Embedded Orthogonal Space-Time Codes for High Rate and Low Decoding Complexity

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***Abstract* — We propose a new family of high-rate space-time block codes called *embedded orthogonal space-time (EOS)* codes. The family is parameterized by the number of transmit antennas, which can be any positive integer, and by the rate, which can be as high as half the number of transmit antennas. The proposed codes are based on a new concept called *embedding*, whereby information symbols of a traditional space-time code are replaced by codewords from a second space-time code. The EOS codes use orthogonal designs as this second code, which induces orthogonality in an effective channel matrix and leads to reduced-complexity decoding. The EOS codes have lower decoding complexity than previously reported space-time codes for any number of transmit antennas, and for any rate. Furthermore, simulation results show that the EOS codes outperform previous constructions for certain number of antennas and certain rates, when performance is measured by error probability in quasistatic Rayleigh fading.**

***Index Terms* — transmit diversity, space-time coding.**

I. Introduction

The choice of a space-time code depends strongly on the size of the receiver antenna array relative to the size of the transmitter antenna array. At one extreme, when there are more receive antennas than transmit antennas, good candidates include the threaded algebraic space-time (TAST) [1] codes and the perfect space-time codes [2][3], both of which achieve maximum diversity order and full transmission rate. At the other extreme, when there is only a single receive antenna, good candidates include the Alamouti code [4], the quasiorthogonal space-time block codes [5] and the semi-orthogonal algebraic space-time (SAST) block codes [6]. This paper examines the problem of designing so-called *high-rate* codes whose rate is larger than one but less than full, appropriate in between the two extremes.

There are two main approaches to the construction of high-rate codes. The first approach is *puncturing*, in which a maximal-rate space-time code is punctured to obtain the high-rate code. For example, TAST and perfect space-time codes for four transmit antennas of rate $R \in \{1, 2, 3, 4\}$ are easily obtained by puncturing $4 - R$ threads of the rate-4 code. The second approach is *multiplexing*, in which lower rate space-time codes are multiplexed or combined to form the high-rate code. For example, the rate-two space-time code in [7] for the four-input two-output channel is constructed as a combination of two quasiorthogonal blocks.

In this paper, we introduce the concept of *embedding*. a third approach to the construction of high-rate codes in which complex orthogonal designs [8][9] assume the role of complex symbols in the encoding process.

Based on the embedding concept we propose a new family of codes called embedded-orthogonal space-time (EOS) codes, defined for an arbitrary number of antennas and for any rate up to half the number of antennas. When compared to previously reported space-time codes, the proposed family of codes is lower in decoding complexity. Furthermore, the proposed family of codes is better performing for certain number of transmit antennas and certain rates.

The remainder of the paper is organized as follows. In Section II, we present the system model and review the construction of the TAST codes. In Section III, we present the proposed family of embedded orthogonal space-time codes and describe their decoder. In Section IV, we present numerical results. In Section V, we conclude the paper.

II. System Model and Background

A. System Model

A space-time code with M transmit antennas transmitting P complex information symbols over T symbol periods can be represented by a $T \times M$ matrix:

$$\mathbf{C} = \begin{bmatrix} c_1[1] & c_2[1] & \cdots & c_M[1] \\ c_1[2] & c_2[2] & & c_M[2] \\ \vdots & \vdots & & \vdots \\ c_1[T] & c_2[T] & \cdots & c_M[T] \end{bmatrix}, \quad (1)$$

where $c_m[t]$ denotes the symbol transmitted from antenna $m \in \{1, \dots M\}$ at time $t \in \{1, \dots T\}$. The rate of the space-time code is $R = P/T$ symbols per signaling interval. If the channel remains constant over the duration of the codeword, the received signal $y_n[t]$ at receive antenna $n \in \{1, \dots N\}$ at time t is:

$$y_n[t] = \sum_{m=1}^{M} h_{m,n} c_m[t] + w_n[t], \quad (2)$$

where $w_n[t]$ is the complex AWGN at receive antenna n at time t, and $h_{m,n}$ is the channel coefficient between the m-th transmit antenna and n-th receive antenna.

