Noise-Predictive Minimum-Mean-Squared-Error Decision-Feedback Detection for Multiple-Input Multiple-Output Channels

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Abstract

The decision-feedback detector is a nonlinear detection strategy for multiple-input multiple-output channels. It can significantly outperform a linear detector, provided that the inputs are detected using the so-called BLAST input ordering, which approximately minimizes the joint error probability. The performance of the decision-feedback (DF) detector can be further enhanced by using the minimum-mean-squared-error (MMSE) criterion. The MMSE-DF detector may be implemented as the cascade of an MMSE linear detector, which mitigates interference at the expense of correlating the noise, followed by a noise predictor, which exploits the correlation in the noise to reduce its variance. We show that the noise-predictive implementation facilitates a low-complexity algorithm for determining the MMSE-BLAST input ordering. As a result, the ordered MMSE noise-predictive DF detector requires fewer computations than previously reported BLAST ordered DF detection implementations. We also propose an approximation of the ordered MMSE-DF detector that can outperform the ordered zero-forcing DF detector. Finally, we show by simulation that for some multiple-input multiple-output systems the ordered MMSE-DF detector has no performance benefit over the ordered zero-forcing DF

1 Introduction

In multiple-input multiple-output communications, the detector that minimizes the joint error probability is the maximum-likelihood (ML) detector. Unfortunately, the complexity of the ML detector increases exponentially with the number of channel inputs, and is often prohibitively complex. The decision-feedback (DF) detector trades performance for reduced complexity; it is outperformed by the ML detector, but requires fewer computations. The DF detector emerges as a popular detection strategy in a wide range of multiple-input multiple-output (MIMO) applications. For example, in the context of a wireless point-to-point link with antenna arrays at both the transmitter and receiver the DF detector is known as the BLAST nulling and cancelling detector [1]; in CDMA applications it is known as the DF multiuser detector [2]; and in packet transmission it is known as a generalized DFE [3].

The performance of the DF detector is strongly impacted by the order in which the inputs are detected. Unfortunately, optimizing the detection order is a difficult problem that often dominates the overall receiver complexity. It is common and practical to define as optimal the detection order that maximizes the worst case post-detection SNR. This ordering, known as the BLAST ordering, approximately minimizes the joint error probability of the DF detector. The BLAST ordering algorithm of [4] uses repeated computations of a matrix pseudoinverse to find this ordering with a complexity of $O(N^4)$, where N is the number of channel inputs. Three $O(N^3)$ reduced-complexity ordering algorithms have also been proposed: the zero-forcing (ZF) noise-predictive algorithm of [5], the ZF decorrelating algorithm of [6], and the minimum-mean-squared-error (MMSE) square-root algorithm of [7]. Other algorithms settle for a suboptimal ordering in order to reduce complexity [8−9].

In [2] an architecture for implementing the DF detector based on linear prediction of the noise was presented. The noise-predictive DF detector consists of a linear detector followed by a linear prediction mechanism that reduces the noise variance before making a decision. In [5], we showed that this noise-predictive DF detector facilitates a low-complexity algorithm for determining the BLAST ordering. The resulting ordered zero-forcing noise-predictive DF (O-ZF-NP-DF) detector is mathematically equivalent to the ordered ZF-DF detectors of [4] and [6], but is least complex BLAST ordered ZF-DF detector reported so far.

In this paper, we propose the BLAST ordered minimum-mean-squared-error noise-predictive DF (O-MMSE-NP-DF) detector, which is the MMSE version of the O-ZF-NP-DF detector. Through simulations we find that the performance difference between the BLAST ordered MMSE-DF detector and the BLAST ordered ZF-DF detector varies from enormous to zero, depending on the system parameters. In addition, we propose an approximate BLAST ordered MMSE-NP-DF (A-MMSE-NP-DF) detector that requires less a priori knowledge than the O-MMSE-NP-DF detector, while maintaining most of the performance advantage over the O-ZF-NP-DF detector.

We begin by establishing the channel model and describing the MMSE noise-predictive DF (MMSE-NP-DF) detector in Section 2. Section 3 describes a low-complexity implementation of the optimally ordered MMSE-NP-DF detector, and Section 4 analyzes its complexity. Section 5 proposes the A-MMSE-NP-DF detector. Finally, the performance of the two proposed detectors and the O-ZF-NP-DF detector are compared in Section 6.

