

# A Single-Symbol-Decodable Space-Time Block Code with Full Rate and Low Peak-to-Average Power Ratio

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**Abstract**—Three desirable properties of a four-antenna space-time block code are full rate, full diversity, and single-symbol decodability. Previously reported space-time codes that achieve all three properties do so at the expense of the peak-to-average power ratio (PAPR). A fourth desirable property of a space-time block code is that its PAPR be the same as that of the underlying quadrature-amplitude modulation alphabet. In this letter we introduce space-time codes for three and four transmit antennas that achieve all four properties; these codes use a diversity technique based on constellation stretching. Numerical results for quasistatic Rayleigh-fading channels show that, despite their low PAPR, the proposed codes are comparable in SNR performance to the best-performing single-symbol decodable space-time codes for three and four transmit antennas.

**Index Terms**—Quasiorthogonal codes, transmit diversity.

## I. INTRODUCTION

THE orthogonal space-time block codes of Alamouti [1] and Tarokh *et al.* [2] attain full diversity with low decoding complexity, but they suffer a rate loss when there are more than two transmit antennas. A quasiorthogonal code relaxes the orthogonality constraint to enable full rate, at the expense of an increase in decoding complexity. For example, quasiorthogonal codes for four antennas were proposed independently by Jafarkhani [3], Tirkkonen-Boarin-Hottinen [4] and Papadias-Foschini [5]; these full-rate codes have two drawbacks: they are not full diversity, and they require pair-wise complex symbol decoding. The first drawback can be eliminated by constellation rotation. For example, full-rate and full-diversity quasiorthogonal codes with rotation were proposed by Tirkkonen [6], Sharma-Papadias [7] and Su-Xia [8]. While these quasiorthogonal codes outperform orthogonal codes at all spectral efficiencies for four transmit antennas, they still require pair-wise complex symbol decoding.

Recent work has shown that low decoding complexity is possible even with nonorthogonal codes. For example, a combination of constellation rotation and coordinate interleaving was proposed by Yuen *et al.* [9] and Khan-Rajan [10] to achieve full rate and full diversity for four transmit antennas with maximum-likelihood (ML) decoding that requires only pair-wise decoding of real symbols. Since a pair of real symbols defines a single complex symbol, these codes are said to be *single-symbol decodable* (SSD). It was also shown

in [10] that full rate and full diversity single-symbol decodable codes exist for only four antennas or less. Since the codes in [9] and [10] are full rate and single-symbol decodable, their worst-case ML decoding complexity for  $M$ -ary QAM is  $4M$  metric computations, the lowest among all previously reported full rate and full diversity space-time codes for four antennas.

Another important property of a space-time block code is its peak-to-average power ratio (PAPR). The PAPR is important for several reasons. First, a high-PAPR signal is susceptible to clipping and other nonlinear distortion by the power amplifier, leading to both detection errors and out-of-band interference. Second, power consumption of the power amplifier depends mainly on the peak power rather than average power, and hence, high PAPR results in high power consumption. Finally, the transmission of a high-PAPR signal requires a power amplifier with a large back off, making it inefficient, bulky and expensive [11].

In this letter, we present single-symbol-decodable space-time block codes based on the coordinate interleaving technique of [9][10] but coupled with constellation stretching rather than constellation rotation. The proposed code is not only single-symbol decodable, but it also maintains the same PAPR as the underlying QAM alphabet. While this letter was under review, we discovered that the concept of constellation stretching was proposed previously and independently by Marsch *et al.* in [12], albeit with a different motivation and construction. Nevertheless, our letter is unique in at least two respects: we derive the optimal stretching factor that maximizes the coding gain, and we quantify the PAPR benefits of the proposed code, which is the main motivation behind our construction.

The remainder of the letter is organized as follows. Section II presents the system model, relevant definitions, and a brief overview of existing single-symbol-decodable space-time block codes. Section III describes the proposed code along with its decoder, and compares it to existing codes. Section IV presents numerical results and Section V summarizes the conclusions.