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B. Review of the TAST Code

We begin by reviewing the TAST code [1], whose rate is nominally M, but with puncturing can achieve any rate in the range $R \in \{1, \ldots M\}$. This code encodes RM information symbols $\{a_{\ell,m}\}$, drawn from a q-ary QAM alphabet, that are organized into R *threads*, where $\mathbf{a}_\ell = [a_{\ell,1}, a_{\ell,2}, \ldots a_{\ell,M}]^T$ denotes the information symbols for the ℓ-th *thread*, $\ell \in \{1, \ldots R\}$. The rate-R TAST code can be written as [1]:

$$\mathbf{C}_{\text{TAST}} = \sum_{\ell=1}^{R} \text{diag}(\mathbf{u}_\ell)\mathbf{J}^{\ell-1}, \tag{3}$$

where

- $\mathbf{u}_\ell = \mathbf{G}\mathbf{a}_\ell = [u_{\ell,1}, u_{\ell,2}, \ldots u_{\ell,M}]^T$
- $\mathbf{G}$ is an $M \times M$ unitary *rotation* or *generator* matrix
- $\text{diag}(\mathbf{u})$ is the diagonal matrix with $\mathbf{u}$ on the diagonal
- $\mathbf{J} = [\phi\mathbf{e}_M, \mathbf{e}_1, \ldots, \mathbf{e}_{M-1}]$
- $\mathbf{e}_m$ is the m-th column of the $M \times M$ identity matrix
- ϕ is a unit-magnitude complex number.

The design of a TAST code is a two-step design process. The first step is to choose an algebraic rotation matrix $\mathbf{G}$. The second step is to choose ϕ to ensure full diversity. The two-step design process simplifies the design problem, since the algebraic rotation matrices $\mathbf{G}$ that maximize the coding gain of an encoded thread have been thoroughly investigated in the literature [10].

III. Embedded Orthogonal Space-Time Codes

In this section we describe the proposed EOS codes and compare to the TAST codes of the previous section.

A. Encoding

The design of the proposed embedded orthogonal space-time block codes is a three-step process, based on the choice of three parameters:

- a rate R_1 complex orthogonal design $\mathcal{G}$ of size $T_1 \times M_1$, called the *embedded* code, designed for M_1 antennas;
- an algebraic rotation matrix $\mathbf{G}$;
- a complex number ϕ.

Before we discuss the construction of EOS codes, we discuss the choice of these parameters. First, we embed orthogonal codes because they induce orthogonality between the columns of the effective channel matrix, resulting in reduced-complexity decoding. Second, we choose real rotation matrices because this enables reduced-complexity decoding without sacrificing coding gain. Finally, we choose ϕ to ensure a nonvanishing determinant and/or ensure full diversity.

We will present the EOS codes by drawing similarities to the TAST construction of (3). Instead of defining $\mathbf{a}_\ell$ as the vector of M symbols for the ℓ-th thread, we define:

$$\mathbf{A}_\ell = \begin{bmatrix} \mathcal{G}_{\ell,1} \\ \mathcal{G}_{\ell,2} \\ \vdots \\ \mathcal{G}_{\ell,M_2} \end{bmatrix} \tag{4}$$

as the $T_1 M_2 \times M_1$ matrix of M_2 embedded codewords for the ℓ-th *thread*, where $\mathcal{G}_{\ell,m}$ is the m-th embedded orthogonal codeword of the ℓ-th thread. By comparing $\mathbf{a}_\ell$ and $\mathbf{A}_\ell$, we see that the information symbols $a_{\ell,m}$ have been replaced by orthogonal codewords $\mathcal{G}_{\ell,m}$.

