This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

Space-Time Active Rotation (STAR): A New Layered Space-Time Architecture

Arumugam Kannan and John R. Barry

School of ECE, Georgia Institute of Technology
Atlanta, GA 30332-0250 USA
{aru, barry}@ece.gatech.edu

***Abstract* — We propose *space-time active rotation* (STAR), a new layered space-time architecture for fading multiple-input multiple-output channels. The vertically layered V-STAR architecture is similar in spirit to the popular V-BLAST architecture, but by rotating the set of active antennas it achieves better performance with comparable complexity. We derive the outage probability and bounds on the diversity order of a V-STAR system. We propose an ordering algorithm that minimizes the outage probability of a successive cancellation decoder for the V-STAR system. Over a 4-input 4-output Rayleigh-fading channel, we show that V-STAR outperforms V-BLAST by 17.7 dB, and that V-STAR outpeforms transmitter-optimized V-BLAST by 2.7 dB. The outage probability of V-STAR with successive cancellation decoding is only 1.4 dB away from the optimum outage probability achieved by an unconstrained decoder. This gap drops to 0.6 dB for an 8-input 8-output channel.**

I. Introduction

Wireless communication systems with multiple transmit and receive antennas are rapidly growing in popularity due to their high spectral efficiencies and robustness to multipath fading. A large body of research has focussed on harnessing the multiplexing and diversity benefits promised by the theory of MIMO systems [3][4]. Achieving these benefits at an affordable computational complexity is still an active topic of research.

The vertically layered V-BLAST architecture was one of the earliest systems proposed to achieve a high multiplexing gain at low decoding complexity [1]. The V-BLAST receiver uses *successive cancellation* (SC) decoding, which is much less complex than the optimum maximum-likelihood (ML) decoder. However, the performance of SC decoding is significantly inferior that of ML decoding.

The performance of V-BLAST can be improved by modifying the transmitter so better suit the SC decoder. For example, optimal allocation of the rate and energy across the transmit antennas considerably improves the performance of SC decoding [2]. Antenna selection strategies are also effective in improving the performance of SC decoding [5][6]. In closed-loop methods, where a full or partial feedback path is available from the receiver to the transmitter, the optimal rate and energy allocation or the optimal subset of antennas is fed back to the transmitter. Closed-loop antenna selection yields significant performance improvements [5].

This paper considers open-loop strategies for the case when there is no feedback from the receiver to the transmitter. Current open-loop methods are reasonably effective in improving the performance of SC decoding [6][8]. However, the performance of the best available open-loop system based on the V-BLAST architecture [8] is still around 4.1 dB away from the optimum ML decoder for a 4×4 MIMO system operating at 8 b/s/Hz. This gap suggests the need to evolve new and improved layered space-time architectures, rather than mere optimization of the V-BLAST architecture.

In this paper, we propose a new layered space-time architecture for open-loop systems, called *space-time active rotation* (STAR). In the STAR architecture, the duration over which the channel response is constant is divided into t *blocks*. The first antenna is inactive during the first block, the second antenna is inactive during the second block, and so on. Thus, the set of active antennas rotates. The STAR receiver is based on SC decoding, which maintains the low complexity of layer processing. We will see that the vertically layered V-STAR system significantly outperforms the V-BLAST system.

This paper is organized as follows. In Section II, we describe the channel model. In Section III, we describe the STAR architecture. In Section IV, we derive its capacity and outage probability with SC decoding. In Section V, we derive the ordering algorithm that minimizes outage probability for the SC decoder. In Section VI, we present the simulation results to support the analysis. In Section VII we make concluding remarks.

II. Channel Model

We consider a MIMO system with t transmit and r receive antennas, with the assumption that $r \geq t$. We consider the scenario where the channel is flat-fading and quasi-static,

This research was supported in part by National Science Foundation grants 0431031 and 0121565.

 0-7803-9415-1/05/$20.00 © 2005 IEEE

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

which is a popular channel model for slow fading, fixed wireless environments. The received vector at the k^{th} signaling interval is given by

$$\boldsymbol{r}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \,. \tag{1}$$

where elements of the $r \times 1$ noise vector $\mathbf{n}_k$ are independent, circularly symmetric Gaussian random variables with zero mean and variance N_0, so that $\mathrm{E}[\mathbf{n}_k \mathbf{n}_l^*] = \delta_{k-l} N_0 \mathbf{I}_r$, where A^* denotes conjugate transpose of A. The $r \times t$ channel matrix $\mathbf{H}$ is assumed to be a random Rayleigh fading matrix, its entries being independent, circularly symmetric complex Gaussian random variables with zero mean and unit variance. Let $\boldsymbol{h}_i$ denote the i^{th} column of the channel matrix $\mathbf{H}$. The receiver has perfect knowledge of the channel $\mathbf{H}$, but the transmitter does not.

