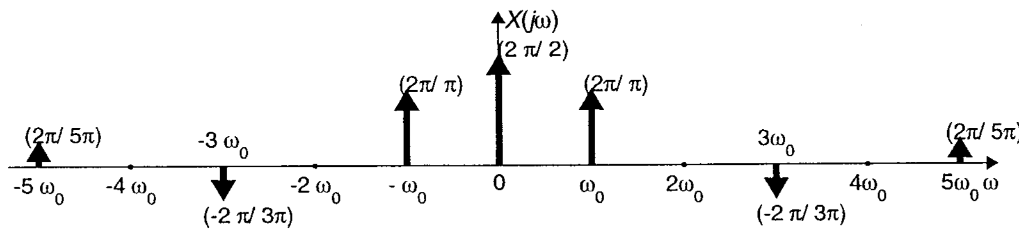
$5e^{j\pi/2} + 5\sqrt{2}\,e^{-j\pi/4} = 5j + 5\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right) = 5$

$\Rightarrow x[n] = 5\cos(0.3\pi n)$

PROBLEM fall-04-F.2:



The periodic input to the above LTI system has the Fourier transform $X(j\omega)$ drawn below:



where the dark arrows denote impulses.

(a) If the frequency response of the filter is given by

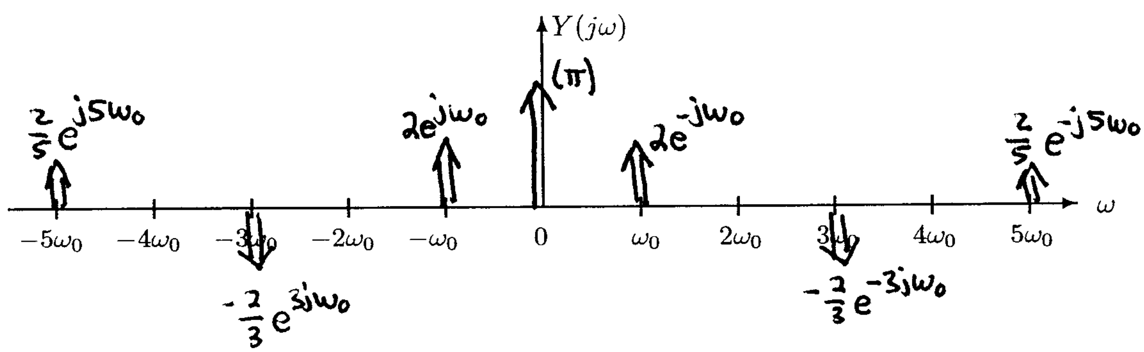
$$H(j\omega) = \begin{cases} e^{-2j\omega} & 5\omega_0/2 < |\omega| < 7\omega_0/2 \\ 0 & \text{otherwise} \end{cases}$$

determine $y(t)$. Your answer should be written as a real time function, i.e., there should be no j's in your answer.

H(jw) is a bandpass filter that only passes the cosine at frequency $3w_0$. Since the filter has a delay of two, then

$$y(t) = -\frac{2}{3\pi}\cos(3w_0(t-2))$$

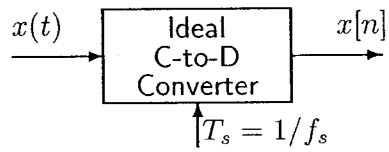
(b) If $y(t) = x(t-1)$ sketch $Y(j\omega)$, the spectrum of $y(t)$, on the axes below.



PROBLEM fall-04-F.3:

The individual parts of this problem are independent.