2 MMSE Noise-Predictive DF Detection

In this section we derive an implementation of the MMSE-DF detector based upon linear prediction. This detector is an improvement over the ZF-DF detector because it strikes an optimal balance between interference surpression and noise enhancement [10]. Linear prediction is used to estimate the error after the MMSE linear detector. This estimate is subtracted from the MMSE linear detector output to reduce its variance. The error being estimated consists of colored additive Gaussian noise and residual intersymbol interference (ISI), however, throughout this paper we will refer to this error simply as noise. Fig. 1 shows the block diagram of the *minimum-mean-squared-error noise-predictive* DF (MMSE-NP-DF) detector which employs this linear-prediction strategy; the filters $\boldsymbol{c}_i$ and $p_{i,j}$ will be defined shortly. The notion of ordering (the permutation block) is neglected momentarily by assuming an identity permutation.

In this paper we consider the following model of a MIMO channel with N inputs $\boldsymbol{a} = [\, a_1, \ldots a_N]^T$ and M outputs $\boldsymbol{r} = [r_1, \ldots r_M]^T$:

$$\boldsymbol{r} = \mathbf{H}\boldsymbol{a} + \boldsymbol{w}\,, \tag{1}$$

where $\mathbf{H}$ is a complex $M \times N$ channel matrix and where $\boldsymbol{w} = [w_1, \ldots w_M]^T$ is additive noise. We assume that the columns of $\mathbf{H}$ are linearly independent, which implies that there are at least as many outputs as inputs, $M \geq N$. We assume that the noise components are uncorrelated with complex variance σ^2, so that $\mathrm{E}[\boldsymbol{w}\boldsymbol{w}^*] = \sigma^2\mathbf{I}$, where $\boldsymbol{w}^*$ denotes the conjugate transpose of $\boldsymbol{w}$. Further, we assume that the inputs are chosen from the same unit-energy alphabet $\mathcal{A}$ and are

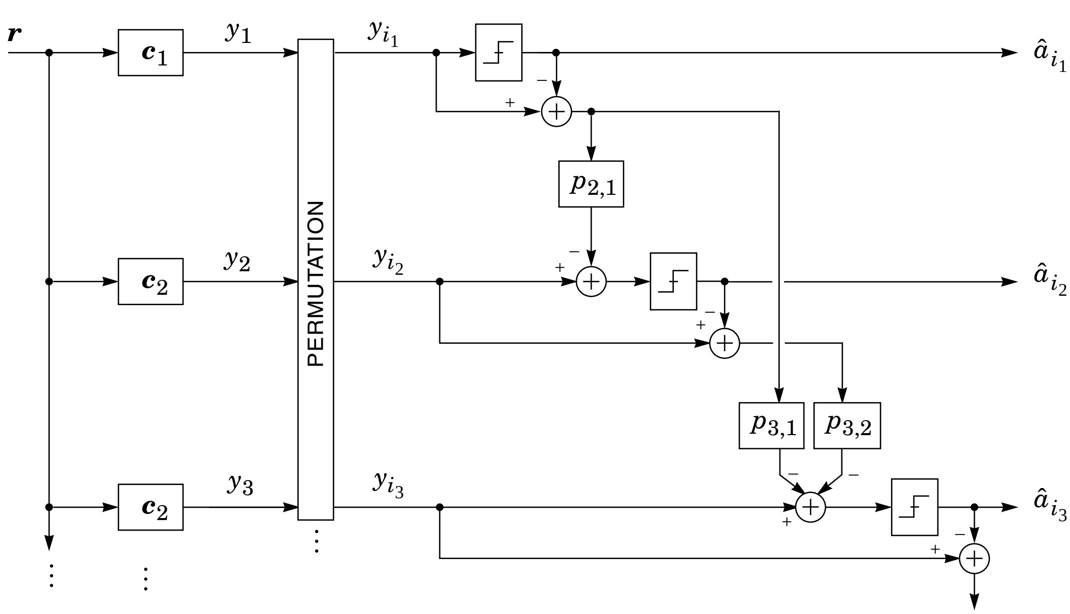

Fig. 1. The noise-predictive DF detector.

uncorrelated, so that $\mathrm{E}[\boldsymbol{a}\boldsymbol{a}^*] = \mathbf{I}$.

