Generalized Partial Response Targets for Perpendicular Recording with Jitter Noise

Piya Kovintavewat, Inci Ozgunes, Erozan Kurtas, John R. Barry, and Steven W. McLaughlin

Abstract— **In this paper, we propose new generalized partial response (GPR) targets for perpendicular recording whose transition response is modeled as an error function [1], and compare their performance with the partial response (PR) targets both in the presence and in the absence of jitter noise. Regardless of any jitter noise amount, results indicate that the GPR targets outperform the PR targets, especially at high linear recording densities. We also determine that the dominant error sequence for this perpendicular recording is the same for all targets when jitter noise is low. The system performance can therefore be further improved by designing and using codes to avoid this dominant error sequence. Another significant point is the fact that the dominant error sequence of perpendicular recording is different from longitudinal recording, thus requiring design of different types of codes than the ones used for longitudinal recording. Finally, we show that the effective signal-to-noise ratio (SNR) can be equivalently used instead of the bit-error-rate (BER) as a measure to compare the performance of different targets.**

Keywords— **Error events, generalized partial response targets, jitter noise, perpendicular recording.**

I. Introduction

RESEARCH on perpendicular recording has been interesting due to the potential for increase in storage capacity as compared to longitudinal recording. Unlike a longitudinal recording channel, a perpendicular recording channel contains significant information at low frequencies including d.c.. Even though the same detection process used in longitudinal recording, which is a combination of a partial response equalizer and the Viterbi detector (VD), can still be used for perpendicular recording, the partial response targets must be specifically designed for the perpendicular channel for optimal performance.

The PR target of the form $(1+D)^n$, where D is the delay operator and n is integer, is suitable for perpendicular channel, however, not optimal. We show that a generalized partial response (GPR) target with arbitrary coefficients yields a better performance than a full d.c. response PR target with integer coefficients [1], even in the presence of significant jitter noise. The GPR target and its corresponding equalizer are designed to minimize the mean-squared error (MSE) between the equalizer output and the desired output, subject to the monic constraint [2].

In addition to designing new GPR targets and comparing their performance with the PR targets, in the presence and in the absence of jitter noise, we investigate the nature of the error events and determine the dominant error sequence for this perpendicular channel. We also validate the use of effective SNR instead of BER as a convenient measure of performance, considering that computation of BER takes considerable amount of simulation time.

This paper is organized as follows. After describing the system model in Section II, Section III briefly describes how to design the GPR target. The concept of the effective SNR is described in Section IV. Simulation results for the system with and without jitter noise are presented in Section V. Conclusions are summarized in Section VI.

II. System Model

Fig. 1 shows the system model for perpendicular recording. A binary input sequence $a_k \in \{\pm 1\}$ with bit period T

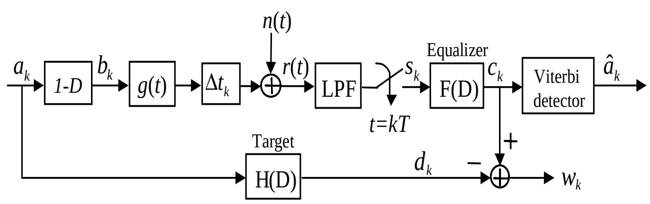

Fig. 1. System model with target design.

is filtered by ideal differentiator $1-D$ to form a transition sequence $b_k \in \{-2, 0, 2\}$ where $b_k = \pm 2$ corresponds to a positive or negative transition and $b_k = 0$ corresponds to the absence of a transition. The sequence b_k then passes through the channel represented by the transition response

$$g(t) = erf(t\sqrt{\ln 16}/PW_{50}), \tag{1}$$

where $erf(\cdot)$ is an error function defined as $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz$, and PW_{50} is the width of the derivative of $g(t)$ at half its maximum. We define a normalized recording density as ND $= PW_{50}/T$. The jitter noise Δt_k is modeled as a random shift in the transition position whose probability distribution function is truncated Gaussian with zero mean and variance σ_j^2, where σ_j is specified as a percentage of T. Thus, when we specify that jitter equals $x\%$, it means σ_j is $x\%$ of T. We model this jitter noise as a truncated Gaussian pulse such that $|\Delta t_k|$ will not exceed half the bit period T.