(a) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the sampling rate is 8000 samples/sec, then determine two possible positive values of the input frequency of $x(t)$ that are less than 8000 Hz.:



ANS 1: = 1000 Hz	ANS 2: = 7000 Hz

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;
xx = (3/pi) * cos(pi*1.25*nn + pi/3);
soundsc(xx,fsamp)
```

Although the sinusoid was not written to have a frequency of 2100 Hz, it is possible to play out the vector xx so that it sounds like a 2100 Hz tone. Determine the value of `fsamp` that should be used to play the vector xx as a 2100 Hz tone. Write your answer as an integer.

fsamp = 5600 Hz

$$\cos\left(\frac{5\pi}{4}n + \frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{4}n - \frac{\pi}{3}\right)$$

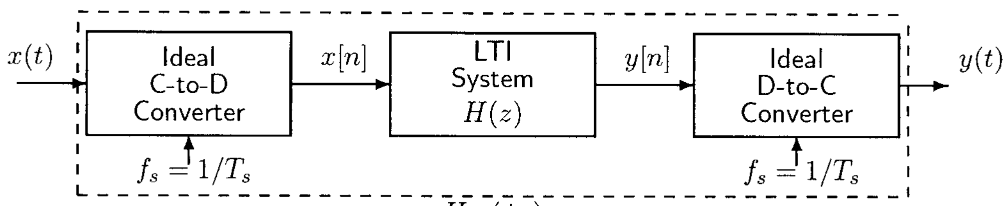
$$\frac{3\pi}{4} f_s = 2100(2\pi) \implies f_s = \frac{2100(8)}{3} = 5600$$

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j4000\pi t} + e^{-j3000\pi t}\}$.

ANS = 4000 samples/sec.

PROBLEM fall-04-F.4:

Consider the following system for discrete-time filtering of a continuous-time signal:

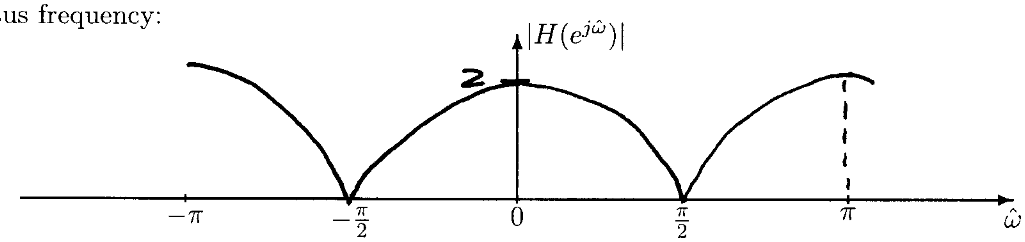


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,0,1,0,1],xx)
```

where xx is an array of samples of $x[n]$ and yy holds samples of $y[n]$.

(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}} = e^{-j3\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}})$$

$$= 2e^{-j3\hat{\omega}}\cos(\hat{\omega})$$

(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 3\cos(100\pi t - \pi/4) + 2\cos(400\pi t + \pi/3)$$

For this input signal, determine the output signal $y(t)$ when the sampling rate is $f_s =$ **300 samples/sec.** Your answer should be expressed as a sum of cosines.

$$x[n] = 3\cos\left(\frac{100\pi n}{300} - \frac{\pi}{4}\right) + 2\cos\left(\frac{400\pi n}{300} + \frac{\pi}{3}\right)$$

$$= 3\cos\left(\frac{\pi n}{3} - \frac{\pi}{4}\right) + 2\cos\left(\frac{4\pi}{3}n + \frac{\pi}{3}\right)$$

$$= 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{4}\right) + 2\cos\left(\frac{2\pi}{3}n - \frac{\pi}{3}\right)$$

$$\Rightarrow y[n] = 3\left(2\cos\left(\frac{\pi}{3}\right)\right)\cos\left(\frac{\pi}{3}n - \pi - \frac{\pi}{4}\right) + 2\left(2\cos\left(\frac{2\pi}{3}\right)\right)\cos\left(\frac{2\pi}{3}n - 2\pi - \frac{\pi}{3}\right)$$

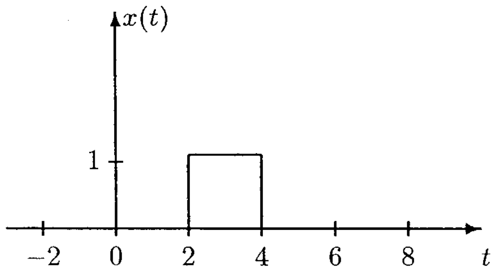
$$= 3\cos\left(\frac{\pi}{3}n - \frac{5\pi}{4}\right) - 2\cos\left(\frac{2\pi}{3}n - \frac{\pi}{3}\right)$$