The total data rate is R, and the average transmit energy per signaling interval is E. Under these assumptions, the average SNR per receive antenna is $S = E/N_0$. We adopt the quasi-static fading model, so that the channel is static over frames of duration T signaling intervals, but fades independently from one frame to the next. The frame duration T is assumed to be large enough for the standard infinite time horizon of information theory to be meaningful.

III. The STAR Layered Architecture

A. Transmitter

The STAR transmitter has two primary components, the *antenna setup* and the *coding rule*. First, we describe the antenna setup of STAR. The static fading frame, over which the channel is assumed to be a constant, is divided into t blocks. During the j^{th} block, the j^{th} antenna is inactive. Thus, the effective channel matrix, $\mathbf{H}^{(j)}$, in the j^{th} block, is formed by the set of all $\boldsymbol{h}_i, j \neq i$, where $\boldsymbol{h}_i$'s are the columns of $\mathbf{H}$. For $t = 4$, the frame is divided into four blocks as illustrated in Fig. 1.

The other component of the transmitter is the *coding rule*. A *layer* is defined as one codeword of the outer scalar error correcting code. The *coding rule* is the way in which the layers are transmitted through the transmit antennas. In general, layered coding is of two types: *independent coding*, where symbols of one layer are transmitted through only one antenna, or *joint coding*, where a layer spans more than one antenna. The transmitter of the V-BLAST architecture has all the antennas active at all times and it employs independent coding on the parallel data streams. This simple transmission strategy is an example of *spatial multiplexing*.

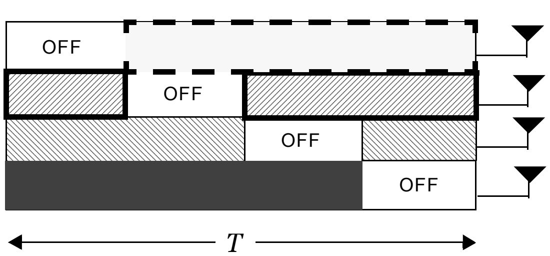

Fig. 1. The vertical space-time active rotation (V-STAR) transmitter with t = 4 transmit antennas.

In this work, we consider the STAR architecture with independent coding, and the system will hence be referred to as V-STAR (vertical STAR). In this setup, t independent streams of data are coded using t scalar SISO codes. Thus, the total number of layers is always equal to t, the number of transmit antennas, and the number of active antennas at any given instant is $t - 1$. In Fig. 1, the different layers are shaded by different patterns to show that they are encoded using independent error correcting codes.

B. Receiver

As with V-BLAST, the layered nature of the V-STAR transmitter yields itself to simplified decoding using a successive cancellation (SC) decoder. SC decoders [1] decode one layer at a time, subtracting out the estimated contribution of previously decoded layers, and nulling out interference from undecoded layers. However, unlike V-BLAST, it will be seen that V-STAR with SC decoding achieves near-optimum outage probability of spatial multiplexing systems.

Define the i^{th} *layer* as the scalar coded data stream transmitted through the i^{th} antenna. The i^{th} layer is a concatenation of t blocks, with the i^{th} block being inactive. The SC decoding process is a t-stage process, where one stage represents the detection and decoding of one layer. In the i^{th} stage, the SC decoder detects each active block $\{j = 1, 2, \ldots, t; j \neq i\}$ in the i^{th} layer using the nulling procedure. The detected blocks are concatenated and decoded using the outer scalar decoder. Subsequently, the decoded layer is used to cancel out the interference before decoding the next layer.