As depicted in Fig. 1, the noise-predictive MMSE-DF detector begins with an MMSE *linear* detector [11], which computes $\boldsymbol{y} = \mathbf{C}\boldsymbol{r}$, where $\mathbf{C} = [\boldsymbol{c}_1, \boldsymbol{c}_2, \dots \boldsymbol{c}_N]^{\mathrm{T}}$ is defined as:

$$\mathbf{C} = \tilde{\mathbf{R}}^{-1}\mathbf{H}^*, \tag{2}$$

where:

$$\tilde{\mathbf{R}} = \mathbf{H}^*\mathbf{H} + \sigma^2\mathbf{I}. \tag{3}$$

This choice for $\mathbf{C}$ minimizes the total MSE $\mathrm{E}[\|\varepsilon\|^2]$, where $\varepsilon = \mathbf{C}\boldsymbol{r} - \boldsymbol{a}$ is the vector of errors after the linear filter. From (1), the error at the output of this filter is:

$$\begin{aligned}\varepsilon &= \tilde{\mathbf{R}}^{-1}(\mathbf{H}^*\mathbf{H} + \sigma^2\mathbf{I} - \sigma^2\mathbf{I})\,\boldsymbol{a} + \mathbf{C}\boldsymbol{w} - \boldsymbol{a} \\ &= -\,\sigma^2\tilde{\mathbf{R}}^{-1}\boldsymbol{a} + \mathbf{C}\boldsymbol{w}.\end{aligned} \tag{4}$$

This error constitutes the “noise” being predicted. The linear predictor must minimize the variance of this noise by exploiting its correlation, defined by the autocorrelation matrix $\mathbf{R}_{\varepsilon\varepsilon} = \mathrm{E}[\,\varepsilon\varepsilon^*]$:

$$\mathbf{R}_{\varepsilon\varepsilon} = \sigma^4\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{R}}^{-1} + \sigma^2\mathbf{C}\mathbf{C}^*. \tag{5}$$

The correlation of the noise can be exploited using linear prediction to reduce its variance. If the first $i-1$ elements of the error vector were known, we could form an estimate $\hat{\varepsilon}_i$ of the i-th element ε_i and subtract this estimate from y_i to reduce its variance. Specifically, given $\{\varepsilon_1, \dots \varepsilon_{i-1}\}$, a linear predictor estimates ε_i according to:

$$\hat{\varepsilon}_i = \sum_{j<i} p_{i,j}\varepsilon_j\,, \tag{6}$$

or equivalently $\hat{\varepsilon} = \mathbf{P}\varepsilon$, where $\mathbf{P}$ is a strictly lower triangular *prediction filter* whose element at the i-th row and j-th column is $p_{i,j}$. This process is complicated by the fact that the receiver does not have access to ε_i directly, but rather to the sum $y_i = a_i + \varepsilon_i$. However, as shown in Fig. 1, the decision about a_i can be subtracted from y_i to yield ε_i as long as the decision is correct. The MMSE-NP-DF detector of Fig. 1 can be summarized succinctly by the following recursion:

$$\hat{a}_i = \mathrm{dec}\Big\{\; y_i - \sum_{j<i} p_{i,j}\,(y_j - \hat{a}_j)\Big\}\,, \tag{7}$$

where $\mathrm{dec}\{x\}$ denotes the quantization of x to the nearest constellation point in $\mathcal{A}$.

We now derive the linear prediction filter $\mathbf{P}$ that minimizes the total MSE $\mathrm{E}[\,\|\,\boldsymbol{e}\,\|^2\,]$, where $\boldsymbol{e} = (\mathbf{I} - \mathbf{P})\varepsilon$ denotes the error of the linear prediction. First we reduce $\mathbf{R}_{\varepsilon\varepsilon}$ as follows:

$$\begin{aligned}\mathbf{R}_{\varepsilon\varepsilon} &= \sigma^4\tilde{\mathbf{R}}^{-2} + \sigma^2\tilde{\mathbf{R}}^{-1}(\mathbf{H}^*\mathbf{H} + \sigma^2\mathbf{I} - \sigma^2\mathbf{I})\tilde{\mathbf{R}}^{-1} \\ &= \sigma^2\tilde{\mathbf{R}}^{-1}.\end{aligned} \tag{8}$$

Since $\tilde{\mathbf{R}}$ is Hermitian and positive definite, $\mathbf{R}_{\varepsilon\varepsilon}$ has the following Cholesky factorization:

$$\mathbf{R}_{\varepsilon\varepsilon} = \sigma^2\mathbf{M}^{-1}\mathbf{D}^{-2}\mathbf{M}^{-*}, \tag{9}$$

where $\mathbf{M}^{-1}$ is a lower triangular matrix with diagonal elements of one, and where $\mathbf{D}^{-2}$ is a real diagonal matrix with positive diagonal elements. The total MSE is related to $\mathbf{R}_{\varepsilon\varepsilon}$ by:

$$\mathrm{E}[\,\|\,\boldsymbol{e}\,\|^2] = \mathrm{trace}\{\,(\,\mathbf{I} - \mathbf{P}\,)\,\mathbf{R}_{\varepsilon\varepsilon}\,(\,\mathbf{I} - \mathbf{P}^*)\,\}. \tag{10}$$