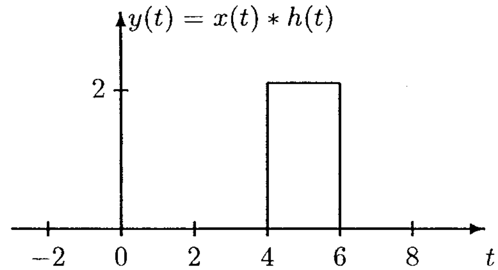
$$\boxed{y(t) = 3\cos\left(100\pi t + \frac{3\pi}{4}\right) - 2\cos\left(200\pi t - \frac{\pi}{3}\right)}$$

PROBLEM fall-04-F.5:

The parts of this problem are completely independent.

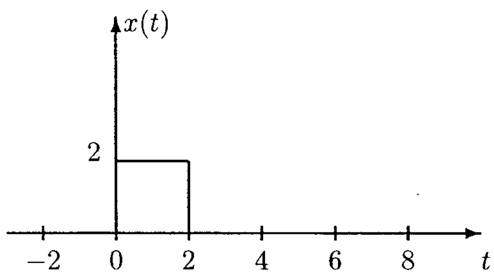
(a) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) =$ $2\,\delta(t-2)$

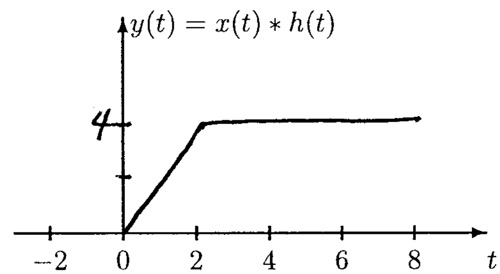




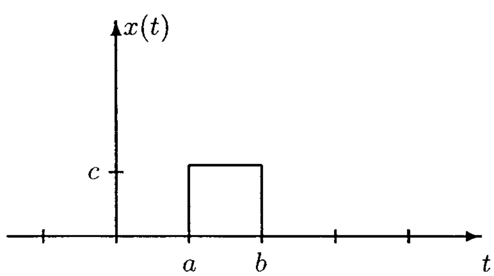
delayed by two
scaled by two

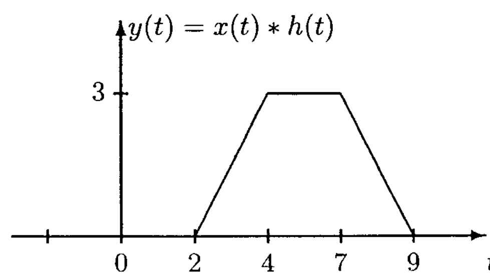
(b) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*





(c) If $h(t) = u(t-1) - u(t-3)$ and $y(t) = x(t) * h(t)$, determine the values of a, b, and c in the graph of $x(t)$ on the left, if $y(t)$ is given by the graph on the right.





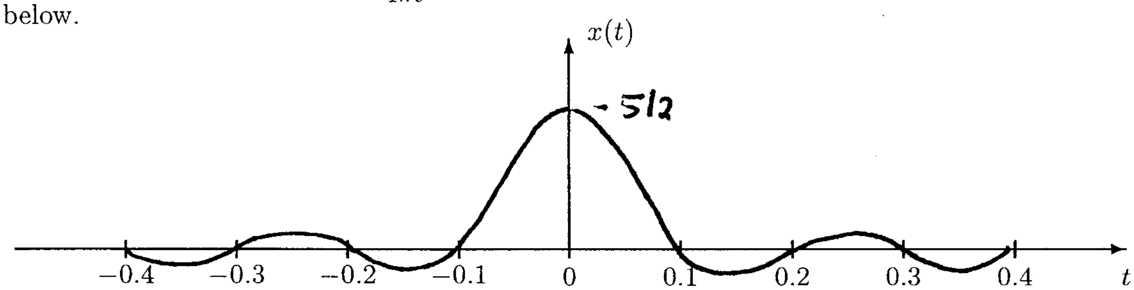
$a =$ 1

$b =$ 6

$c =$ 3/2

PROBLEM fall-04-F.6:

(a) Consider the signal $x(t) = \dfrac{\sin(10\pi t)}{4\pi t}$. Make a carefully labeled sketch of $x(t)$ in the space below.