To detect the j^{th} block in the i^{th} layer, the SC decoder nulls out the undecoded streams using the *nulling vector* $\boldsymbol{w}_i^{(j)}$, defined as the first row of the Moore-Penrose inverse of matrix $[\boldsymbol{h}_i, \boldsymbol{h}_{i+1}, \ldots, \boldsymbol{h}_t \setminus \boldsymbol{h}_j]$. The $1 \times T/t$ decision vector

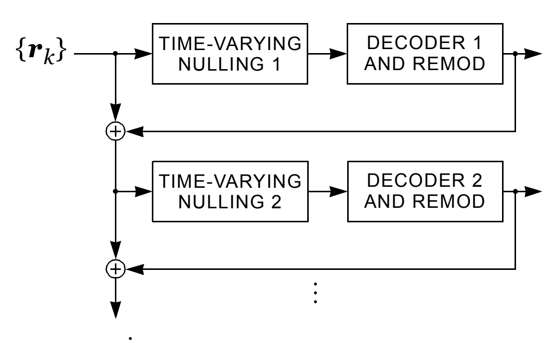

Fig. 2. The V-STAR successive cancellation decoder.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

in the j^{th} block of the i^{th} layer is obtained as $\boldsymbol{y}_i^{(j)} = \boldsymbol{w}_i^{(j)}\boldsymbol{r}^{(j)}$, where $\boldsymbol{r}^{(j)}$ is the $r \times T/t$ received vector during the j^{th} block. Thus, the channel model reduces to

$$\boldsymbol{y}_i^{(j)} = \boldsymbol{x}_i^{(j)} + \boldsymbol{w}_i^{(j)}\boldsymbol{n}^{(j)} \tag{2}$$

The equivalent channel (2) is an AWGN channel with noise variance $N_0\|\boldsymbol{w}_i^{(j)}\|^2$. The output of the SC detector forms the input to the outer scalar decoder. The instantaneous signal to noise ratio of this block is $\rho_i^{(j)} = E/(tN_0\|\boldsymbol{w}_i^{(j)}\|^2) = S/(t\|\boldsymbol{w}_i^{(j)}\|^2)$.

The performance of the SC decoder depends on the order of detection of the layers. The optimal order is channel dependent. The order can be described by the permutation $\pi(1, 2, \ldots, t)$, where π_k is the k^{th} detected layer. Let Π be the matrix whose k^{th} column is the $\pi_k{}^{th}$ column of $\mathbf{I}$. Once the optimal order is computed based on $\mathbf{H}$, fixed ordered SC decoding is performed on $\mathbf{H}\Pi$ instead of $\mathbf{H}$.

IV. Outage Probability and Bounds on Diversity Order

In this section, we compute the instantaneous capacity and the outage probability of the system. Note that capacity here refers to the information carrying capacity of the *equivalent channel* formed by the MIMO channel in conjunction with the soft output SC decoder. The capacity of the i^{th} layer, equivalently the i^{th} parallel channel is:

$$C_i(\boldsymbol{H}) = \underset{p(x)}{\arg\min}\ \frac{1}{t} \cdot I(\boldsymbol{X}, \boldsymbol{Y}|\boldsymbol{H}) \tag{3}$$

where, $\boldsymbol{X} = \{\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_t\}$ is the concatenation of the transmitted blocks of the i^{th} layer and $\boldsymbol{Y} = \{\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_t\}$ are the corresponding blocks at the output of the soft output SC decoder. Given $\mathbf{H}$, the mutual information of the blocks are independent of each other since the noise is independent across time, so that

$$C_i(\boldsymbol{H}) = \underset{p(x)}{\arg\min}\ \frac{1}{t} \cdot \sum_{j=1}^{t} I(\boldsymbol{X}_i, \boldsymbol{Y}_i|\boldsymbol{H}) \tag{4}$$

Each active block is of length T/t. After detection, each block is effectively an AWGN channel with instantaneous SNR equal to $\rho_i^{(j)}$. Hence, the capacity of the j^{th} block in the i^{th} layer is $\log_2(1 + \rho_i^{(j)})$. The inactive blocks have zero capacity. Thus, the capacity of the i^{th} layer is

$$C_i(H) = \frac{1}{t} \sum_{j \neq i} \log_2(1 + \rho_i^{(j)}) \tag{5}$$

Thus, the layer capacity is the arithmetic mean of the capacities of the blocks. Since each data stream is assumed to have a capacity-achieving code, it is incorrectly decoded if and only if an *outage* occurs, i.e., if $C_i(\mathbf{H}) < R/t$. If *all* data streams are outage-free, the SC decoder is also error-free. However, if *any* of the streams is in outage, the SC decoder is in outage. Consequently, the *outage probability* is

$$P_{STAR}(S, R) = \Pr[\bigcup_{i=1}^{t} \left\{ C_i(\boldsymbol{H}) < \frac{R}{t} \right\}] \tag{6}$$

The outage probability is a lower bound on the achievable frame error rate of the system. The bound can be approached by using a powerful error control code such as an LDPC or a Turbo code as the outer code. An important metric to quantify the performance of wireless communication systems is the diversity order, d, defined as the asymptotic slope of the outage probability:

$$d = \lim_{S \to \infty} \frac{-\log P_{STAR}(S, R)}{\log S} \tag{7}$$

We now prove the following results about the diversity order of the V-STAR system.