It is easy to show [12] that the best choice for $(\mathbf{I} - \mathbf{P})$ cancels $\mathbf{M}^{-1}$:

$$\mathbf{P} = \mathbf{I} - \mathbf{M}. \tag{11}$$

Therefore, the effective front-end filter of the noise-predictive MMSE-DF (NP-MMSE-DF) detector is given by:

$$\begin{aligned}(\mathbf{I} - \mathbf{P})\mathbf{C} &= \tilde{\mathbf{M}}\,\tilde{\mathbf{R}}^{-1}\mathbf{H}^* \\ &= \tilde{\mathbf{D}}^{-2}\tilde{\mathbf{M}}^{-*}\mathbf{H}^*.\end{aligned} \tag{12}$$

This forward filter is identical to the forward filter of the conventional MMSE-DF detector defined in [13]. With this forward filter, the corresponding feedback filter is $-\mathbf{P}$, which is identical to the feedback filter of the conventional MMSE-DF detector defined in [13]. Therefore, we conclude that the NP-MMSE-DF detector is equivalent to the conventional MMSE-DF detector.

3 Ordered MMSE Noise-Predictive DF Detector

To implement the MMSE-NP-DF detector of Fig. 1, the receiver must first determine the linear detection filter **C**, the symbol detection order, and the linear prediction filter **P**. In this section we show how to calculate both the detection order and the prediction filter assuming that **C** and $\sigma\tilde{\mathbf{R}}^{-1}$ are already known.

We first describe a low-complexity algorithm for finding the best detection order. As implied by Fig. 1, this sorting algorithm occurs after $\boldsymbol{y} = \mathbf{C}\boldsymbol{r}$ has been calculated. The permutation in the block diagram of Fig. 1 gives the detector the flexibility to use any symbol detection order, but we consider only the BLAST ordering. Let i_k denote the index of the k-th symbol to be detected, so that $\{i_1, i_2, \ldots i_N\}$ is a permutation of $\{1, 2, \ldots N\}$.

The noise-predictive view of the DF detector leads to a simple algorithm for finding the ordering that is optimal with respect to the MSE of each symbol when symbol decisions are assumed to be correct. As proven in [1], this BLAST ordering can be found in a recursive fashion by choosing each i_k so as to maximize the post-detection SNR of the k-th symbol, or equivalently minimize its MSE. The MSE for the first detected symbol is equal to $[\mathbf{R}_{\varepsilon\varepsilon}]_{i_1,i_1}$, for convenience we define a matrix $\mathbf{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots \boldsymbol{b}_N]^T$ such that $\mathbf{R}_{\varepsilon\varepsilon} = \sigma^2\mathbf{B}\mathbf{B}^*$:

$$\mathbf{B} = \left[\begin{array}{cc} \mathbf{C} & \sigma\tilde{\mathbf{R}}^{-1} \end{array}\right], \tag{13}$$

In terms of this new matrix, the MSE of the first detected symbol is $\sigma^2\|\boldsymbol{b}_{i_1}\|^2$. Therefore, we choose the symbol with minimum MSE by:

$$i_1 = \underset{j \in \{1, \ldots N\}}{\operatorname{argmin}} \|\boldsymbol{b}_j\|^2. \tag{14}$$

In other words, the row of **B** with the smallest norm determines which symbol to detect first.

Once i_1 is chosen, and assuming $\hat{a}_{i_1}$ is correct, the MSE for the second symbol is:

$$\begin{aligned} &\mathrm{E}[\,|\,\varepsilon_{i_2} - \hat{\varepsilon}_{i_2}|^2] = \mathrm{E}[\,|\,\varepsilon_{i_2} - p_{2,1}\varepsilon_{i_1}|^2] \\ &= \mathrm{E}[\,|\,\boldsymbol{c}_{i_2}\boldsymbol{w} - \sigma^2\tilde{\boldsymbol{r}}_{i_2}\boldsymbol{a} - p_{2,1}\boldsymbol{c}_{i_1}\boldsymbol{w} + \sigma^2 p_{2,1}\tilde{\boldsymbol{r}}_{i_1}\boldsymbol{a}|^2] \\ &= \sigma^2\|\boldsymbol{c}_{i_2} - p_{2,1}\boldsymbol{c}_{i_1}\|^2 + \sigma^2\|\tilde{\boldsymbol{r}}_{i_2} - p_{2,1}\tilde{\boldsymbol{r}}_{i_1}\|^2 \\ &= \sigma^2\|\boldsymbol{b}_{i_2} - p_{2,1}\boldsymbol{b}_{i_1}\|^2, \end{aligned} \tag{15}$$