## II. SYSTEM MODEL AND PRIOR SSD CODES

### A. System Model and Definitions

We consider a transmitter with  $N$  antennas transmitting  $K$  complex information symbols over  $T$  symbol periods, so that the rate- $K/T$  space-time block code is represented by a  $T \times N$  matrix  $\mathbf{X}$ . Letting  $y_i$  denote the sample received at time  $i$  by a receiver with a single antenna, we can write:

$$[y_1, \dots, y_T]^T = \mathbf{X}\mathbf{h} + \mathbf{w}, \quad (1)$$

where  $\mathbf{h}$  is a vector of channel gains between the  $N$  transmit antennas and the receive antenna, and where  $\mathbf{w}$  is complex

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Fig. 1. Encoding architecture.

Gaussian noise with *i.i.d.*  $\mathcal{CN}(0, N_0)$  entries. We assume a quasistatic Rayleigh-fading channel, so that the entries of  $\mathbf{h}$  are *i.i.d.*  $\mathcal{CN}(0, 1)$  and constant over the duration of the  $T$  symbol periods. The results are easily extended to the case of multiple receive antennas.

The PAPR for the  $m$ -th transmit antenna of a space-time code is [13]:

$$\text{PAPR}_m = \frac{\max_{t \in \{1, \dots, T\}} |X_{t,m}|^2}{T^{-1} \sum_{t \in \{1, \dots, T\}} E(|X_{t,m}|^2)}, \quad (2)$$

where the maximum and the expectation operators are taken over all possible codeword matrices. In all the space-time codes considered in this paper, the PAPR is the same for all the antennas, so that the subscript on PAPR in (2) may be dropped. The coding gain of a full-diversity space-time code is [10]:

$$\Gamma = \min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \det((\mathbf{X} - \tilde{\mathbf{X}})^*(\mathbf{X} - \tilde{\mathbf{X}}))^{1/N}, \quad (3)$$

where the minimum is taken over all distinct codeword matrices  $\mathbf{X} \neq \tilde{\mathbf{X}}$ , and where  $\mathbf{X}^*$  denotes the Hermitian transpose of  $\mathbf{X}$ .

### B. Single-Symbol-Decodable Space-Time Codes

Single-symbol-decodable space-time block codes for four transmit antennas were proposed in [9] and [10]. In both cases the encoder decomposes into a concatenation of three steps, as shown in Fig. 1. The encoder starts with a vector  $\mathbf{x} \in \mathcal{A}^4$  of information symbols chosen from a conventional  $M$ -ary QAM alphabet  $\mathcal{A}$ . The first step is to distort the alphabet in some way; the codes of [9] and [10] rotate each alphabet by an angle of  $\phi$ , producing  $\mathbf{a} = e^{j\phi} \mathbf{x}$ , as shown in Fig. 2(b). The purpose of the rotation is to ensure full diversity. The second step is to interleave the coordinates of  $\mathbf{a}$ , yielding  $\mathbf{s} = \Pi(\mathbf{a})$ . The interleavers of [9] and [10] act on the real and imaginary parts separately, so that  $[s_1^R, s_1^I, s_2^R, s_2^I, s_3^R, s_3^I, s_4^R, s_4^I] = [a_1^R, a_1^I, a_2^R, a_2^I, a_3^R, a_3^I, a_4^R, a_4^I] \Pi$ , where  $\Pi$  is an  $8 \times 8$  permutation matrix (so that its columns are a permutation of the columns of the identity matrix, with the possibility of sign inversion), and where  $s_i^R$  and  $s_i^I$  denote the real and imaginary parts, respectively, of  $s_i$ . For the Yuen-Guan-Tjhung and Khan-Rajan codes, the interleaver is used to achieve full diversity while maintaining single-symbol decodability. The final step is to encode  $\mathbf{s}$  using a conventional space-time block encoder  $\mathbf{G}(\cdot)$ , yielding  $\mathbf{X} = \mathbf{G}(\mathbf{s})$ .