The readback signal, $r(t)$, can be written as

$$r(t) = \sum_{k=-\infty}^{\infty} b_k g(t - kT + \Delta t_k) + n(t), \tag{2}$$

where $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. The readback signal $r(t)$ is filtered by a 7-th order Butterworth low-pass

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filter and then sampled at a symbol rate, assuming perfect timing. The received sequence, s_k, is equalized such that the output sequence, c_k, resembles the desired sequence, d_k. Eventually, the VD performs sequence detection to determine the most likely input sequence.

III. GPR Target Design

In this paper, we are interested in designing the GPR target based on the minimum mean-squared error (MMSE) approach [2], subject to the monic constraint because it yields the best performance among other constraints [2].

Let $\mathsf{H} = [h_0\ h_1 \cdots h_{L-1}]^T$ represent the GPR target and let $\mathsf{F} = [f_{-K} \cdots f_0 \cdots f_K]^T$ represent the equalizer, where h_k and f_k denote the filter coefficients of $H(D)$ and $F(D)$, respectively, and $[\cdot]^T$ represents the transpose operation (refer to Fig. 1). Let w_k be the difference between the output of the equalizer, c_k, and the desired output of the equalizer, d_k. Given sequences s_k and a_k, the equalizer and the target are designed such that $E\{w_k^2\}$ is minimized in the minimum mean squared sense using (3)

$$E\{w_k^2\} = E\{[(s_k * f_k) - (a_k * h_k)]^2\} \tag{3}$$

where, $*$ denotes the convolution operator, and $E\{\cdot\}$ is the expectation operator.

In this paper, $K = 10$ is employed in the GPR design with an assumption that the center tap is at $k = 0$. During the minimization process, we use the monic constraint $h_0 = 1$ to avoid reaching the trivial solutions of $\mathsf{H} = \mathsf{F} = \mathsf{0}$.

By minimizing (3) subject to the monic constraint, one obtains [2]

$$\lambda = \frac{1}{\mathsf{I}^T(\mathsf{A} - \mathsf{M}^T\mathsf{R}^{-1}\mathsf{M})^{-1}\mathsf{I}} \tag{4}$$

$$\mathsf{H} = \lambda(\mathsf{A} - \mathsf{M}^T\mathsf{R}^{-1}\mathsf{M})^{-1}\mathsf{I} \tag{5}$$

$$\mathsf{F} = \mathsf{R}^{-1}\mathsf{M}\,\mathsf{H}, \tag{6}$$

where λ is the Lagrange multiplier, I is an L-element column vector whose first element is one and the rest is zero, A is an L-by-L autocorrelation matrix of a sequence a_k, M is an N-by-L cross-correlation matrix of sequences s_k and a_k where N is the number of equalizer coefficients ($N = 2K+1$), and R is an N-by-N autocorrelation matrix of a sequence s_k.

IV. Effective SNR

When comparing the performance of different targets, BER is the ultimate indicator of performance. However, determining BER, especially when BER is less than 10^{-6}, requires a considerable amount of computation time. Instead, the effective signal-to-noise ratio ($\mathrm{SNR_{eff}}$) can be considered as a criterion to determine which target is the best, because it correlates well with the BER and it can be computed much faster than BER. To compute $\mathrm{SNR_{eff}}$ we need to determine the dominant error event as well as the autocorrelation matrix of w_k, R_{ww}. This can be accomplished by using only one data sector, as opposed to several data sectors required for computation of BER.

In this paper, $\mathrm{SNR_{eff}}$ is defined as [3]

$$\mathrm{SNR_{eff}} = \frac{(\varepsilon^{\mathsf{T}}\varepsilon)^2}{\varepsilon^{\mathsf{T}}\mathsf{R}_{\mathrm{ww}}\varepsilon} = \frac{d^2_{effmin}}{\sigma_w^2} \tag{7}$$

where ε is a column vector of the dominant error event. For example, if the dominant error event is such that $\varepsilon(D) = 1 - 2D + 3D^2$, then $\varepsilon = [1, -2, 3]^T$. Let the error sequence $\varepsilon_a(D) = a_1(D) - a_2(D)$, where $a_1(D)$ and $a_2(D)$ are two input sequences of the same length. The error event is then defined as $\varepsilon(D) = \varepsilon_a(D)H(D)$. The performance of the VD is largely determined by the error sequence $\varepsilon_a(D)$ that results in the error event $\varepsilon(D)$ having the smallest effective distance, d_{effmin}, rather than the Euclidean distance [3]. The error event $\varepsilon(D)$ and error sequence $\varepsilon_a(D)$ having the smallest effective distance is referred to as the dominant error event and dominant error sequence, respectively.