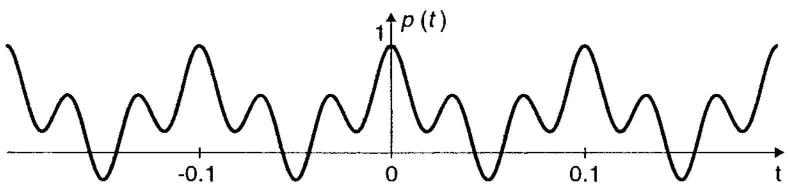


(b) Determine the Fourier transform of $y(t) = x(2t - 0.2)$, using $x(t)$ from part (a).

$$y(t) = x(2t-0.2) = \frac{\sin(10\pi(2t-0.2))}{4\pi(2t-0.2)} = \frac{\sin(20\pi(t-0.1))}{8\pi(t-0.1)}$$

$$\Rightarrow \quad Y(j\omega) = \frac{1}{8}\left[u(\omega+20\pi) - u(\omega-20\pi)\right]e^{-j0.1\omega}$$

(c) Now consider the periodic signal $p(t)$ plotted below:



$T_0 = 0.1$

$f_0 = 10$

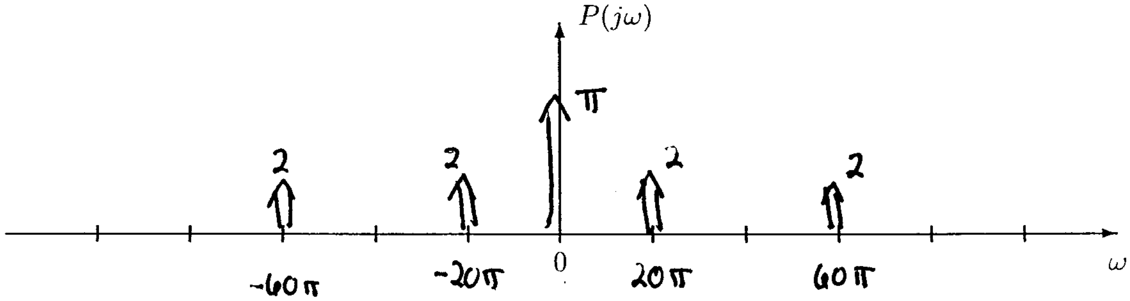
The Fourier series for this input can be simplified to the following form:

$$p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_0 t) + \frac{2}{\pi}\cos(3\omega_0 t)$$

$\omega_0 =$ 20π rad/sec

First determine the value of ω_0 and put your result in the box. Then, **either** write an equation for $P(j\omega)$, the Fourier transform of $p(t)$, in the space below, **or** plot it on the axes below. **You must label your plot carefully to receive full credit.**

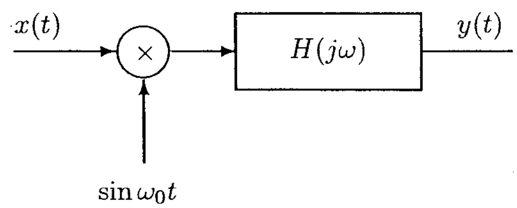
$$P(j\omega) = \pi + 2\delta(\omega-20\pi) + 2\delta(\omega+20\pi) + 2\delta(\omega-60\pi) + 2\delta(\omega+60\pi)$$

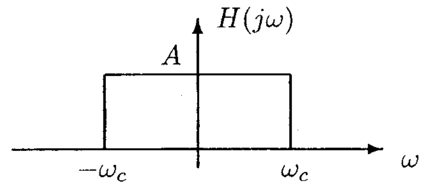


PROBLEM fall-04-F.7:

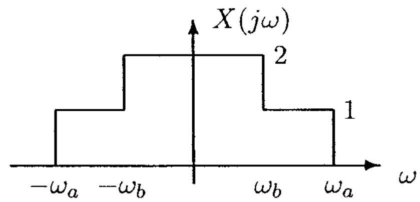
The two parts of this problem are independent.