In terms of Fig. 1, the Khan-Rajan code of [10] is specified by  $\phi = \frac{1}{2} \tan^{-1}(2)$ ,  $\Pi = [\mathbf{e}_1, \mathbf{e}_6, \mathbf{e}_3, \mathbf{e}_8, \mathbf{e}_5, \mathbf{e}_2, \mathbf{e}_7, \mathbf{e}_4]$ , where  $\mathbf{e}_i$  is the  $i$ -th column of the  $8 \times 8$  identity matrix, and:

$$\mathbf{G}(\mathbf{s}) = \sqrt{2} \begin{pmatrix} \mathbf{A}(s_1, s_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}(s_3, s_4) \end{pmatrix}, \quad (4)$$

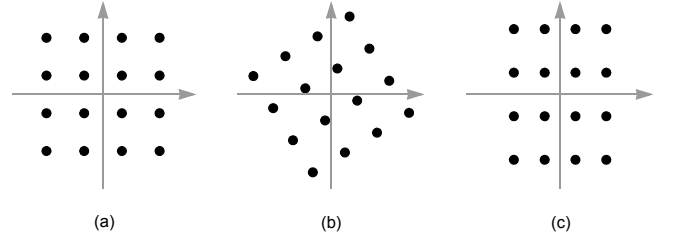


Fig. 2. The 16-QAM alphabet: (a) original; (b) rotated; (c) stretched.

where

$$\mathbf{A}(s_1, s_2) = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (5)$$

is the Alamouti space-time code [1]. The constant  $\sqrt{2}$  ensures that the average transmit energy per transmitted symbol (namely the denominator in (2)) is identical to the energy of the underlying alphabet  $\mathcal{A}$ . As noted in [9], a drawback of this code is that half of its entries are zero, which leads to a high PAPR.

The Yuen-Guan-Tjhung code of [9] can also be defined in terms of Fig. 1, with a rotation angle of  $\phi = \pi/4 - \frac{1}{2} \tan^{-1}(2)$ , an interleaver of  $\Pi = [\mathbf{e}_1, \mathbf{e}_5, \mathbf{e}_3, \mathbf{e}_7, -\mathbf{e}_2, \mathbf{e}_6, -\mathbf{e}_4, \mathbf{e}_8]$ , and:

$$\mathbf{G}(\mathbf{s}) = \begin{pmatrix} \mathbf{A}(s_1, s_2) & \mathbf{A}(s_3, s_4) \\ \mathbf{A}(s_3, s_4) & \mathbf{A}(s_1, s_2) \end{pmatrix}. \quad (6)$$

Both the Khan-Rajan code and the Yuen-Guan-Tjhung code require decoding of only two real symbols at a time, or equivalently, a single complex symbol.

## III. PROPOSED CODE CONSTRUCTION AND DECODING

### A. Proposed Code Construction and Optimization

We propose a new code based on the same architecture of Fig. 1 but differing in an important way from [9] and [10]: rather than distorting the alphabet by a rotation, so that  $a_i = e^{j\phi} x_i$ , we instead propose to distort it by stretching it vertically, using:

$$a_i = \sqrt{\frac{2}{1+K^2}} (Kx_i^R + jx_i^I), \quad (7)$$

where  $K \in (0, 1]$  is a stretching parameter to be specified later. The constant  $\sqrt{2/(1+K^2)}$  ensures that the stretched alphabet has the same energy as the original. Scaling the real and imaginary parts of the symbol by different amounts stretches the square constellation into a rectangle, as illustrated in Fig. 2(c). After stretching, we propose the interleaver  $\Pi = [\mathbf{e}_1, \mathbf{e}_6, \mathbf{e}_3, \mathbf{e}_8, \mathbf{e}_2, \mathbf{e}_5, \mathbf{e}_4, \mathbf{e}_7]$  and the same encoder  $\mathbf{G}(\mathbf{s})$  from (6) that is used in the Yuen-Guan-Tjhung code. The proposed interleaver has three desirable properties: it maintains single-symbol decodability, it guarantees full diversity, and it results in a PAPR that is identical to that of the underlying QAM alphabet. (The interleaver choice satisfying these three properties is not unique.) Thus, the proposed code for four transmit antennas is:

$$\mathbf{X} = \sqrt{\frac{2}{1+K^2}} \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{pmatrix}, \quad (8)$$

where  $[s_1, s_2, s_3, s_4] = [Kx_1^R + jx_3^I, Kx_2^R + jx_4^I, x_1^I + jKx_3^R, x_2^I + jKx_4^R]$ .

**Proposition 1:** The stretching parameter  $K \in (0, 1]$  that maximizes the coding gain (3) for the code in (8) with QAM is  $K = 1/\sqrt{2}$ . The resulting asymptotic coding gain is  $\frac{2}{3}d_{\min}^2$ , where  $d_{\min}$  is the minimum distance of the QAM alphabet  $\mathcal{A}$ .