The proposed rate-R EOS code of size $T \times M$, where $T = T_1 M_2$ and $M = M_1 M_2$, is:

$$\mathbf{C}_{\text{EOS}} = \sum_{\ell=1}^{R_2} \text{blkdiag}(\mathbf{U}_\ell)(\mathbf{J}^{\ell-1} \otimes \mathbf{I}), \tag{5}$$

where

- $R_2 = \lceil R/R_1 \rceil$
- $\mathbf{U}_\ell = (\mathbf{G} \otimes \mathbf{I}_{M_1 \times M_1})\mathbf{A}_\ell = \begin{bmatrix} \mathbf{U}_{\ell,1} \\ \mathbf{U}_{\ell,2} \\ \vdots \\ \mathbf{U}_{\ell,M_2} \end{bmatrix}$
- $\otimes$ is the Kronecker product;
- $\mathbf{G}$ is an $M_2 \times M_2$ real unitary generator matrix
- $\text{blkdiag}(\mathbf{U})$ is the $T_1 M_2 \times M_1 M_2$ *block* diagonal matrix with the $T_1 \times M_1$ subblocks of $\mathbf{U}$ on the diagonal
- $\mathbf{J} = [\phi\mathbf{e}_{M_2}, \mathbf{e}_1, \ldots, \mathbf{e}_{M_2-1}]$.

The orthogonality embedding concept is evident by comparing (3) to (5). In particular, the transmitted symbols $a_{\ell,m}$ in (3) are replaced with orthogonal block codes $\mathcal{G}_{\ell,m}$ in (5). Alternatively, the orthogonal codewords are *embedded* into the code in (3) to yield the EOS codeword of (5).

We note that if the ratio R/R_1 is not an integer, then the rate of the EOS code in (5) is higher than R. A rate-R EOS code can then be obtained by puncturing the embedded codewords in the R_2-th thread. We will see an example of a punctured fractional-rate EOS code later in the Section.

We next discuss the specific choice of complex orthogonal designs. In this paper, we focus on the rate-1 Alamouti code for two antennas and rate-$3/4$ code for four antennas:

$$\mathcal{G} = \begin{bmatrix} a_1 & a_2 \\ -a_2^* & a_1^* \end{bmatrix} \text{ and } \mathcal{G} = \begin{bmatrix} a_1 & 0 & -a_2^* & a_3^* \\ 0 & a_1 & -a_3 & -a_2 \\ a_2 & a_3^* & a_1^* & 0 \\ -a_3 & a_2^* & 0 & a_1^* \end{bmatrix}. \tag{6}$$

The rate-$3/4$ orthogonal code for three transmit antennas can be obtained by deleting the fourth column of $\mathcal{G}$ for four antennas. Although we can embed arbitrary orthogonal codes, we will only consider the embedding of orthogonal designs for two, three or four antennas. This is because the orthogonal code rate tends to $1/2$ as the number of antennas increases, and the code

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length becomes prohibitively large.

For clarity of exposition, we give two examples for the construction of the rate-2 and rate-1.5 EOS codes for four and eight transmit antennas, respectively.

Example 1. We construct the rate-2 EOS code for four transmit antennas. Since $M = M_1 M_2 = 4$, there is only one choice for the orthogonal code, which is the Alamouti code with $M_1 = 2$.

With $M_1 = 2$ and $R_1 = 1$, we have that $M_2 = 2$ and $R_2 = 2$, and hence, $\mathbf{G}$ is the 2×2 real generator matrix:

$$\mathbf{G} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ -\alpha_2 & \alpha_1 \end{bmatrix} = \begin{bmatrix} 0.851 & 0.526 \\ -0.526 & 0.851 \end{bmatrix}, \quad (7)$$

where $\alpha_1 = \cos(\theta)$, $\alpha_2 = \sin(\theta)$ and $\theta = \frac{1}{2}\tan^{-1}(2)$. The generator matrix in (7) maximizes the coding gain compared to all real unitary generator matrices. The rate-two EOS code for four transmit antennas is then given by:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_{1,1} & \mathbf{U}_{2,1} \\ \phi\mathbf{U}_{2,2} & \mathbf{U}_{1,2} \end{bmatrix} = \begin{bmatrix} \alpha_1 \mathcal{G}_1 + \alpha_2 \mathcal{G}_2 & \alpha_1 \mathcal{G}_3 + \alpha_2 \mathcal{G}_4 \\ \phi(-\alpha_2 \mathcal{G}_3 + \alpha_1 \mathcal{G}_4) & -\alpha_2 \mathcal{G}_1 + \alpha_1 \mathcal{G}_2 \end{bmatrix}, \quad (8)$$