Lemma 4.1 The diversity order d of V-STAR with SC decoding is $d = \min\{d_1, d_2, \ldots, d_t\}$, where d_i is the diversity order of the i^{th} layer.

Proof: Using the union bound, the outage probability can be bounded as follows.

$$Pr\left\{ C_i(\boldsymbol{H}) < \frac{R}{t} \right\} < P_{STAR} < \sum_{i=1}^{t} Pr\left\{ C_i(\boldsymbol{H}) < \frac{R}{t} \right\} \tag{8}$$

The above inequality is true for any i. We choose i such that $d_i = \min\{d_1, d_2, \ldots, d_t\}$ so that the diversity order of the lower bound is $\min_i\{d_i\}$. As $S \to \infty$, the upper bound is dominated by the term with the lowest diversity order. Since the lower and upper bounds have the same diversity order, $d = \min_i\{d_i\}$.

Theorem 1. The diversity order d of V-STAR with SC decoding is bounded as

$$\min_i\{\max_j\{d_i^{(j)}\}\} \leq d \leq r \tag{9}$$

where $d_i^{(j)}$ is the diversity order of the j^{th} block in the i^{th} layer.

Proof: The capacity of the i^{th} layer can be upper bounded as follows for any block j in layer i.

$$C_i(\boldsymbol{H}) > \frac{1}{t} \cdot \log_2(1 + \rho_i^{(j)}) \tag{10}$$

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

Choose the block j^* in the i^{th} layer such that $d_i^{(j*)} = \max_j\{d_i^{(j)}\}$, then, using (10) the following holds true,

$$Pr\left(C_i(\boldsymbol{H}) < \frac{R}{t}\right) < \alpha \cdot S^{-max_j(d_i^{(j)})} \quad (11)$$

for some constant α. Combining (11) with Lemma 4.1, we get $d \geq \min_i\{\max_j\{d_i^{(j)}\}\}$. The second part can be proved using the fact that $P_{STAR}(S, R) \geq P_{SM}(S, R)$, where $P_{SM}(S, R)$ is the minimum possible outage probability of any spatial multiplexing system.

$$P_{SM}(S, R) = Pr\left(\bigcup_{\gamma \in \Gamma}\left\{\log det\left(I + \frac{S}{t}\boldsymbol{H}^{\gamma}\boldsymbol{H}^{\gamma *}\right) < R^{\gamma}\right\}\right) \quad (12)$$

where Γ is the set of all non-empty subsets of antennas $\{1, 2, \ldots, t\}$ with γ denoting each element of Γ. $\mathbf{H}^{\gamma}$ is the matrix formed the columns of $\mathbf{H}$ specified by γ and R^{γ} is the total rate transmitted through the subset of antennas specified by γ [2]. It is well known [4] that the diversity order of $P_{SM}(S, R)$ is r.

The above result could prove to be useful in determining the diversity order of V-STAR with SC decoding if the diversity order of any one of the blocks is known.

V. Receiver Design: ordering Algorithm

The performance of the SC decoder depends on the order in which the layers are detected. For every instance of $\mathbf{H}$, the receiver determines the optimal order of detection, π, which minimizes the outage probability. The expression for outage probability given by (6) can be re-written as

$$Pr\left(\bigcup_{i=1}^{t}\left\{C_i(\boldsymbol{H}) < \frac{R}{t}\right\}\right) = Pr\left(min_i\{C_i(\boldsymbol{H})\} < \frac{R}{t}\right) \quad (13)$$

Thus, the order of detection must be chosen to maximize the minimum among layer capacities, to minimize the outage probability. We propose a simple, greedy ordering algorithm along the lines of [1] and we prove that it minimizes the outage probability.