*Proof:* Substituting from the definition of  $\mathbf{G}(\mathbf{s})$  in (6), the asymptotic coding gain of (3) is:

$$\begin{aligned} \Gamma(K) &= \min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \det((\mathbf{X} - \tilde{\mathbf{X}})^*(\mathbf{X} - \tilde{\mathbf{X}}))^{1/4} \\ &= \min \det \begin{pmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{pmatrix}^{1/4} \\ &= \min(\alpha^2 - \beta^2)^{1/2}, \end{aligned} \quad (9)$$

where we have introduced  $\alpha = \sum_{i=1}^4 |\Delta s_i|^2 = \sum_{i=1}^4 |\Delta a_i|^2$ ,  $\beta = 2\Re\{\Delta s_1 \Delta s_3^* + \Delta s_2 \Delta s_4^*\} = 2\sum_{i=1}^4 \Delta a_i^R \Delta a_i^I$ ,  $\Delta s_i = s_i - \tilde{s}_i$ , and  $\Delta a_i = a_i - \tilde{a}_i$ . The expressions for  $\alpha$  and  $\beta$  in terms of  $\Delta a_i$  follow from  $\mathbf{s} = \Pi(\mathbf{a})$ . The minimization in (9) is over all nonzero vectors  $[\Delta a_1, \dots, \Delta a_4]$ . But

$$\begin{aligned} \alpha^2 - \beta^2 &= (\alpha + \beta)(\alpha - \beta) \\ &= \left( \sum_i (|\Delta a_i|^2 + 2\Delta a_i^R \Delta a_i^I) \right) \times \\ &\quad \left( \sum_j (|\Delta a_j|^2 - 2\Delta a_j^R \Delta a_j^I) \right) \\ &= \left( \sum_i (\Delta a_i^R + \Delta a_i^I)^2 \right) \times \\ &\quad \left( \sum_j (\Delta a_j^R - \Delta a_j^I)^2 \right). \end{aligned} \quad (10)$$

Because both factors in (10) are the sum of squares, the nonzero vector  $[\Delta a_1, \dots, \Delta a_4]$  that minimizes the product will have only one nonzero element, say  $\Delta a_i$ , so that the gain (9) reduces to:

$$\Gamma(K) = \min |(\Delta a_i^R)^2 - (\Delta a_i^I)^2|, \quad (11)$$

where now the minimization is over all nonzero error symbols  $\Delta a_i$ . But from (7) we can write:

$$\Delta a_i = \sqrt{\frac{2}{1+K^2}}(Km + jn)d_{\min} \quad (12)$$

for some pair of integers  $(m, n) \in \{-\sqrt{M}+1, \dots, \sqrt{M}-1\}^2$ , where  $d_{\min}$  is the minimum Euclidean distance of the (unstretched) alphabet  $\mathcal{A}$ . Plugging (12) into (11) yields the following expression for the coding gain, as a function of  $K$ :

$$\Gamma(K) = 2d_{\min}^2 \cdot \min_{(m,n) \in \mathcal{M}} \frac{|K^2 m^2 - n^2|}{1+K^2}, \quad (13)$$

where we have introduced  $\mathcal{M} \triangleq \{-\sqrt{M}+1, \dots, \sqrt{M}-1\}^2 - \{(0,0)\}$  as the set of all possible integer pairs  $(m, n)$ , with the  $(0,0)$  pair excluded.

The remaining problem is to find the value of  $K$  that maximizes (13). We proceed in two steps: first, we establish the bound  $\Gamma(K) \leq \frac{2}{3}d_{\min}^2$ ; then, we show that  $K = 1/\sqrt{2}$  achieves the bound with equality.