where $\phi = e^{i\pi/2}$ and $\mathcal{G}_k$ is the k-th Alamouti space-time block codeword. By comparison, the rate-2 TAST or perfect code for two transmit antennas is:

$$\mathbf{C} = \begin{bmatrix} u_{1,1} & u_{2,1} \\ \phi u_{2,2} & u_{1,2} \end{bmatrix} = \begin{bmatrix} \alpha_1 a_1 + \alpha_2 a_2 & \alpha_1 a_3 + \alpha_2 a_4 \\ \phi(-\alpha_2 a_3 + \alpha_1 a_4) & -\alpha_2 a_1 + \alpha_1 a_2 \end{bmatrix}, \quad (9)$$

where $\phi = e^{i\pi/2}$ for the perfect code and $\phi = e^{i\pi/6}$ for the TAST code. By comparing (8) and (9), we see that the information symbols a_k of the perfect code in (9) have been replaced with Alamouti blocks $\mathcal{G}_k$ in (8). Each $\mathcal{G}_k$ is a 2×2 matrix containing two complex information symbols, so the matrix of (8) is 4×4 and encodes a total of 8 symbols.

Example 2. We construct the rate-1.5 EOS code for eight transmit antennas. For $M = M_1 M_2 = 8$, there are two choices for the embedded orthogonal code; the rate-1 Alamouti code with $M_1 = 2$, or the rate-$3/4$ orthogonal code with $M_1 = 4$.

We first consider the rate-$3/4$ orthogonal code as the embedded code. This implies that $M_2 = 2$ and $R_2 = 2$, as was the case in Example 1. Therefore, the rate-1.5 EOS code for eight transmit antennas is:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_{1,1} & \mathbf{U}_{2,1} \\ \phi\mathbf{U}_{2,2} & \mathbf{U}_{1,2} \end{bmatrix} = \begin{bmatrix} \alpha_1 \mathcal{G}_1 + \alpha_2 \mathcal{G}_2 & \alpha_1 \mathcal{G}_3 + \alpha_2 \mathcal{G}_4 \\ \phi(-\alpha_2 \mathcal{G}_3 + \alpha_1 \mathcal{G}_4) & -\alpha_2 \mathcal{G}_1 + \alpha_1 \mathcal{G}_2 \end{bmatrix}, \quad (10)$$

where ϕ, α_1 and α_2 are as in Example 1 and $\mathcal{G}_k$ is the k-th rate-$3/4$ orthogonal code of (6). Orthogonality embedding is again evident by comparing (9) and (10). In particular, the information symbols of the perfect code in (9) have been replaced with the rate-$3/4$ orthogonal code in (10).

Next, we consider the rate-1 Alamouti code as the embedded code. We have $M_2 = 4$ and $R_2 = 2$, and hence, $\mathbf{G}$ is the 4×4 real generator matrix given by:

$$\mathbf{G} = \begin{bmatrix} 0.405 & 0.542 & -0.656 & -0.335 \\ 0.273 & 0.498 & 0.169 & 0.806 \\ 0.335 & -0.656 & -0.542 & 0.405 \\ 0.806 & -0.169 & 0.498 & -0.273 \end{bmatrix}. \quad (11)$$

The rate-2 EOS code for $M = 8$ is then given by:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_{1,1} & \mathbf{U}_{2,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{1,2} & \mathbf{U}_{2,2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{U}_{1,3} & \mathbf{U}_{2,3} \\ \phi\mathbf{U}_{2,4} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{1,4} \end{bmatrix}. \quad (12)$$

where $\phi = e^{i\pi/2}$. In order to obtain the rate-1.5 EOS code, we puncture the second embedded thread $\mathbf{U}_2$ to obtain:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_{1,1} & \overline{\mathbf{U}}_{2,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{1,2} & \overline{\mathbf{U}}_{2,2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{U}_{1,3} & \overline{\mathbf{U}}_{2,3} \\ \phi\overline{\mathbf{U}}_{2,4} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{1,4} \end{bmatrix}, \quad (13)$$

where $\overline{\mathbf{U}}_2 = (\mathbf{G} \otimes \mathbf{I})\overline{\mathbf{A}}_2$, $\overline{\mathbf{A}}_2 = [\overline{\mathcal{G}}_{2,1}^T, \ldots, \overline{\mathcal{G}}_{2,4}^T]^T$, and $\overline{\mathcal{G}}_k$ is the rate-$1/2$ punctured Alamouti block code defined by $\overline{\mathcal{G}}_k = \mathrm{diag}([a_{k,1}, a_{k,1}^*])$.

