Rate-Allocation Strategies for Closed-Loop MIMO-OFDM

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***Abstract*— A closed-loop MIMO-OFDM transmitter can exploit channel knowledge using eigenbeamforming to create MN parallel channels, where M is the number of antennas at both ends and N is the number of OFDM tones. We find that the penalty due to a *flat-frequency constraint*, which forces each tone to convey the same amount of information, becomes negligible as M grows. We propose low-complexity bit-allocation strategies by combining the flat-frequency constraint with previously-reported spatial bit-allocation strategies [1]. A *fixed (nonadaptive)* allocation across space and frequency performs remarkably well with M as small as 4.**

I. Introduction

It is well-known that the solution to the classical rate-allocation problem for a bank of scalar AWGN channels is given by the water-pouring procedure [2]. Parallel channels arise in a closed-loop wideband multi-input multi-output (MIMO) channel when orthogonal-frequency division multiplexing (OFDM) creates a bank of narrowband MIMO channels across frequency, and when eigenbeamforming transforms each narrowband MIMO channel into scalar channels across space [3].

On MIMO-OFDM fading channels, the capacity-achieving rate allocation is based on water-pouring over space, frequency, and time [4]. Complexity can be significantly reduced by adopting a *power control* strategy, which performs water-pouring in space and frequency but not time [5][6]. With power control, each OFDM block has the same fixed total rate regardless of temporal channel changes.

Even with the power-control strategy, the rate-allocation complexity can still be high when the number of OFDM tones is large. To further reduce complexity, we introduce a *flat-frequency constraint*, in which each narrowband MIMO channel is restricted to have the same rate budget. In other words, with the flat-frequency constraint, water-pouring is performed over space but not frequency and not time. Although the flat-frequency constraint is grossly suboptimal for the case of a single-input single-output channel, we show that it is nearly optimal for the case of a MIMO channel.

This paper also examine *bit-allocation problem*, where each rate assigned to one of the parallel channels is constrained to a discrete and finite set. The best allocation strategy would enumerate all possible combinations of bit allocations and choose the one that has the minimum SNR requirement. Unfortunately the complexity of this exhaustive search is prohibitively high when the number of scalar channels is large in MIMO-OFDM. We propose low-complexity bit-allocation strategies for MIMO-OFDM by combining the strategies of [1] with the flat-frequency constraint. The reduction of bit-allocation complexity is significant because the number of parallel channels can be very large. We find that even a nonadaptive strategy, which uses a *fixed spatial allocation* on top of the flat-frequency constraint, performs surprisingly well.

The rest of paper is organized as follows. Section II describes the channel model for MIMO-OFDM. In Section III we introduce the flat-frequency constraint and assess its penalty. In Section IV, we propose low-complexity bit-allocation strategies and evaluate their performance. Finally we conclude in Section V.

II. System Model

A. Channel Model

We consider a wideband MIMO system with M transmit and M receive antennas, where we assume the same number of antennas at each end for simplicity. A frequency-selective channel is characterized by L significant delayed paths. Let $\boldsymbol{x}_k$ be an $M \times 1$ complex transmitted signal vector and $\boldsymbol{y}_k$ be an $M \times 1$ received signal vector in the baseband during the k-th signaling interval. Then the discrete-time baseband model is:

$$\boldsymbol{y}_k = \sum_{l=0}^{L-1} \mathbf{H}_l \boldsymbol{x}_{k-l} + \boldsymbol{n}_k, \tag{1}$$

where $\mathbf{H}_l$ is an $M \times M$ matrix representing the l-th tap of the discrete-time MIMO channel response [7]. The noise $\boldsymbol{n}_k$ is an $M \times 1$ white Gaussian vector with zero mean and $\mathrm{E}[\boldsymbol{n}_k \boldsymbol{n}_k^*] = N_0 \mathbf{I}_M$, where the asterisk denotes the Hermitian transpose and $\mathbf{I}_M$ is an $M \times M$ identity matrix.