To establish the bound, let us introduce  $\mathcal{L} \triangleq \{(1,0), (1,1)\} \subset$

$\mathcal{M}$ . Because  $\mathcal{L}$  is a subset of  $\mathcal{M}$ , it clearly follows that  $\min_{\mathcal{M}}\{\cdot\} \leq \min_{\mathcal{L}}\{\cdot\}$ , so that the coding gain of (13) can be bounded by:

$$\begin{aligned} \Gamma(K) &\leq 2d_{\min}^2 \cdot \min_{(m,n) \in \mathcal{L}} \frac{|K^2 m^2 - n^2|}{1+K^2} \\ &= 2d_{\min}^2 \cdot \min \left\{ \frac{|K^2 1^2 - 0^2|}{1+K^2}, \frac{|K^2 1^2 - 1^2|}{1+K^2} \right\} \\ &\leq 2d_{\min}^2 \cdot \max_{K>0} \min \left\{ \frac{K^2}{1+K^2}, \frac{1-K^2}{1+K^2} \right\} \quad (14) \\ &= \frac{2}{3}d_{\min}^2. \quad (15) \end{aligned}$$

The inequality of (14) follows from the fact that  $g(K) \leq \max_{K>0}\{g(K)\}$  for any function  $g(K)$  and for any  $K > 0$ . The equality in (15) follows because, for  $K > 0$ , the functions  $(K^2)/(1+K^2)$  and  $(1-K^2)/(1+K^2)$  are monotonically increasing and decreasing, respectively, so that the maximum of the minimum occurs at the intersection, namely at  $K = 1/\sqrt{2}$ .

The bound  $\Gamma(K) \leq \frac{2}{3}d_{\min}^2$  of (15) is in fact *achievable* with  $K = 1/\sqrt{2}$ , since from (13):

$$\begin{aligned} \Gamma(1/\sqrt{2}) &= 2d_{\min}^2 \cdot \min_{(m,n) \in \mathcal{M}} \frac{|m^2/2 - n^2|}{1 + \frac{1}{2}} \\ &= \frac{2}{3}d_{\min}^2, \end{aligned} \quad (16)$$

where the second equality follows from the fact that  $|m^2 - 2n^2|$  must be a nonnegative integer, and it cannot be zero because  $\sqrt{2}$  is irrational; therefore,  $\min_{(m,n) \in \mathcal{M}} |m^2 - 2n^2| = 1$ . ■

We make two important remarks regarding the PAPR of the proposed code. First, stretching the alphabet according to (7) does not alter its PAPR. Indeed, it is easily verified that the stretched alphabet has the same PAPR as the original one, since:

$$\max_{\mathcal{A}} (|Kx_i^R + jx_i^I|^2) = (1+K^2)(\sqrt{M}-1)^2, \quad (17)$$

$$E(|Kx_i^R + jx_i^I|^2) = (1+K^2)\frac{(M-1)}{3}. \quad (18)$$

Taking the ratio of these two yields  $3(\sqrt{M}-1)/(\sqrt{M}+1)$ , which is precisely the PAPR of the underlying QAM alphabet.

Second, our particular choice for coordinate interleaving does not alter the PAPR; this latter fact can be seen by observing that the elements of (8) all have the same PAPR as the stretched alphabet. Together, these two facts imply that the PAPR of the proposed code is identical to that of the underlying QAM alphabet, regardless of the value of  $K$ .

The proposed space-time code for three antennas is obtained by deleting the fourth column of  $\mathbf{X}$  in (8). The proof that the space-time code obtained by deleting the fourth column is still fully diverse follows directly from Theorem 3 and Theorem 4 in [14]. For two antennas, an equivalent form of the Alamouti code [1] can be obtained by deleting columns three and four (or two and three) and setting  $K = 1$ .

We remark that although the concept of constellation stretching may be applied to PSK alphabets, it is easily verified that the PAPR of the resulting code matrix in (8) would be higher than that of the underlying PSK alphabet. Therefore, we only consider QAM alphabets.

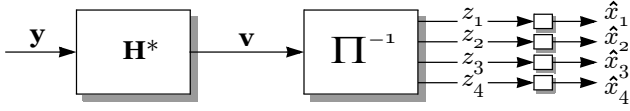


Fig. 3. The ML decoder for the proposed space-time code decomposes into a matched filter bank, a deinterleaver, and a bank of four independent complex-symbol quantizers.

TABLE I  
A COMPARISON OF DECODING COMPLEXITY AND PAPR OF  
FOUR-ANTENNA SPACE-TIME BLOCK CODES.