V. Simulation Results

In simulations, we refer to the input signal-to-noise ratio as "electronics SNR" or, simply, SNR and define it as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{{V_p}^2}{\sigma^2}\right) \quad \text{(dB)}. \tag{8}$$

where, $V_p = g(\infty) = 1$ is the peak amplitude of the isolated transition and $\sigma^2 = N_0/(2T)$ is input AWGN power.

Each BER point was computed using as many 4096 bit data sectors as needed to collect 500 error bits, while each $\mathrm{SNR_{eff}}$ point was computed using only one data sector. For convenience, we denote the "GPRn" target as the n-tap GPR target with the monic constraint. For each ND, the SNR used to design the target and its corresponding equalizer was chosen to minimize the SNR required to achieve the desired BER.

Fig. 2 (a) compares the performance of different targets as a function of ND in the absence of jitter noise. Apparently, GPR targets can outperform PR targets, especially at higher recording densities. This is because the GPR target provides a better match to the channel response than the PR targets. In Fig. 2 (b), we pick ND=2.5, and this

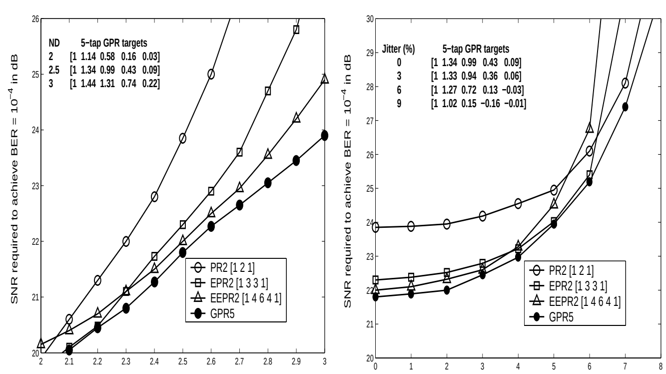

Fig. 2. (a) Required electronics SNR vs ND without jitter noise, and (b) required electronics SNR vs jitter noise amount at ND = 2.5.

time compare the performance of different targets as a function of the jitter noise amount. Again, it is clear that the

GPR target requires a lower SNR to achieve BER $= 10^{-4}$ than PR targets for all jitter noise amounts.

We would like to point out that, even though the PR2 target requires a lower SNR than longer PR targets when jitter noise amount is large (which might be because the PR target with a fewer number of coefficients is less sensitive to the jitter noise than that with a larger number of coefficients), this is not the case for the GPR targets because they still provide a good performance as the target length increases.

We also investigate the error events for the perpendicular channel. Table I shows the error sequences and their relative frequency of occurrence for the system operating at ND $= 2.5$ and BER $= 10^{-4}$, where "J" denotes σ_j/T. Note that "+" represents "2" and "-" denotes "-2", and all error sequences have a corresponding symmetrical sequence, i.e., $\varepsilon_a(D) = -\varepsilon_a(D)$.

TABLE I
ERROR SEQUENCE FOR DIFFERENT TARGETS AND σ_j/T

Error Sequences	PR2 J=0%	GPR5 J=0%	GPR5 J=3%	GPR5 J=6%	GPR5 J=9%
+	4.90%	3.19%	3.36%	5.84%	41.53%
+-	67.54%	83.25%	79.66%	35.21%	9.62%
+-+	5.79%	0.35%	1.98%	38.31%	21.53%
+-+-	0.51%	0.58%	1.14%	6.66%	15.73%
+-+-+	0.13%	0.23%	0.48%	2.66%	6.31%
+-0+-	15.53%	8.75%	8.94%	3.03%	0.00%
+-0+-0+-	1.34%	0.73%	0.84%	0.22%	0.00%
Others	4.26%	2.72%	3.60%	8.06%	5.28%

For low jitter cases (0 to 3%), the dominant error sequence for longitudinal recording was shown to be {2, -2, 2} [2], while we found that the dominant error sequence for perpendicular recording, for all targets, is {2, -2}, which corresponds to two consecutive transitions being shifted by one bit period. Additionally, the number of dominant error sequences tends to increase as the jitter noise amount increases. Performance can be further improved by designing and utilizing codes that avoid all dominant error sequences [4]. Another significant point is that due to the different nature of error events, post-processors that work well with longitudinal recording might not work as well with perpendicular recording.