(a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine..

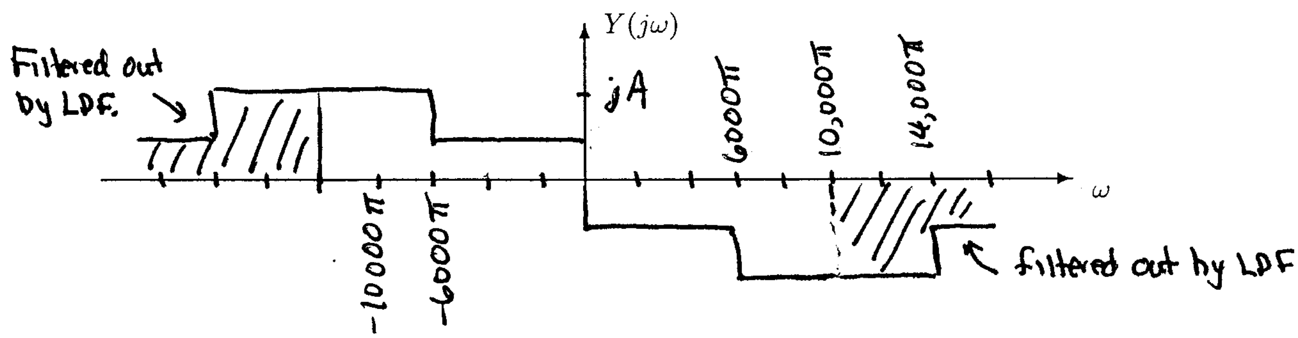




Assume that $x(t)$ has the spectrum, $X(j\omega)$ shown below

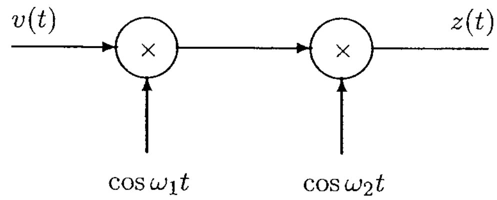


Sketch the spectrum, $Y(j\omega)$ of the output of the scrambler on the axes below if $\omega_a/(2\pi) =$ 5 kHz, $\omega_b/(2\pi) = 2$ kHz, $\omega_c/(2\pi) = 5$ kHz, and $\omega_0/(2\pi) = 5$ kHz. Be sure to LABEL YOUR PLOT.

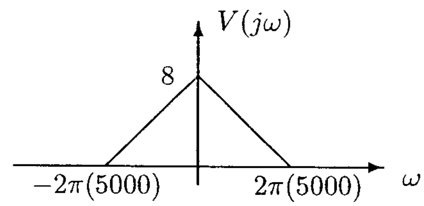


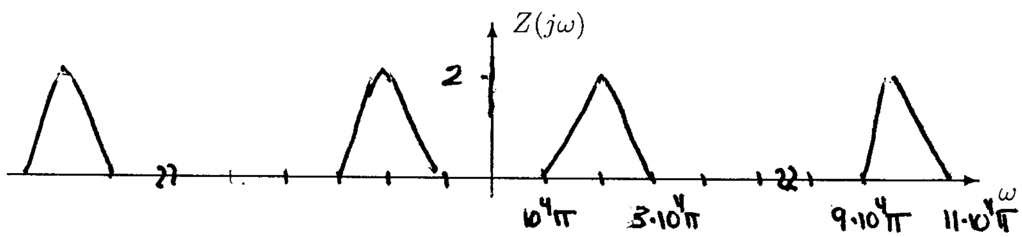
$\sin \omega_0 t \longrightarrow -j\pi\,\delta(\omega-\omega_0) + j\pi(\omega+\omega_0) \;:\; \omega_0 = 10\pi$

(b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.