Code	No. symbols jointly detected	PAPR (dB)		
		4-QAM	16-QAM	64-QAM
Su-Xia [8]	4	0	2.55	3.68
Yuen <i>et al.</i> [9]	2	1.61	4.16	5.29
Khan-Rajan [10]	2	5.79	8.34	9.47
Proposed	2	0	2.55	3.68

### B. Single-Symbol ML Decoding of Proposed Code

Substituting the definition of the proposed code from (8) into (1), the received vector, after conjugating two of its elements, can be rewritten as

$$\begin{pmatrix} y_1 \\ y_2^* \\ y_3 \\ y_4^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2^* \\ w_3 \\ w_4^* \end{pmatrix}$$

or  $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$ . (19)

The ML decision is the symbol vector  $\mathbf{s}$  that minimizes

$$\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \|\mathbf{y}\|^2 + \mathbf{s}^* \mathbf{H}^* \mathbf{H} \mathbf{s} - 2\Re\{\mathbf{s}^* \mathbf{v}\}, \quad (20)$$

where we have introduced the vector  $\mathbf{v} = \mathbf{H}^* \mathbf{y}$ , which can be interpreted as the output of a filter matched to  $\mathbf{H}$ . Let  $\eta = \sum_i |h_i|^2$  denote the squared norm of any column of  $\mathbf{H}$  and  $\rho = 2\Re\{h_1 h_3^* + h_2 h_4^*\}$  denote the inner product between the first and third columns of  $\mathbf{H}$  (or equivalently, the second and fourth columns of  $\mathbf{H}$ ). Then  $\mathbf{H}^* \mathbf{H}$  is a circulant matrix given by:

$$\mathbf{H}^* \mathbf{H} = \begin{pmatrix} \eta & 0 & \rho & 0 \\ 0 & \eta & 0 & \rho \\ \rho & 0 & \eta & 0 \\ 0 & \rho & 0 & \eta \end{pmatrix}. \quad (21)$$

Substituting (21) into (20) yields:

$$\begin{aligned} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 &= \|\mathbf{y}\|^2 + \eta \sum_{i=1}^4 |s_i|^2 \\ &\quad + 2\rho \Re\{s_1 s_3^* + s_2 s_4^*\} - 2\Re\{\mathbf{s}^* \mathbf{v}\}. \end{aligned} \quad (22)$$

Already we can see that the symbols  $s_1$  and  $s_3$  can be decoded separately from  $s_2$  and  $s_4$ , since there is no interference between the symbol pair  $(s_1, s_3)$  and the pair  $(s_2, s_4)$ . This result is expected due to the quasiorthogonal structure of the code matrix (8), where the subspace spanned by the first and third columns is orthogonal to the subspace spanned by the second and fourth columns.

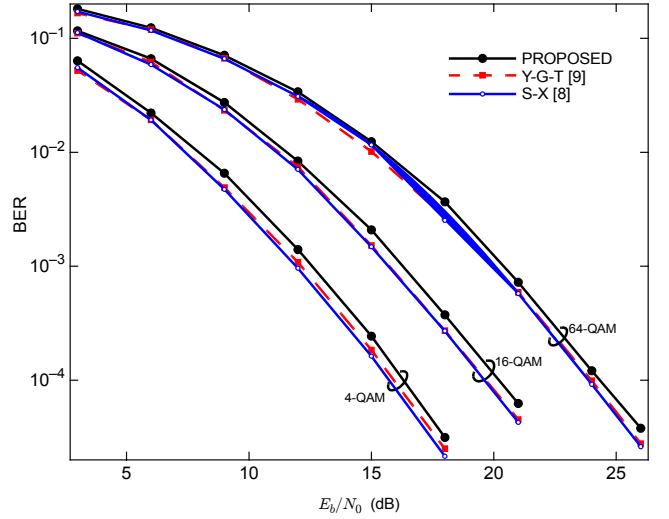


Fig. 4. Performance for three transmit antennas.