We propose the following ordering algorithm which greedily maximizes the layer capacity at every stage of detection. For a given channel realization $\mathbf{H}$, π_k, the k^{th} layer detected is chosen according to

$$\pi_k = \underset{i \notin \{\pi_1, \ldots, \pi_{k-1}\}}{\arg max} \frac{1}{t}\sum_{j \neq i}\log_2\left(1 + \frac{S}{t}\left\|\boldsymbol{h}_i - \hat{\boldsymbol{h}}_i(\bar{\Omega})\right\|^2\right) \quad (14)$$

where,

$$\Omega = \{i, j, \pi_1, \ldots, \pi_{k-1}\} \quad (15)$$

$$\bar{\Omega} = \{1, 2, \ldots, t\} - \Omega \quad (16)$$

and $\hat{\boldsymbol{h}}_i(\bar{\Omega})$ is the projection of $\boldsymbol{h}_i$ on the plane spanned by the r-dimensional column vectors $\boldsymbol{h}_\omega \forall \omega \in \bar{\Omega}$. Contrast this to V-BLAST where,

$$\pi_k = \underset{i \notin \{\pi_1, \ldots, \pi_{k-1}\}}{\arg max} \log_2\left(1 + \frac{S}{t}\left\|\boldsymbol{h}_i - \hat{\boldsymbol{h}}_i(\bar{\Omega})\right\|^2\right) \quad (17)$$

where $\Omega = \{i, \pi_1, \ldots, \pi_{k-1}\}$.

Note that, in V-STAR, the capacity of the i^{th} layer is averaged over t blocks in a static fading frame, as opposed to V-BLAST. The performance of V-BLAST might be degraded due to one '*bad*' antenna, whereas V-STAR guards against such an occurrence. We now claim that the proposed ordering algorithm is optimal in terms of minimizing the outage probability.

Theorem 2. The minimum among all layer capacities is maximized, if, at every stage of decoding $i = \{1, 2, \ldots, t\}$, the layer detected at that stage is chosen such that it has the maximum capacity among the $t - i + 1$ possibilities for that layer.

Proof: (**Sketch**) This result can be proved along the same lines as the proof of optimal ordering in [1], by replacing the metric of interest namely the SNR of the i^{th} layer $\rho_i = \frac{S}{t}\left\|\boldsymbol{h}_i - \hat{\boldsymbol{h}}_i(\bar{\Omega})\right\|^2$ by the capacity of the i^{th} layer given by

$$C_i = \frac{1}{t}\sum_{j \neq i}\log_2\left(1 + \frac{S}{t}\left\|\boldsymbol{h}_i - \hat{\boldsymbol{h}}_i(\bar{\Omega})\right\|^2\right) \quad (18)$$

The definition of constraint sets would remain the same as in [1]. The constraint set for π_k, the k^{th} detected layer is $\{\pi_{k+1}, \pi_{k+2}, \ldots, \pi_t\}$. Following the steps of the proof in [1], we establish that the greedy ordering algorithm yields the globally optimum order.

VI. Simulation Results

In this section, we present simulation results for V-STAR. We consider a 4 × 4 MIMO system operating at a data rate R = 8 b/s/Hz, distributed equally among the transmit antennas.

Fig. 3 shows that the outage probability of V-STAR is 1.4 dB away from the minimum possible outage probability of the 4 × 4 spatial multiplexing systems. Fig. 3 also shows that STAR outperforms the unoptimized BLAST system by 17.7 dB. Moreover, V-STAR outperforms the transmitter

 0-7803-9415-1/05/$20.00 © 2005 IEEE

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

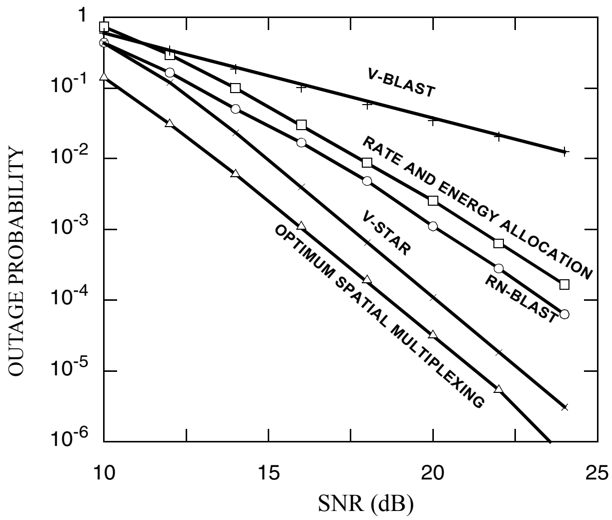

Fig. 3. Comparison of outage probability of various spatial multiplexing systems with SC decoding over a 4 × 4 MIMO channel at R = 8 b/s/Hz.

optimization methods for V-BLAST namely optimal rate and energy allocation in [2] by 4.2 dB and RN-BLAST [8] by 2.7 dB at an outage probability of 10^{-3}.