where $\tilde{\boldsymbol{r}}_j$ is the j-th row of $\tilde{\mathbf{R}}^{-1}$. The last line of (15) results from straight forward algebraic manipulation. When the prediction coefficient $p_{2,1}$ is chosen to minimize the MSE, the term $p_{2,1}\boldsymbol{b}_{i_1}$ reduces to the projection of $\boldsymbol{b}_{i_2}$ onto the subspace spanned by $\boldsymbol{b}_{i_1}$, which we denote as $\hat{\boldsymbol{b}}_{i_2}$. Hence, the optimal i_2 satisfies:

$$i_2 = \underset{j \neq i_1}{\operatorname{argmin}} \|\boldsymbol{b}_j - \hat{\boldsymbol{b}}_j\|^2. \tag{16}$$

Repeating the above procedure recursively leads to the following simple and succinct description of an optimal ordering algorithm:

$$i_k = \underset{j \notin \{i_1, \ldots i_{k-1}\}}{\operatorname{argmin}} \|\boldsymbol{b}_j - \hat{\boldsymbol{b}}_j\|^2, \tag{17}$$

where $\hat{\boldsymbol{b}}_j$ is the projection of $\boldsymbol{b}_j$ onto the span of $\{\boldsymbol{b}_{i_1}, \ldots \boldsymbol{b}_{i_{j-1}}\}$. This is a key result that is the basis of this new ordered DF detector. In words, *finding the BLAST ordering amounts to choosing the rows of the augmented matrix* **B**, *where the best choice for the k-th row is the unchosen row that is closest to the subspace spanned by the rows already chosen*.

A computationally efficient implementation of the sorting algorithm of (17) is given in Fig. 2. It is based on an adaptation of the Householder QR decomposition [14]. The algorithm accepts the matrix **B** as an input, and it produces two outputs: the BLAST ordering $\{i_1, \ldots i_N\}$, and an intermediate matrix **F** that can be used to determine the linear prediction filter **P**. The Householder procedure of the sorting algorithm operates on the rows of **B**, $\{\boldsymbol{b}_{i_1}, \ldots \boldsymbol{b}_{i_k}\}$. During the first iteration ($k = 1$), line (A-4) chooses the row nearest to the null space, then line (A-8) removes the portions of the remaining rows of **B** that are parallel to $\boldsymbol{b}_{i_1}$. When this is done the remaining row elements of the first column of **B** are zero, so we no longer need them. Therefore, the effective dimensions of **B** are $(N - k + 1)\times(M - k + 1)$ during the k-th iteration. In the next iteration ($k = 2$) each of the candidate rows of **B** is orthogonal to $\boldsymbol{b}_{i_1}$. Consequently, the remaining row closest to the subspace spanned by the previously chosen row is the row with minimum norm. As before, line (A-8) ensures that the remaining rows of **B** are orthogonal to $\boldsymbol{b}_{i_2}$. The iterations continue until $k = N - 1$, when the BLAST ordering is determined.

Function A. Input: **B**, Output: $\{i_1, i_2, \dots i_N\}$, and **F**

(A-1) $\mathcal{U} = \{1, 2, \dots N\}$ = the set of unchosen rows.

(A-2) $E_j = \|\boldsymbol{b}_j\|^2, j \in \mathcal{U}$.

(A-3) for $k = 1$ to N–1,

(A-4) $i_k = \underset{j \in \mathcal{U}}{\arg\min} \; E_j$

(A-5) $\mathcal{U} = \mathcal{U} - i_k$; remove chosen row from $\mathcal{U}$

(A-6) $\boldsymbol{v} = \boldsymbol{b}_{i_k}$; $d = \sqrt{E_{i_k}}$; $v_1 = v_1 + dv_1 / |v_1|$

(A-7) $\boldsymbol{x} = \mathbf{B}\boldsymbol{v}^*$

(A-8) $\mathbf{B} = \mathbf{B} - \boldsymbol{x}\boldsymbol{v} / x_{i_k}$

(A-9) $E_j = E_j - |b_{j,1}|^2, j \in \mathcal{U}$

(A-10) Delete first column from **B**; store it as k-th column of **F**

(A-11) end

(A-12) $i_N = \mathcal{U}$

Fig. 2. The noise-predictive sorting algorithm using Householder orthogonalization.