The decoding complexity reduces even further when we substitute  $\mathbf{s} = \Pi(\mathbf{a}) = [a_1^R, a_1^I, a_2^R, a_2^I, a_3^R, a_3^I, a_4^R, a_4^I] \Pi$  into (22), yielding:

$$\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \|\mathbf{y}\|^2 + \sum_{i=1}^4 f(z_i | a_i), \quad (23)$$

where  $f(z_i | a_i)$  is defined as follows:

$$f(z_i | a_i) = \eta |a_i|^2 + \rho \Im\{a_i^2\} - 2\Re\{a_i^* z_i\}, \quad (24)$$

where we have introduced  $\mathbf{z} = \Pi^{-1}(\mathbf{v}) = [v_1^R + jv_3^R, v_2^R + jv_4^R, v_3^I + jv_1^I, v_4^I + jv_2^I]$ . Observe that each  $f(z_i | a_i)$  depends on  $\mathbf{z}$  and  $\mathbf{a}$  only through  $z_i$  and  $a_i$ . Therefore, after matched filtering to form the vector  $\mathbf{v}$  and deinterleaving to form the vector  $\mathbf{z}$ , the ML decoder reduces to separate decoding of the four transmitted symbols. Specifically, as illustrated in Fig. 3, the ML decision about  $a_i$  (and ultimately  $x_i$ ) can be found by minimizing the corresponding  $f(z_i | a_i)$  metric of (24). We have thus shown that the proposed code is single-complex-symbol decodable. We note that the cross-interference between the real and imaginary components of  $a_i$  in  $f(z_i | a_i)$  prevents us from achieving single-real-symbol decoding.

In Table I we compare the proposed code with other single-symbol decodable codes in terms of decoding complexity and PAPR for four transmit antennas. We see that the PAPR of the proposed code is significantly smaller than that of the Khan-Rajan code [10], and that it has the same PAPR as the Su-Xia code [8] but at a reduced decoding complexity.

## IV. NUMERICAL RESULTS

In this section we compare the bit-error rate performance of the proposed space-time code with the Su-Xia code [8] and Yuen-Guan-Tjhung code [9] over a quasistatic Rayleigh-fading channel with additive Gaussian noise. The Khan-Rajan code [10] performs identically to the Yuen-Guan-Tjhung code with four transmit antennas, and slightly worse with three antennas. Therefore, we did not include the performance of Khan-Rajan code in our comparison. In Fig. 4 we show the performance for three antennas, while in Fig. 5 we show the performance

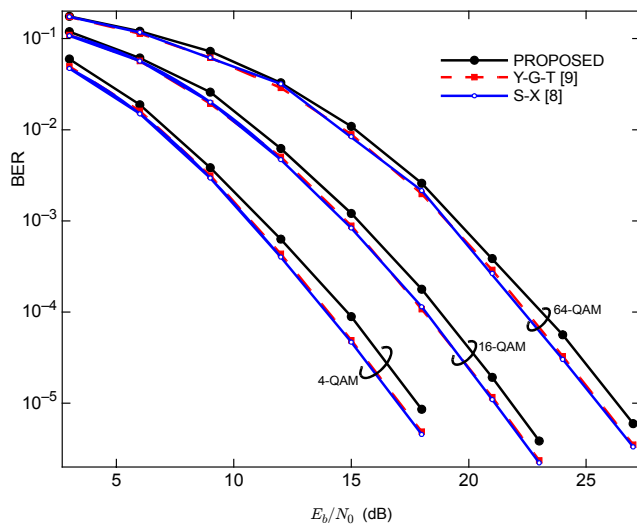


Fig. 5. Performance for four transmit antennas.

for four antennas. Results are shown for 4-QAM, 16-QAM and 64-QAM Gray-coded alphabets.

Fig. 4 shows that, for three antennas, the proposed code performs only 0.5 dB worse than the Yuen-Guan-Tjhung code, and 0.6 dB worse than the Su-Xia code. For four antennas, the proposed code suffers a 0.7 dB and 0.8 dB loss compared with the Yuen-Guan-Tjhung code and Su-Xia code, respectively. The proposed code, however, has a lower decoding complexity than the Su-Xia code and maintains the same PAPR. The proposed code has a significantly lower PAPR than the Yuen-Guan-Tjhung code while suffering only a slight performance loss.

## V. CONCLUSION

We introduced a novel space-time block code for three and four transmit antennas that combines coordinate interleaving with constellation stretching. The stretching parameter was chosen analytically to maximize the coding gain. The resulting code has a quasiorthogonal structure that makes it single-symbol decodable, meaning that the jointly maximum-likelihood decoder may be implemented using a bank of

independent quantizers, one for each complex information symbol. The proposed codes for three and four antennas achieve full rate, full diversity, and single-symbol decodability without sacrificing the peak-to-average power ratio.

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