We summarize the three design parameters for the construction of EOS codes for up to eight transmit antennas in Table I. For the sake of completeness, we give the numerical value of the 3×3 real generator matrix:

$$\mathbf{G} = \begin{bmatrix} -0.328 & -0.591 & -0.737 \\ -0.737 & -0.328 & 0.591 \\ -0.591 & 0.737 & -0.328 \end{bmatrix}. \quad (14)$$

In describing the embedded code, we use the notation $\mathcal{G}(M_1, R_1)$ to denote the rate-R_1 orthogonal code designed for M_1 antennas, and we use and $\overline{\mathcal{G}}(2, 1/2)$ to denote the rate-1/2 punctured Alamouti code.

B. Decoding

We will assume that $N \geq R$ to ensure reliable detection at the receiver. The received vector at the n-th receive antenna during time slots $t \in \{1, \ldots, T\}$ can be written as:

$$\mathbf{y}_n = \mathbf{C}\mathbf{h}_n + \mathbf{w}_n, \quad (15)$$

where $\mathbf{h}_n$ is $[h_{1,n}, \ldots, h_{M,n}]^T$ and $\mathbf{y}_n$ and $\mathbf{w}_n$ are the $T \times 1$ vectors of received signals and noise at the n-th antenna, respectively. The received signal from all antennas at all time intervals is then given by:

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$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = (\mathbf{I}_N \otimes \mathbf{C}) \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_N \end{bmatrix} + \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_N \end{bmatrix} = \overline{\mathbf{C}}\mathbf{H} + \mathbf{W}. \quad (16)$$

In order to facilitate the use of efficient decoding algorithms, like the sphere decoding algorithm, the system of equations in (16) can be expressed in the form:

$$\mathbf{Y} = \mathbf{H}_e \mathbf{a} + \mathbf{W}. \quad (17)$$

where $\mathbf{a} = [a_1, ..., a_{RT}]^T$ is the $P \times 1$ vector of transmitted complex symbols, $P = RT$, and $\mathbf{H}_e$ is the effective channel matrix. We discuss the remainder of the decoding algorithm in terms of equivalent real-valued system.

Let $\hat{\mathbf{y}}_n = [\Re(y_{1,n}), \Im(y_{1,n}), ..., \Re(y_{T,n}), \Im(y_{T,n})]^T$ and $\hat{\mathbf{w}}_n = [\Re(w_{1,n}), \Im(w_{1,n}), ..., \Re(w_{T,n}), \Im(w_{T,n})]^T$ denote the real-valued representation of $\mathbf{y}_n$ and $\mathbf{w}_n$, respectively, where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts. Then, the system in (16) can be written as:

$$\hat{\mathbf{Y}} = \begin{bmatrix} \hat{\mathbf{y}}_1 \\ \dots \\ \hat{\mathbf{y}}_N \end{bmatrix} = \hat{\mathbf{H}}\hat{\mathbf{a}} + \begin{bmatrix} \hat{\mathbf{w}}_1 \\ \dots \\ \hat{\mathbf{w}}_N \end{bmatrix} = \hat{\mathbf{H}}\hat{\mathbf{a}} + \hat{\mathbf{w}}, \quad (18)$$

where $\hat{\mathbf{H}}$ is the real-valued effective channel matrix and $\hat{\mathbf{a}} = [\Re(a_1), \Im(a_1), ..., \Re(a_P), \Im(a_P)]^T$ is the $2P \times 1$ vector of PAM information symbols.