The elements of $\mathbf{H}_l$ are possibly correlated, which is represented by a correlation matrix $\mathbf{R}_l$. If $\mathbf{R}_l = \mathbf{R}_l^{1/2} \mathbf{R}_l^{1/2*}$, the l-th channel matrix can be written as:

$$\mathbf{H}_l = \mathbf{R}_l^{1/2} \mathbf{Q}_l, \quad l = 0, 1, \ldots, L-1, \tag{2}$$

where $\mathbf{Q}_l$ is an uncorrelated $M \times M$ matrix with i.i.d. complex-valued elements. In (2), the deterministic matrix $\mathbf{R}_l^{1/2}$ models the spatial fading correlation at the receiver. If there is no spatial correlation at the receiver, $\mathbf{R}_l$ is simply an identity matrix. The uncorrelated fading happens when there are many scatterers around the receiver providing sufficient scattering from all directions. When there exists spatial correlation, we use the correlation model in [7], in which the delay spread

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channel is represented by L significant scatterer clusters at the transmitter side. In this case the element at the m-th row and n-th column of correlation matrix can be approximated as:

$$[\mathbf{R}_l]_{m,n} \approx \sigma_l^2 e^{-j2\pi(m-n)\Delta\cos(\bar{\theta}_l)} e^{-\frac{1}{2}(2\pi(m-n)\Delta\sin(\bar{\theta}_l)\sigma_{\theta_l})^2}, \tag{3}$$

where Δ is the antenna spacing relative to wavelength. The two parameters, $\bar{\theta}_l$ and $\sigma_{\theta_l}^2$, denote the average arrival angle and the variance of cluster angle spread, respectively, for the l-th cluster. The path gains $\{\sigma_l^2\}$ are basically dependent on the power delay profile and the shaping filter [3]. In fact the approximation in (3) is accurate only for small cluster angle spread, but it provides the correct trend for large spread. The rank of $\sum_l \mathbf{R}_l$ critically impacts the maximum achievable rate of MIMO-OFDM. Note that $\mathbf{R}_l$ collapses to a rank-1 matrix when $\sigma_{\theta_l} = 0$, that is, when there is no cluster angle spread. In such case a large increase in capacity is expected as L grows [7].

In this paper we only consider Rayleigh fading, where each element of $\mathbf{Q}_l$ is circularly-symmetric complex Gaussian with zero mean and unit variance. We assume that $\sum_{l=0}^{L-1} \text{trace}(\mathbf{R}_l) = M$ regardless of L for the sake of normalization. The path gains σ_l in (3) are equal for all l.

B. MIMO-OFDM

An OFDM system converts a wideband MIMO channel into a bank of parallel narrowband MIMO channels, avoiding the need for time-domain equalization at the receiver. Let N denote the number of tones (subcarriers) in OFDM. Then the narrowband MIMO channel at the n-th tone is given by:

$$\mathbf{G}_n = \sum_{l=0}^{L-1} \mathbf{H}_l e^{-j2\pi ln/N}. \tag{4}$$

Let $\mathbf{G}_n = \mathbf{U}_n \text{diag}\{\boldsymbol{s}_n^{1/2}\}\mathbf{V}_n^*$ be a singular-value decomposition of $\mathbf{G}_n$, where $\mathbf{U}_n$ and $\mathbf{V}_n$ are unitary, and where elements of $\boldsymbol{s}_n = [s_{0,n}, \ldots, s_{M-1,n}]$ are real and nonnegative eigenvalues of $\mathbf{G}_n\mathbf{G}_n^*$ that are ordered from largest to smallest. When the eigenbeamforming transmitter and receiver filter by $\mathbf{V}_n$ and $\mathbf{U}_n^*$ for each n, respectively, a bank of scalar channels is created across space and frequency [3]:

$$z_{m,n} = \sqrt{s_{m,n}} a_{m,n} + w_{m,n}, \qquad \begin{matrix} n = 0, 1, \ldots, N-1 \\ m = 0, 1, \ldots, M-1 \end{matrix}, \tag{5}$$

where $a_{m,n}$ is the data symbol across m-th spatial channel of n-th tone, and $z_{m,n}$ is the corresponding received signal affected by the noise $w_{m,n}$, which is white in space and frequency.

III. Flat-Frequency Constraint

In this section, We investigate the power-control strategy in MIMO-OFDM using the channel model of (5). The power-control strategy fixes the total rate per OFDM block, and rates are distributed to the parallel channels of (5) such that the transmitter power requirement for Shannon's error-free communications is minimized. The rate-allocation problem for the power-control strategy is solved by water-pouring over space and frequency. The power-control strategy has lower complexity than the capacity-achieving water-pouring strategy over space, frequency, and time, but it is not optimal as it does not consider temporal water-pouring. On MIMO channels, however, its penalty is known to be very small, especially for large antenna arrays [1][6].