Given the output **F** of the sorting algorithm just described, calculating the linear prediction filter **P** is straightforward. To avoid confusion, let Π denote an $N \times N$ permutation matrix whose j-th column is the i_j-th column of the identity matrix. In Fig. 1, the ordering is accounted for by permuting the rows of the linear detector, so that the cascade of the linear filter and the permutation leads to an effective front-end filter of:

$$\mathbf{C}' = \Pi^* \mathbf{C}. \tag{18}$$

This order also affects the autocorrelation matrix:

$$\mathbf{R}_{\varepsilon\varepsilon}' = \Pi^* \mathbf{R}_{\varepsilon\varepsilon} \Pi \tag{19}$$

When performed on $\mathbf{R}_{\varepsilon\varepsilon}'$, the decomposition of (9) yields the matrices $\mathbf{D}'$ and $\mathbf{M}'$. From (11), and in these new terms, the ordered prediction filter is:

$$\mathbf{P} = \mathbf{I} - \mathbf{M}'. \tag{20}$$

The output matrix **F** is closely related to $(\mathbf{M}')^{-1}$. To calculate $(\mathbf{M}')^{-1}$ from **F**, divide the j-th column of **F** by the element $f_{i_j,j}$, then permute the rows such that the result is lower triangular. Next, simply invert $(\mathbf{M}')^{-1}$ using back substitution and use (20) to get **P**. Fig. 3 gives the pseudocode for calculating **P** from **F**.

In [5] we proposed an algorithm to implement the O-ZF-NP-DF detector that is functinally equivalent to Function A. Here, we have shown that the O-ZF-NP-DF detector is a special case of the O-MMSE-NP-DF detector, where the final N columns of the augmented matrix **B** are zero and are not used. Therefore, the sorting algorithm for both detectors may be implemented using Function A, by adjusting the input.

In summary, the BLAST ordered MMSE noise-predictive DF (O-MMSE-NP-DF) detector implementation has four steps. First, the MMSE linear detection filter is applied to the received vector. Next, the optimal symbol order is calculated from the MMSE linear detection filter using the Householder sorting algorithm. Then, the linear prediction filter is calculated from the output of the sorting algorithm. After these calculations the detector can be implemented using (7), as illustrated in Fig. 1.

Function B. Input: **F**, Output: **P**

(B-1) $\mathbf{P} = \mathbf{0}_{N \times N}$

(B-2) for $k = 2$ to N,

(B-3) for $j = (k-1)$ downto 1,

(B-4) $t_{k,j} = f_{i_k,j} / f_{i_j,j}$

(B-5) $p_{k,j} = t_{k,j} - \sum_{m=j+1}^{k-1} t_{k,m} \, p_{m,j}$

(B-6) end

(B-7) end

Fig. 3. Calculation of the prediction filter **P** from the output of the noise-predictive sorting algorithm **F**. In the end, $\mathbf{T} = (\mathbf{M}')^{-1}$.

4 Complexity Analysis

We now give a complete description of the complexity of the O-MMSE-NP-DF detector. In this section we continue to assume that the MMSE linear detection filter **C**, and $\sigma\tilde{\mathbf{R}}^{-1}$ have been perfectly estimated before detection begins. In practice linear detectors can be estimated using adaptive techniques [11, p. 306], [15]. The complexity of these estimations is not counted in the complexity of the detector.

Several notes are appropriate regarding the complexity analysis. First, we measure

Table 1: O-MMSE-NP-DF detector complexity

	Number of Operations
(A-2)	$2MN + N^2 - N$
(A-6)	$5N - 5$
(A-7)	$MN^2 + 2N^3/3 + MN + N^2/2 - 2M - N/6 - 1$
(A-8)	$MN^2 + 2N^3/3 + 2MN + 3N^2/2 - 3M + 5N/6 - 3$
(A-9)	$N^2 - N - 2$
Function B	$N^3/3 - N^2/2 + N/6$
Eq. (7)	$(2MN + N^2 - N - 1)L$
Total Complexity	$2MN^2 + 5N^3/3 + 5MN + 9N^2/2 - 5M + 23N/6 - 11 + (2MN + N^2 - N - 1)L$

complexity as the total number of complex additions, subtractions, multiplications, divisions, and square-roots required each time the detector is calculated. Second, in the context of DF detectors, MIMO systems with N and M as low as two are of interest. As a result, lower-order complexity terms are not always negligible. Finally, the complexity of the quantization operation is ignored since it is the same for all DF detectors, and it depends on the symbol constellation.