The maximum-likelihood receiver chooses $\hat{\mathbf{a}}$ to minimize $\|\hat{\mathbf{Y}} - \hat{\mathbf{H}}\hat{\mathbf{a}}\|^2$. Let $\hat{\mathbf{H}} = \mathbf{QR}$ be a Q-R decomposition, where $r_{l,p} = \mathbf{q}_l^T \mathbf{h}_p$ is the inner product between the l-th column of $\mathbf{Q}$ and the p-th column of $\hat{\mathbf{H}}$. The ML decoding of (18) is equivalent to minimizing $\|\mathbf{V} - \mathbf{R}\hat{\mathbf{a}}\|^2$, where $\mathbf{V} = \mathbf{Q}^T\hat{\mathbf{Y}}$.

TABLE I. DESIGN PARAMETERS FOR EOS CODES.

M	R	Embedded Code	$\mathbf{G}$	ϕ
2	1	$\mathcal{G}(2, 1)$	[1]	1
4	0.75	$\mathcal{G}(4, 3/4)$	[1]	1
	1	$\mathcal{G}(2, 1)$	(7)	1
	1.5	$\mathcal{G}(2, 1)$, $\overline{\mathcal{G}}(2, 1/2)$	(7)	$e^{i\pi/2}$
	2	$\mathcal{G}(2, 1)$	(7)	$e^{i\pi/2}$
6	0.75	$\mathcal{G}(3, 3/4)$	(7)	1
	1	$\mathcal{G}(2, 1)$	(14)	1
	1.5	$\mathcal{G}(3, \underline{3}/4)$ $\mathcal{G}(2, 1)$, $\mathcal{G}(2, 1/2)$	(7) (14)	$e^{i\pi/2}$ $e^{i\pi/12}$
	2	$\mathcal{G}(2, 1)$	(14)	$e^{i\pi/12}$
	3	$\mathcal{G}(2, 1)$	(14)	$e^{i\pi/12}$
8	0.75	$\mathcal{G}(4, 3/4)$	(7)	1
	1	$\mathcal{G}(2, 1)$	(11)	1
	1.5	$\mathcal{G}(4, \underline{3}/4)$ $\mathcal{G}(2, 1)$, $\mathcal{G}(2, 1/2)$	(7) (11)	$e^{i\pi/2}$ $e^{i\pi/2}$
	2	$\mathcal{G}(2, 1)$	(11)	$e^{i\pi/2}$
	3	$\mathcal{G}(2, 1)$	(11)	$e^{i\pi/2}$
	4	$\mathcal{G}(2, 1)$	(11)	$e^{i\pi/2}$

The proposed EOS codes are not *separable* for $R > 1$, meaning that we cannot separate the decoding into two or more *independent* groups of symbols. Hence, the reduction in worst-case decoding complexity is determined by the reduction in decoding complexity of the last decoded thread.

First, consider the worst-case complexity of an embedded Alamouti thread, which is obtained by setting $R_2 = 1$ in (5), for q-ary QAM alphabet. It can be shown that the symbols can be decoded in four independent groups, with each group containing $T/2$ $\sqrt{q}$-PAM symbols. Therefore, the worst-case complexity of an exhaustive search decoder is $\mathcal{O}(4\sqrt{q}^{T/2}) = \mathcal{O}(4q^{T/4})$. However, the worst-case complexity of a tree-based sphere decoder drops from $\mathcal{O}(4\sqrt{q}^{T/2})$ to $\mathcal{O}(4\sqrt{q}^{T/2-1})$. To see why, consider the problem of finding the "best" leaf node for a tree with $T/2$ levels. While one could in principle exhaustively search all $\sqrt{q}$ possibilities in turn, the problem can be solved efficiently using a *slicer*, whose complexity does not grow with the size of the alphabet; rather, the worst-case complexity of a PAM slicer is $\mathcal{O}(1)$ and not $\mathcal{O}(\sqrt{q})$. Specifically, a PAM slicer requires a single multiply, a single rounding operation, a single addition, and a single hard-limiting operation, none of which depends on q.