(a) flat-frequency constraint

space \ frequency								
	4	3	2	2	4	3	3	4
	0	1	2	2	0	1	1	0

(b) no constraint

space \ frequency								
	6	3	2	2	3	2	3	5
	2	0	1	1	0	0	1	1

Fig. 1. An example of rate allocations in MIMO-OFDM with $M = 2$ antennas and $N = 8$ tones for $R = 32$ bits per OFDM block: (a) with flat-frequency constraint; and (b) without flat-frequency constraint.

Let R denote the total rate per OFDM block. The rate-allocation problem for the power-control strategy is solved by the following strategy:

Strategy 1 (water-pouring in space and frequency): Choose the set of rates $\{r_{m,n}\}$ so as to minimize the instantaneous SNR requirement:

$$\frac{1}{N}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\frac{2^{r_{m,n}}-1}{s_{m,n}}, \tag{6}$$

subject to $\sum_m \sum_n r_{m,n} = R$.

We now introduce a *flat-frequency constraint*, where each tone is forced to have the same rate budget. In other words, each tone must satisfy $\sum_{m=0}^{M-1} r_{m,n} = R/N$ for all n, so that the total rate per tone is "flat" in frequency. For instance, Fig. 1 illustrates rate allocations with and without a flat-frequency constraint for the case of $M = 2$ antennas (space) and $N = 8$ OFDM tones (frequency) when $R = 32$. Notice that each column sums up to $R/N = 4$ with the flat-frequency constraint, which is the rate budget for each tone. The rate-allocation problem with the flat-frequency constraint is solved by the following strategy:

Strategy 2 (water-pouring in space only): For each tone, choose the set of rates $\{r_{m,n}\}$ so as to minimize the instantaneous SNR requirement:

$$\sum_{m=0}^{M-1}\frac{2^{r_{m,n}}-1}{s_{m,n}}, \tag{7}$$

subject to the flat-frequency constraint that $\sum_m r_{m,n} = R/N$ for all n.

This strategy ignores the variability of the channel frequency response and performs water-pouring only over space. With the flat-frequency constraint, water-pouring for large (MN) parallel channels is replaced by N repetitions of water-pouring for rather small (M) parallel channels. In a single-antenna system, Strategy 2 leads to an equal allocation, that is, r_n is identical for all n, and suffers significant performance degradation. In contrast, on MIMO channels, the water-pouring over space helps decrease the degradation. In this paper, we

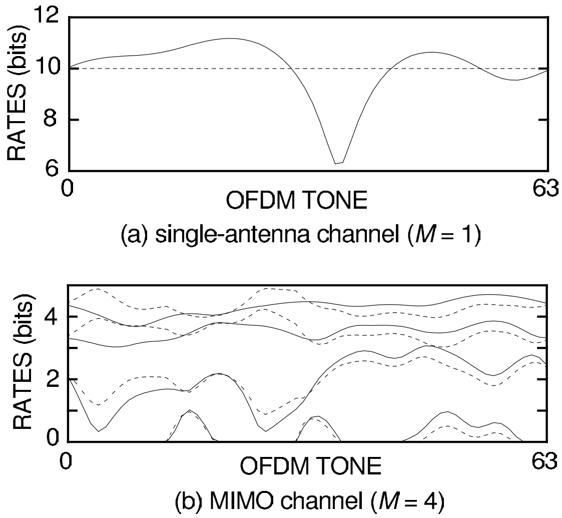

Fig. 2. An example of rate-allocation results of 1×1 and 4×4 systems for $N = 64$, where the solid lines are for Strategy 1 and the dotted lines are for Strategy 2 with the flat-frequency constraint.

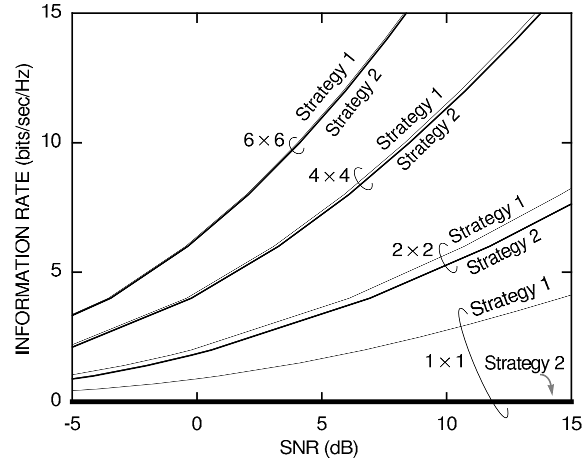

Fig. 3. Achievable rates of Strategy 1 and Strategy 2 for $M \in \{1, 2, 4, 6\}$ antennas at each end when $L = 4$ and $N = 64$ on spatially uncorrelated Rayleigh-fading channels.

investigate how small this penalty can be reduced on MIMO channels.