The complexity analysis begins with the proposed ordered MMSE noise-predictive DF (O-MMSE-NP-DF) detector. The line-by-line complexity of Function A, the total complexity of Function B, and the total complexity of the detection process (after $\{\mathbf{c}_i\}$ and $\{p_{i,j}\}$ are known) are given in Table 1, where we assume that the detection filters are recalculated every L symbol periods. The complexity of the O-MMSE-NP-DF detector is approximately 1.9 times as complex as the O-ZF-NP-DF detector [5], and roughly 15% *less* complex than the ZF-MDDF detector [6].

5 Approximate MMSE-DF Detection

In this section we show the performance of the MMSE-NP-DF detector when it has perfect knowledge of $\mathbf{C}$ and σ, but not of $\sigma\tilde{\mathbf{R}}^{-1}$. We show through simulation that this lack of information is not detrimental to the performance of the O-MMSE-NP-DF detector. In fact, an approximate ordered MMSE-NP-DF (A-MMSE-NP-DF) detector can significantly outperform the O-ZF-NP-DF detector in some cases. The A-MMSE-NP-DF detector is also only fractionally more complex than the O-ZF-NP-DF detector.

The matrix $\sigma\tilde{\mathbf{R}}^{-1}$ that must be estimated is proportional to the autocorrelation matrix of the error following the MMSE linear detector, $\mathbf{R}_{\varepsilon\varepsilon}$. It can be written as:

$$\sigma\tilde{\mathbf{R}}^{-1} = \sigma\left(\mathbf{CC}^* + \sigma^2\tilde{\mathbf{R}}^{-2}\right). \tag{21}$$

The most significant elements of $\sigma\tilde{\mathbf{R}}^{-1}$ come from the diagonal elements of $\mathbf{CC}^*$. Therefore we propose the following estimate:

$$\sigma\tilde{\mathbf{R}}^{-1} \approx \sigma\,\mathrm{diag}\{\mathbf{CC}^*\}, \tag{22}$$

where $\mathrm{diag}\{\mathbf{CC}^*\}$ is a diagonal matrix whose diagonal elements are the squared row norms of $\mathbf{C}$.

Using this estimation technique the last N columns of the augmented matrix $\mathbf{B}$ will have many zeros. If this fact is exploited, the complexity of the A-MMSE-NP-DF detector can be significantly less than the complexity of the O-MMSE-NP-DF detector itself.

6 Simulation Results

In this section we compare the performance of the three detectors: the A-MMSE-NP-DF detector (given $\mathbf{C}$ and σ), the O-MMSE-NP-DF detector (given $\mathbf{B}$), and the O-ZF-NP-DF detector (given the channel pseudoinverse = $\mathbf{C}_{ZF}$). We simulate various $M \times N$ MIMO systems where the detection filters are recalculated for every symbol period ($L = 1$). For each simulated channel the elements of $\mathbf{H}$ are statistically independent Gaussian random variables whose variance is normalized to one. In this way, we simulated 10^5 Rayleigh fading channels. We use SNR per symbol, per receive antenna to quantify the amount of transmit power used by the system: $\mathrm{SNR} = 1/\sigma^2$.

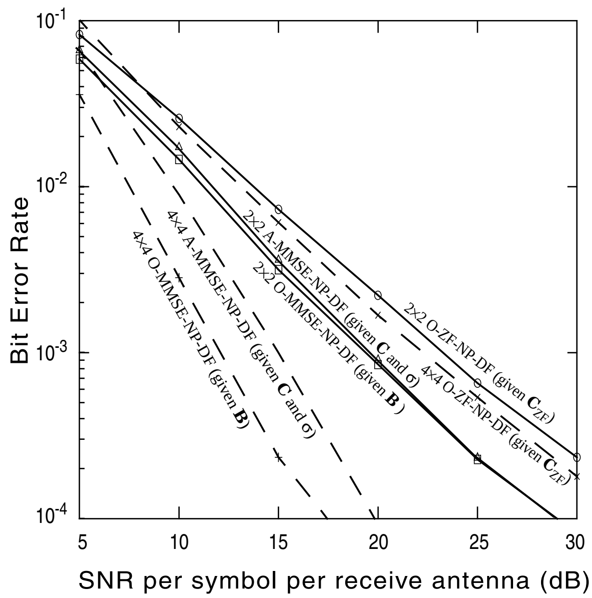

Fig. 4. Bit error rate of the A-MMSE-NP-DF detector (given $\mathbf{C}$ and σ), the O-MMSE-NP-DF detector (given $\mathbf{B}$), and the O-ZF-NP-DF detector (given $\mathbf{C}_{ZF}$) using 4-QAM, in various $M \times N$ MIMO systems with Rayleigh fading.