Similarly, the ML decoding of an embedded $\mathcal{G}(3, 3/4)$ or $\mathcal{G}(4, 3/4)$ thread can be done over six independent groups, with each group containing $T/4$ $\sqrt{q}$-PAM symbols. Therefore, the worst-case complexity is $\mathcal{O}(6\sqrt{q}^{T/4-1})$.

The worst-case decoding complexity for a rate-R embedded Alamouti code is $\mathcal{O}(\sqrt{q}^{2(R-1)T} \times 4\sqrt{q}^{T/2-1})$, where the first term comes from the decoding complexity of the first $R-1$ threads, and the second term comes from the decoding complexity of the last thread. Similarly, the worst-case decoding complexity for a rate-R embedded $\mathcal{G}(3, 3/4)$ or $\mathcal{G}(4, 3/4)$ code is $\mathcal{O}(\sqrt{q}^{2(R-3/4)T} \times 6\sqrt{q}^{T/4-1})$. The worst-case complexity of the perfect, TAST, quasiorthogonal, SAST, and proposed EOS code is summarized in Table II. As can be seen from Table II, the EOS codes have the lowest decoding complexity compared to previous constructions.

IV. NUMERICAL RESULTS

In this section we compare the BER performance of the proposed space-time codes with the best performing space-time codes of [1-9] over quasistatic Rayleigh-fading channel with additive Gaussian noise, assuming maximum-likelihood decoding. In Fig. 1 we compare the performance of the EOS code to existing codes when there are four transmit antennas for various rates and alphabets.

The two curves on the left compare the EOS code to the TAST [1] code when the rate is $R = 1.5$ with a 4-QAM alphabet. The proposed EOS code is not only 3 dB better than the TAST code at BER $= 10^{-4}$, but also lower in decoding complexity. The three curves on the right compare the performance when the rate is $R = 2$ and the alphabet is 16-QAM. We see that, at BER $= 10^{-4}$, the proposed space-time

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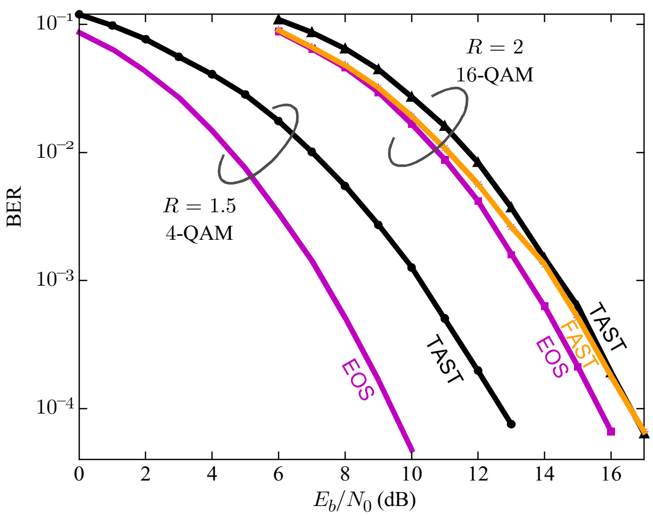

Fig. 1. Performance comparison for $M = 4$ antennas.

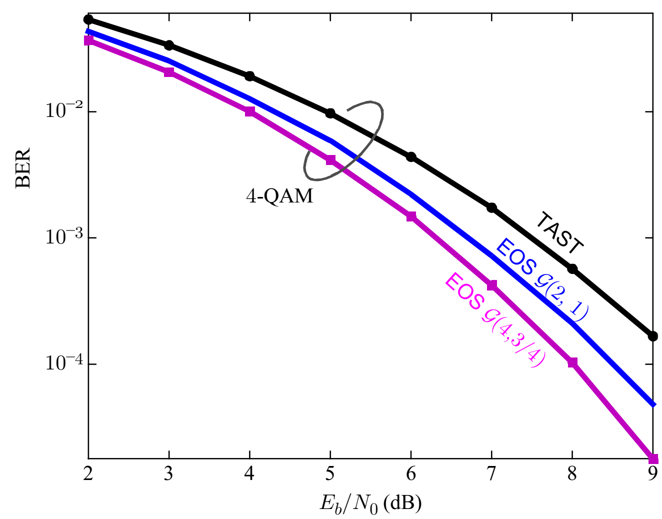

Fig. 2. Performance comparison for $M = 8$ antennas.

code outperforms the fast-decodable code [7] and TAST code by 0.3 dB and 1 dB, respectively.