Next, we illustrate the fact that BER and $\mathrm{SNR_{eff}}$ correlate well, especially when the jitter noise is low (this might not be true as jitter noise increases because there are more than one dominant error sequences when jitter noise is high and our $\mathrm{SNR_{eff}}$ does not take this into account). The BER and $\mathrm{SNR_{eff}}$ performances of the GPR5 target are compared in Fig. 3 (a), (b) (at ND $= 2.5$). Clearly, the $\mathrm{SNR_{eff}}$ performance coincides with the BER performance. Fig. 3 (a) also show that, at low jitter noise, $\mathrm{SNR_{eff}}$ can be used to estimate BER using BER $\approx CQ(\frac{1}{2}\sqrt{\mathrm{SNR_{eff}}})$ [2], where C is a constant independent of σ_w^2. For instance, at $\sigma_j/T = 0\%$, the estimated BER labeled as "Q($\cdot$)" is in agreement with the actual BER obtained from simulation when $C = 2.3$.

In Fig. 4 (a), the BER versus $\mathrm{SNR_{eff}}$ plots illustrate that regardless of which target it corresponds to, if $\mathrm{SNR_{eff}}$ is the same, the BER will be approximately the same, especially at low jitter cases. As a result, $\mathrm{SNR_{eff}}$ can be used instead of BER as a criterion to compare different targets for a

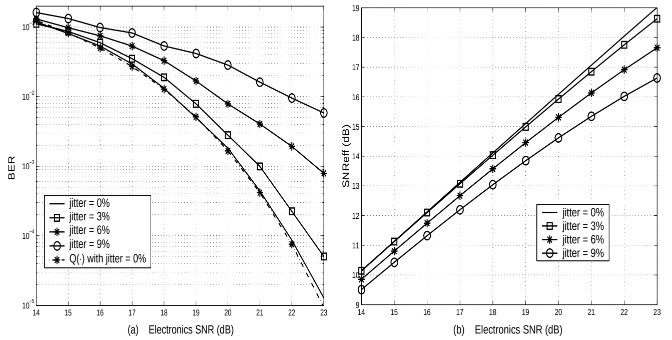

Fig. 3. (a) BER and (b) $\mathrm{SNR_{eff}}$ performances of the GPR5 target.

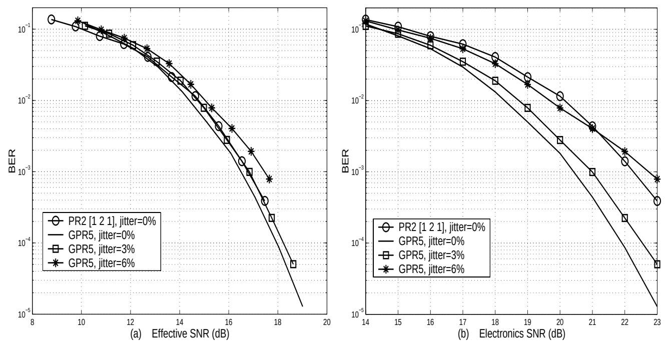

Fig. 4. (a) BER vs. Electronics SNR (dB), and (b) BER vs. $\mathrm{SNR_{eff}}$ of different targets with various jitter amounts at ND $= 2.5$.

given input SNR. However, keep in mind that to achieve the same BER or $\mathrm{SNR_{eff}}$, different targets may require different amounts of input SNR as illustrated in Fig. 4 (b).

VI. Conclusion

The new transition response for perpendicular recording is modeled as an error function [1]. Irrespective of any jitter noise level, the GPR target yields a better performance than the PR target, especially at high recording densities. Apparently, the GPR target is primarily a function of ND, SNR, and the jitter noise amount. One needs to carefully design the GPR target for each situation in order to obtain a good performance. We observed that when jitter noise is low, the dominant error sequence was shown to be the same for all targets and different from longitudinal recording. Designing and using codes that avoid this error sequence will further improve the system performance. We also showed that the $\mathrm{SNR_{eff}}$ can be equivalently used instead of the BER to measure the performance of different targets for a Maximum Likelihood system with VD.

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