Let $f_1 = \omega_1/(2\pi) = 30$ kHz and $f_2 = \omega_2/(2\pi) = 20$ kHz. Sketch the spectrum $Z(j\omega)$ assuming that $V(j\omega)$ has the shape shown in the figure below.





$$z(t) = v(t) \cdot \cos(\omega_1 t) \cdot \cos(\omega_2 t)$$

$$= \frac{1}{2} v(t) \left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t \right]$$

$$= \frac{1}{2} v(t) \left[\cos(20000\pi t) + \cos(100{,}000\pi t) \right]$$

PROBLEM fall-04-F.8:

A discrete-time system is defined by the following system function:

$$H(z) = H(z) = \frac{0.64 + z^{-2}}{1 - 0.64z^{-2}}.$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

$$y[n] = 0.64y[n-2] + 0.64\,x[n] + x[n-2]$$

(b) Fill in numbers for the vectors bb and aa in the following MATLAB computation of the frequency response of the system:

```
bb=[ 0.64, 0, 1 ];      aa=[ 1, 0, -0.64 ];
yy=filter(bb,aa,xx)
```

where xx is the input signal to be filtered.

(c) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.

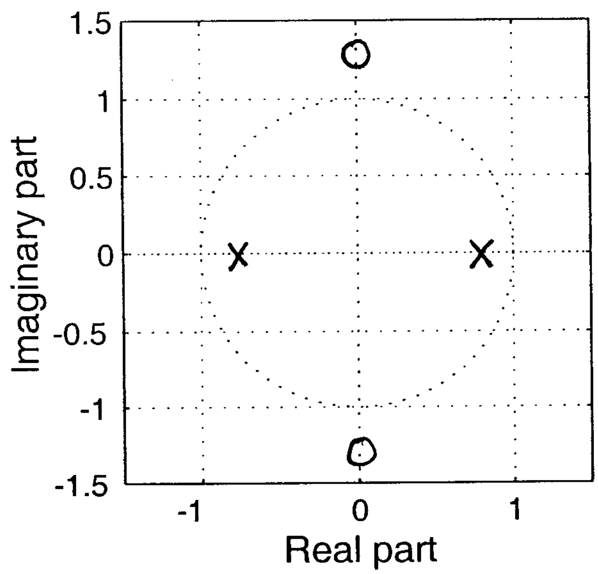
zeros: $z^{-2} = -0.64$

$z = \pm j\frac{1}{0.8} = \pm j1.25$

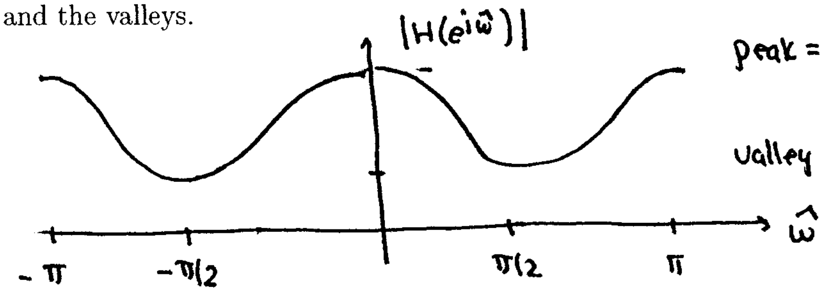
poles: $0.64z^{-2} = 1$

$z^2 = 0.64$

$z = \pm 0.8$



(d) Make a sketch of the magnitude of the frequency response of the system over the range $-\pi < \hat{\omega} \leq \pi$. Indicate where the peaks and valleys are located, and also determine the height of the peaks and the valleys.



peak $= \frac{1+0.64}{1-0.64} = 4.55$

valley $= \frac{1-0.64}{1+0.64} = 0.219$