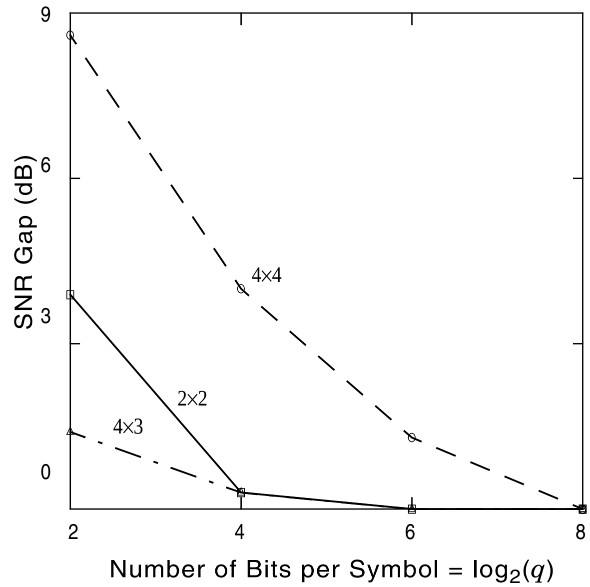

Fig. 5. The SNR improvement at BER = 10^{-3} of the O-MMSE-NP-DF detector (given $\mathbf{B}$) over the O-ZF-NP-DF detector (given $\mathbf{C}$) versus the number of bits per symbol for $M \times N$ systems using q-QAM in Rayleigh fading.

To highlight the effect of the approximation proposed in Section 5, we first consider two systems that have $M = N = 2$, and $M = N = 4$, respectively. Each system uses a 4-QAM constellation. Fig. 4 shows the bit error rate (BER) for these MIMO systems of the three detectors under consideration. To compare the performance of the detectors we consider the SNR they require to reach a BER = 10^{-3}. In the 4×4 MIMO system, using (22) to estimate $\sigma\tilde{\mathbf{R}}^{-1}$ causes an SNR penalty of approximately 2.9 dB for the A-MMSE-NP-DF detector. Even with this penalty the A-MMSE-NP-DF detector still outperforms the O-ZF-NP-DF detector by 7.3 dB! For the 2×2 MIMO system, using the estimate of (22) causes only a 0.3 dB penalty, and the A-MMSE-NP-DF detector (given $\mathbf{C}$ and σ) outperforms the O-ZF-NP-DF detector (given $\mathbf{C}_{ZF}$) by 3.7 dB.

The O-MMSE-NP-DF detector does not maintain this performance improvement over the O-ZF-NP-DF detector for all MIMO systems. The performance gap closes quickly as the minimum distance of the symbol constellation $\mathcal{A}$ decreases. Fig. 5 shows this performance gap versus the number of bits per symbol. We see that the performance gap depends on the number of bits per symbol and the dimensions of the MIMO system.

The O-MMSE-NP-DF detector, the A-MMSE-NP-DF detector, and the O-ZF-NP-DF detector each have the same diversity order. When $M = N$ these DF detectors have an overall diversity of one, and their performance is bounded by the first symbol detected [16]. The performance difference between these DF detectors is less dramatic for MIMO systems with increased diversity. This explains why the SNR improvement for the 4×3 system in Fig. 5 is much smaller than for the 4×4 MIMO system. Likewise, the penalty incurred as a result of using the estimation of (22) is smaller when $M > N$.

7 Conclusion

The noise-predictive DF detector consists of a linear detector and a linear prediction mechanism that reduces noise variance. We showed that the noise-predictive view of the MMSE-DF detector leads to a simple and computationally efficient way of finding the BLAST detection ordering. A key advantage of the noise-predictive detector is that it can begin with knowledge of the MMSE linear detection filter and the autocorrelation matrix of the noise instead of knowledge of the channel itself. In fact, the ordered MMSE noise-predictive decision-feedback (O-MMSE-NP-DF) detector has no need for the channel matrix at all. We also showed through simulation that an approximate ordered MMSE-NP-DF detector can outperform the O-ZF-NP-DF detector given only the MMSE linear detection filter and the noise variance. Finally, we showed with simulations that in some cases there is no performance benefit gained by using the O-MMSE-NP-DF detector instead of the O-ZF-NP-DF detector.

8 References

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