The penalty due to the flat-frequency constraint can be easily measured by comparing the average SNR requirement:

$$\frac{E}{N_0} = \mathrm{E}\left[\frac{1}{N}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\frac{2^{r_{m,n}}-1}{s_{m,n}}\right], \tag{8}$$

where $r_{m,n}$ are determined by either Strategy 1 or Strategy 2, and where the expectation is over the random channel. We define the average SNR penalty of Strategy 2 relative to Strategy 1 as

$$\text{average SNR penalty} = \frac{E/N_0 \text{ for Strategy 2}}{E/N_0 \text{ for Strategy 1}}. \tag{9}$$

Monte-Carlo simulations were performed by generating 10,000 independent sets of channels $\{\mathbf{H}_l\}$ for both spatially uncorrelated and correlated cases.

A. Spatially Uncorrelated Channels

We first consider the case when there is no spatial correlation, so that $\{\mathbf{R}_l\}$ are identity matrices. Spatially uncorrelated channels might not be a very practical model for some applications, but it provides insight into how Strategy 2 performs for various parameters, such as the number of antennas (M) or the number of channel taps (L). As it will be mentioned later, this uncorrelated case serves as an upper bound of the penalty in (9).

For the single-antenna system ($M = 1$), there is only one channel per tone if the flat-frequency constraint is applied. Then each tone inevitably has the same rate for all tones. In fact the elements of channel matrices $\mathbf{G}_n$ for each n are complex Gaussian since they are just linear combinations of Gaussian random variables of $\mathbf{Q}_l$ as shown in (4). Therefore the average SNR requirement in (8) will be infinite on 1×1 Rayleigh fading channels [4].

As M increases, however, each tone has more degrees of freedom (spatial channels) over which to allocate the assigned rate budget, and the water-pouring over space helps decrease the average SNR penalty in (9). For instance, Fig. 2 shows one example of rate-allocation results for $M = 1$ and $M = 4$ with $N = 64$ and $R = 640$. The solid lines represent allocated rates by Strategy 1, whereas the dotted lines correspond to Strategy 2. Obviously, as shown in Fig. 2a, Strategy 2 results in a flat allocation across frequency when $M = 1$. In contrast, for $M = 4$ in Fig. 2b, the allocated rates of Strategy 2 are no longer flat, even though the rates sum up to $R/N = 10$ at each tone. The rates for Strategy 2 are not far from the rate allocations by Strategy 1.

Fig. 3 plots the achievable rate for Strategy 1 and Strategy 2 by measuring (8) for $M \in \{1, 2, 4, 6\}$ when $L = 4$ and $N = 64$. As the number of antennas (M) grows, not only the achievable rate increases, but the gap between two strategies decreases as well. In the case of a single-antenna system, Fig. 3 shows that the flat-frequency constraint results in an infinite SNR penalty. However, average SNR penalty for $M = 2$ dramatically reduces to approximately 1 dB. For $M = 4$ and $M = 6$, the gap becomes even smaller, around 0.2 dB and 0.1 dB, respectively. Fig. 3 shows that performance degradation due to flat-frequency constraint is negligible, especially when more than two antennas are employed at both ends.

We now investigate how the number of channel paths (L) affects the average SNR penalty. The average SNR requirement of Strategy 2 is independent of L and is equal to the flat-fading case since spatial water-pouring is independently performed for each channel matrix $\mathbf{G}_n$, which is statistically identical to the flat-fading case. On the other hand, the average SNR requirement of Strategy 1 decreases as L grows. This is similar to the case when diversity order increases in proportion to L [8]. The more sources of transmitted signals are available at the receiver, the higher the transmission rate can be. From a frequency-domain perspective, as L grows, the frequency response becomes more variable so that water-pouring over frequency becomes more advantageous.

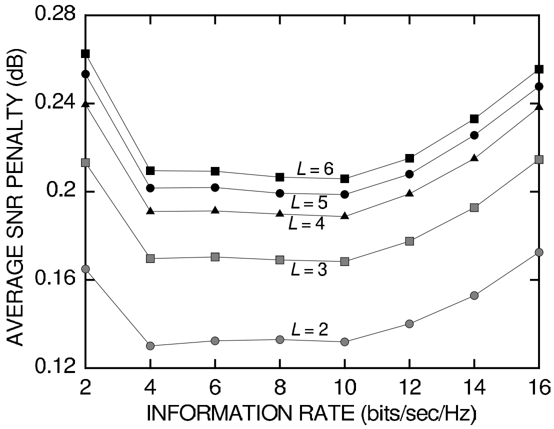

Fig. 4. Average SNR penalty due to the flat-frequency constraint in frequency-selective channels for $L \in \{2, 3, 4, 6\}$ channel taps and no spatial correlation when $M = 4$ and $N = 64$.