In Fig. 2 we compare the EOS code with the embedded orthogonal codes $\mathcal{G}(4,\ 3/4)$ and $\mathcal{G}(2,\ 1)$ to the TAST code for eight transmit antennas, rate $R = 1.5$, and 4-QAM modulation alphabet. The two embedded codes outperform the TAST code by 1.2 dB and 0.8 dB at BER $= 10^{-4}$, respectively, and are also lower in decoding complexity.

TABLE II. WORST-CASE DECODING COMPLEXITY FOR SEVERAL CODES.

M	R	Perfect [2]	TAST [1]	Q-Orth. [5]	SAST [6]	Proposed
2	1	$\mathcal{O}(2q^{0.5})$	$\mathcal{O}(2q^{0.5})$	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
4	0.75	–	–	–	–	$\mathcal{O}(1)$
	1	$\mathcal{O}(2q^{1.5})$	$\mathcal{O}(2q^{1.5})$	$\mathcal{O}(2q^{1.5})$	$\mathcal{O}(4q^{0.5})$	$\mathcal{O}(4q^{0.5})$
	1.5	–	$\mathcal{O}(2q^{3.5})$	–	–	$\mathcal{O}(4q^{2.5})$
	2	$\mathcal{O}(2q^{5.5})$	$\mathcal{O}(2q^{5.5})$	–	–	$\mathcal{O}(4q^{4.5})$
6	0.75	–	–	–	–	$\mathcal{O}(6q^{0.5})$
	1	$\mathcal{O}(q^{5})$	$\mathcal{O}(2q^{2.5})$	$\mathcal{O}(2q^{2.5})$	$\mathcal{O}(4q)$	$\mathcal{O}(4q)$
	1.5	–	$\mathcal{O}(2q^{5.5})$	–	–	$\mathcal{O}(4q^{4})$ $\mathcal{O}(6q^{6.5})^{\dagger}$
	2	$\mathcal{O}(q^{11})$	$\mathcal{O}(2q^{8.5})$	–	–	$\mathcal{O}(4q^{7})$
	3	$\mathcal{O}(q^{17})$	$\mathcal{O}(2q^{14.5})$	–	–	$\mathcal{O}(4q^{13})$
8	0.75	–	–	–	–	$\mathcal{O}(6q^{0.5})$
	1	$\mathcal{O}(2q^{3.5})$	$\mathcal{O}(2q^{3.5})$	$\mathcal{O}(2q^{3.5})$	$\mathcal{O}(4q^{1.5})$	$\mathcal{O}(4q^{1.5})$
	1.5	–	$\mathcal{O}(2q^{7.5})$	–	–	$\mathcal{O}(4q^{5.5})$ $\mathcal{O}(6q^{6.5})^{\ddagger}$
	2	$\mathcal{O}(2q^{11.5})$	$\mathcal{O}(2q^{11.5})$	–	–	$\mathcal{O}(4q^{9.5})$
	3	$\mathcal{O}(2q^{19.5})$	$\mathcal{O}(2q^{19.5})$	–	–	$\mathcal{O}(4q^{17.5})$
	4	$\mathcal{O}(2q^{27.5})$	$\mathcal{O}(2q^{27.5})$	–	–	$\mathcal{O}(4q^{25.5})$

† embedded $\mathcal{G}(3,\ 3/4)$ ‡ embedded $\mathcal{G}(4,\ 3/4)$

V. CONCLUSIONS

We have proposed a family of space-time codes for an arbitrary number of transmit antennas and rates up to half the number of transmit antennas. We introduced the concept of embedded orthogonal space-time codes, in which complex orthogonal designs assume the role of complex information symbols in the encoding process. The proposed family of embedded orthogonal codes has the lowest decoding complexity for any rate up to half the number of transmit antennas. Furthermore, simulation results show that the proposed space-time codes outperform previous constructions for certain number of antennas and certain rates.

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