Fig. 4 shows the performance of STAR with SC decoding in comparison to the optimum outage probability for t = 3, 4 and 8 transmit antennas, with a data rate of 2 b/s/Hz per transmit antenna. At an outage probability of 10^{-3}, the gap to optimum for the three systems are 2.0 dB, 1.4 dB, and 0.6 dB respectively. The SNR gap decreases as the number of antennas increases, which augurs well for high data rate MIMO systems. This trend can be attributed to the fact that the rate penalty incurred by STAR is 1 / t, since the transmitter is inactive for that fraction of the total duration. This redundancy decreases as t increases, hence decreasing the rate penalty.

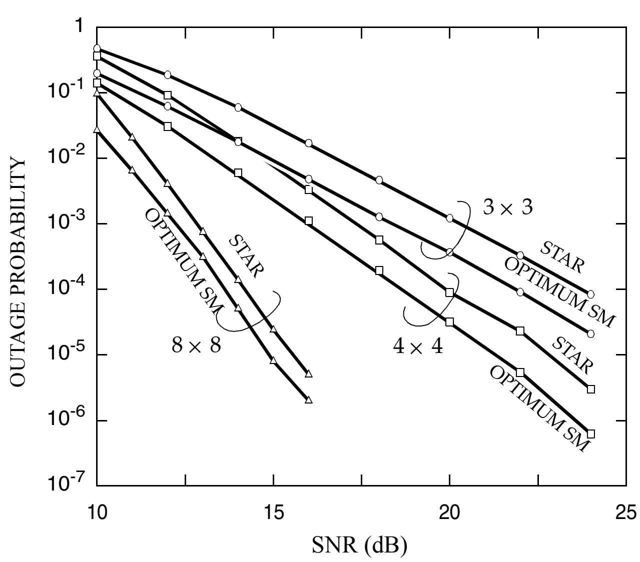

Fig. 4. V-STAR with SC decoding vs. optimum outage probability for spatial multiplexing systems over 3 × 3, 4 × 4 and 8 × 8 MIMO channels at R_j = 2 b/s/Hz/antenna.

VII. Conclusion

This paper introduced the space-time active rotation (STAR) layered space-time architecture. We considered V-STAR with successive cancellation decoding for spatial multiplexing systems. We derived the outage probability and provided bounds on the diversity order of the system. We proposed an ordering algorithm for SC decoding that minimizes the outage probability of STAR. We showed that the proposed system with optimally ordered SC decoding gets to within 1.4 dB of the optimum outage probability of a 4 × 4 spatial multiplexing system. The gap to optimum outage probability was shown to decrease with increasing number of antennas. Our results show that V-STAR with SC decoding is an effective solution to achieve near optimum outage probability of spatial multiplexing systems at a low computational complexity.

References

[1] P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel", *Proc. ISSSE-98*, Pisa, Italy, pp. 295-300, Sept. 29, 1998.

[2] N. Prasad and M. K. Varanasi, "Outage Analysis and Optimization for Multiaccess/V-BLAST Architecture over MIMO Rayleigh Fading Channels", *41st Annual Allerton Conf. on Comm. Control, and Comput.*, Monticello, IL, Oct., 2003.

[3] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov. 1999.

[4] David N. C. Tse, Pramod Viswanath and Lizhong Zheng, "Diversity-multiplexing tradeoff in multiple access channels", *IEEE Transactions on Information Theory*, Vol. 50, No. 9, pp. 1859-1874, Sept. 2004.

[5] Robert W. Heath Jr. and David J. Love, "Multi-mode antenna selection for spatial multiplexing systems with linear receivers", *Proc. Allerton Conf. on Signals and Systems*, Monticello, IL, Oct. 2003.

[6] D. A. Gore, R. W. Heath Jr. and A. J. Paulraj, "Statistical antenna selection for spatial multiplexing systems", *IEEE Int. Conf. On Communications*, vol 1. pp 450-454, 2002.

[7] J. R. Barry, E. A. Lee, D. G. Messerschmitt, *Digital Communication*, Third Edition, *Kluwer Academic Publishers*, 2004.

[8] A. Kannan, B.Varadarajan and J. R. Barry, "Joint optimization of transmitter rate and BLAST ordering to minimize outage probability", in *Proc. IEEE Wireless Communications and Networking Conf.*, New Orleans, vol 1, pp. 550-555, May 2005.