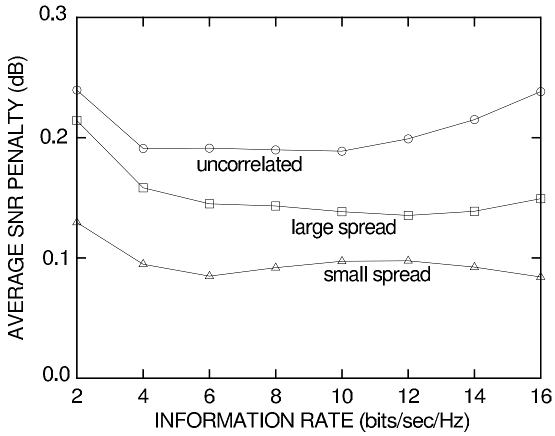

Fig. 5. Comparison between spatially correlated and uncorrelated fading in terms of the average SNR penalty for $M = 4$, $L = 4$, and $N = 64$.

In order to illustrate the effects by L, Fig. 4 plots the average SNR penalty due to the flat-frequency constraint for $L \in \{2, 3, 4, 5, 6\}$ when $M = 4$ and $N = 64$. Clearly it can be seen that the penalty increases as L grows. When $L = 2$, it is less than 0.15 dB. But it becomes more than 0.2 dB as $L = 5$ or $L = 6$. In Fig. 4, the advantage of Strategy 1 is not as impressive as the diversity-order increase since the achievable rate is primarily determined by the rank of $\sum_l \mathbf{R}_l$, which is already full with probability one even for $L = 1$ in uncorrelated fading. Also the advantage in Fig. 4 seems to saturate as L increases. In practice significant paths are often limited to a small number, and the penalty due to the flat-frequency constraint can be kept small as long as a sufficient number of antennas are employed at both ends.

B. Spatially Correlated Channels

When the receiver is located in an open place and no local scattering occurs, spatial fading at the receive antennas will be correlated and this correlated fading can be described by the correlation matrix in (3). In this case the average SNR requirement of (1) is strongly affected by the rank of $\sum_l \mathbf{R}_l$ since it is not necessarily full rank. Thus the decrease in the average SNR requirement of Strategy 1 becomes more conspicuous as L grows. This is true either when the transmitter knows the channel [3] or not [7]. In contrast to uncorrelated fading, where an increase in L does not lower the average SNR requirement of Strategy 2, the average SNR requirement with the flat-frequency constraint also shows a significant decrease as L grows in the case of correlated fading.

For instance, we compare performance in Fig. 5 when fading is either correlated or uncorrelated, and when cluster angle spread is either small ($\sigma_{\theta_l} = 0$) or large ($\sigma_{\theta_l} = 0.25$) if correlated. We assume that there are $L = 4$ clusters, whose average angles $\{\overline{\theta}_l\}$ are $\{0, \pi/4, \pi/3, \pi/2\}$. As shown in Fig. 5, when the spread is smaller, performance degradation due to the flat-frequency constraint is less severe. In this example, as already shown in Fig. 4, uncorrelated case suffers a penalty of approximately 0.2 dB. As the fading becomes more correlated, the penalty decreases up to 0.17 dB for large spread and up to 0.14 dB for small spread. As mentioned before, the uncorrelated fading is the worst case in terms of average SNR penalty.

Generally, when there is spatial correlation, the average SNR penalty increases, just like uncorrelated case, as L grows. However there is a tendency that the penalty is smaller when the angle spread is narrower. When $\sigma_{\theta_l} = 0$ and when there are only $L = 2$ or $L = 3$ clusters, the penalty is nearly zero and Strategy 2 suffers little degradation due to the flat-frequency constraint. We see that performance degradation due to flat-frequency constraint becomes less severe in the presence of spatial correlation of fading.

IV. BIT-ALLOCATION STRATEGIES

The rate-allocation problem becomes the bit-allocation problem when we impose a granularity constraint on the rates, so that $\{r_{m,n}\}$ are restricted to be discrete and finite. With this granularity constraint, the number of possible bit allocations is limited. The best bit-allocation strategy would enumerate all possible allocations and choose the one that has the minimum average SNR requirement. For MIMO-OFDM, however, this exhaustive-search strategy requires high complexity when the number of parallel channels (MN) is large.

We can reduce complexity by imposing a fixed spatial allocation [1] on top of Strategy 2 with a rate budget of R/M per tone. Instead of an exhaustive search, this strategy fixes the allocation for all tones and all channel realizations. Combining the flat-frequency constraint with a fixed spatial allocation per tone leads to a totally *nonadaptive* bit-allocation strategy. If the fixed allocation is carefully chosen to match the anticipated statistics of MIMO fading channels, this fixed-frequency fixed-space strategy performs well and its achievable rate approaches Strategy 1 closely when there are more than two antennas at each end. This is in part due to the ordered nature and reduced variability of the eigenvalues $\{s_{m,n}\}$ of MIMO channels [1].

With $M = 2$ antennas at each end, the fixed-allocation strategy might incur a large penalty as the number of spatial channels is not sufficient. In this case, the binary-search strategy in [1] can be used instead of the fixed-allocation strategy to

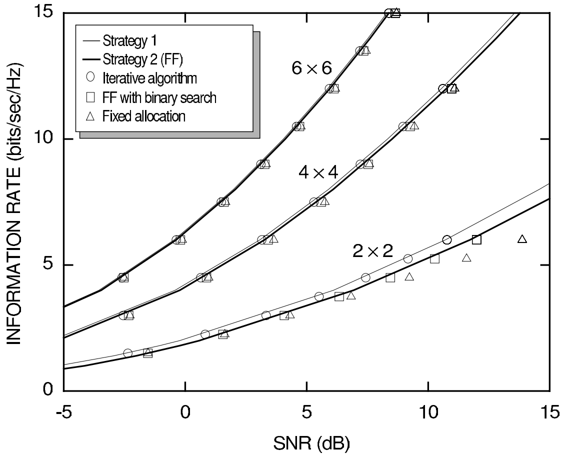

Fig. 6. Performance of the flat-frequency (FF) bit-allocation uncorrelated Rayleigh fading with $M \in \{2, 4, 6\}$ when $L = 4$ and $N = 64$.

improve the performance. The binary-search strategy considers two candidates out of all possible allocations with a budget of R/N and chooses the one with smaller instantaneous SNR requirement in (7). Though the binary search is the simplest form of adaptation, it guarantees good performance for any MIMO channel.

Fig. 6 illustrates the performance of the fixed-allocation and binary-search strategies with the flat-frequency constraint for $M \in \{2, 4, 6\}$, where we assume no spatial correlation with $L = 4$ and $N = 64$. When restricting the spatial allocation to binary search (marked as squares), the flat-frequency strategy incurs an average SNR penalty of between 0.4 dB and 0.9 dB for $M = 2$ compared to the iterative algorithm of [9] (marked as circles), whereas the penalty is negligible when $M = 4$ and $M = 6$. The fixed-frequency fixed-spatial allocation (marked as triangles) is also nearly optimal for $M = 4$ and $M = 6$, while its penalty can be large for $M = 2$. As shown in Fig. 6, the iterative algorithm of [9] is tightly bounded by Strategy 1 (thin lines) while the binary-search strategy with flat-frequency constraint is tightly bounded by Strategy 2 (thick lines). Thus infinite-precision water-pouring is a good indicator for the performance of practical bit-allocation strategies.

V. CONCLUSIONS

We investigated the rate-allocation problem for a closed-loop MIMO-OFDM system using eigenbeamforming. Particular focus is on simple rate-allocation strategies instead of high-complexity water-pouring over space and frequency. First we introduced a flat-frequency constraint, which leads to spatial water-pouring by forcing the same total rate per tone. We showed that the penalty due to the flat-frequency constraint is small on spatially uncorrelated MIMO-OFDM channels. For example, the penalty relative to water-pouring over both space and frequency is only 0.2 dB on 4×4 Rayleigh fading channels with $L = 4$ channel taps. It becomes even smaller when fading is spatially correlated. We further reduce the complexity by imposing a fixed spatial allocation on top of the flat-frequency constraint, which leads to a totally nonadaptive rate allocation. Remarkably this fixed-allocation strategy performs well when the fixed allocation is chosen to match the anticipated statistics of fading channels. These results imply that a closed-loop MIMO system need not perform adaptive modulation in order to approach capacity. Instead, a combination of eigenbeamforming and fixed modulation is sufficient.

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