Optimal LLR Clipping Levels for Mixed Hard/Soft Output Detection

Ernesto Zimmermann*, David L. Milliner†, John R. Barry† and Gerhard Fettweis*
*Vodafone Chair Mobile Communications Systems
Technische Universität Dresden, D-01062 Dresden, Germany
Email: {zimmere,fettweis}@ifn.et.tu-dresden.de
†School of Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta, Georgia 30332–0250
Email: {dlm,barry}@ece.gatech.edu

***Abstract*—Consider a communications system where the detector generates a mix of hard and soft outputs, which becomes fed to a soft-input channel decoder. In such a setting, it is of interest to find the optimal soft representation for the hard detected bits, which minimizes the probability of error at the decoder output. In this contribution we prove that for repetition codes transmitted over the AWGN channel using antipodal signaling, the optimal soft representation is given by the error probability at the detector output. This provides an indication of how "LLR clipping levels" should be chosen, e.g., in the context of list based detection of multiple-input multiple-output (MIMO) signals.**

I. Motivation

Consider the following communications problem: due to constraints on the allowable computational complexity, the receiver cannot make use of the optimal soft output detector. As a consequence, *reliability information* is unavailable for at least some of the detected bits and the exact *magnitude* of the corresponding log-likelihood-ratios (LLRs) is unknown. However, a reasonably precise estimate of the optimal detector's *hard output* can be determined, i.e., the *sign* of the corresponding LLRs is known, although there is a non-vanishing probability of error in the decision. The detector output is fed into a soft-input decoder which is unaware whether a particular bit has been hard or soft detected.

The problem just described is frequently encountered in channels with interference, such as the multiple-input multiple-output (MIMO) channel, the inter-symbol-interference (ISI) channel, and in code division multiple access (CDMA) systems. In such a setup, implementing the optimal a posteriori probability (APP) detector is of prohibitive computational complexity for high spectral efficiencies, and sub-optimal detection schemes must be used. List detectors, such as the list sphere detector [1], the list sequential detector [2], and the M Algorithm [3], are an attractive choice as they allow a flexible trade-off between performance and detection complexity. However, these schemes only produce soft output for those bits in the detected signal vector where a "0-hypothesis" *and* a "1-hypothesis" can be found in the constructed list. For all bits where this is not the case, only hard output is generated.

Evidently, some LLR magnitude (and thus, reliability) has to be assigned to the hard detected bits. A frequently employed concept is to set the LLR magnitude for these bits to a certain predefined value: an approach known as "LLR clipping" [1], [3]. This strategy is extremely simple to implement. It does, however, raise the question of how the LLR clipping level should be chosen in order to maximize performance. There have been numerous approaches to solve this problem: If a list sphere detector is used, the radius of the search sphere may be used to determine a suitable clipping level. Yet, [1] suggests using a fixed LLR clipping level of $\Lambda = 8$ for sufficiently large list sizes. For M Algorithm based detection using small to moderate list sizes, a fixed clipping level of $\Lambda = 3$ was proposed in [3]. The analysis in [4] showed that the mutual information between channel input and detector output is a suitable figure of merit for optimization, and that the optimal LLR clipping level will hence strongly depend on the system setup as well as the chosen detector configuration.

This contribution takes a step back and formulates the selection of the LLR clipping level as a general optimization problem, where the clipping level should be chosen by the detector such that the probability of error at the output of the channel decoder is minimized. Results are provided for the case of repetition codes transmitted over the AWGN channel using antipodal signaling. Specifically, it is proven that for this setup, the optimal LLR clipping level Λ^* is given by:

$$\Lambda^* = \ln \frac{1 - P_b}{P_b}, \tag{1}$$

where P_b is the probability of a bit error at the detector output, i.e., the probability that the sign of the LLR is wrong.

The remainder of this paper is structured as follows: Section II introduces the system setup and basics of soft output detection. In Section III, the proof for the optimality of Λ^* is outlined. Numerical results are provided for illustration in Section IV, before conclusions are drawn in Section V.

II. System Model

Consider a single-input single-output (SISO) system using a BICM transmit strategy as depicted in Figure 1. The vector $\mathbf{u} \in \mathbb{B}^{1\times K}$ of i.i.d. information bits is encoded and interleaved, resulting in the codeword $\mathbf{c} \in \mathbb{B}^{1\times N}$, with $\mathbb{B} = \{0, 1\}$ as the binary field. The rate of the channel code is denoted $R_c = K/N$. The elements c_i of the code bit stream are

978-1-4244-2324-8/08/$25.00 © 2008 IEEE.

mapped onto the channel inputs $x_i \in \{-1, +1\}$ using the assignment $0 \mapsto +1, 1 \mapsto -1$. The transmit signal energy is thus normalized to unity, $E_s = \mathcal{E}\{|x_i|^2\} = 1$.

At the receiver, the detector first estimates log-likelihood ratios L_i for the received code bits. Some of these LLRs are based on hard detected bits, where the bit error probability at the detector output will be denoted P_b in the following. The LLRs generated by the detector are then fed to a soft-input decoder which uses this information to establish estimates $\hat{u}_l$ on the information bits.

We consider transmission over the additive white Gaussian noise (AWGN) channel. In the equivalent discrete-time baseband model, the received signal $\mathbf{y}$ is thus given by:

$$\mathbf{y} = \mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{n} \in \mathbb{R}^{1 \times N}$ represents the receiver noise whose components are zero mean i.i.d. Gaussian random variables with variance $N_0/2$ (double-sided noise power spectral density).

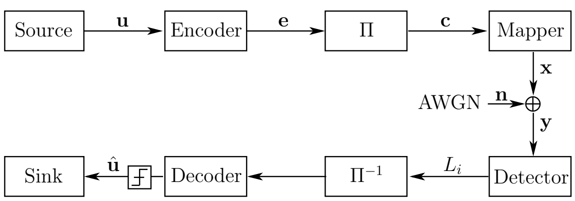

Figure 1. BICM system model.

We consider a non-iterative detection-decoding setup, i.e., no a priori information is provided to the detector from the channel decoder. The soft output of the optimal APP detector for the ith code bit, in the form of an LLR, thus becomes:

$$\begin{aligned} L_i :=& \ln \frac{\Pr(c_i = 0|y_i)}{\Pr(c_i = 1|y_i)} \\ =& \ln \frac{p(y_i|x_i = +1)}{p(y_i|x_i = -1)} = \ln \frac{e^{\frac{-(y_i-1)^2}{N_0}}}{e^{\frac{-(y_i+1)^2}{N_0}}} = 4\frac{E_s}{N_0} y_i, \end{aligned} \tag{3}$$

under the assumptions of statistically independent, uniformly distributed code bits and additive white Gaussian noise.

It is well known that for Gaussian distributed noise, the LLRs also follow a Gaussian distribution (for a given value $+1$ or -1 of the transmit signal) with mean and variance [5]

$$\mu_L = 4\frac{E_s}{N_0} \qquad \sigma_L^2 = 2\mu_L = 8\frac{E_s}{N_0} \tag{4}$$

For antipodal signaling with equiprobable inputs, the LLR distribution can thus be described by the superposition of two Gaussian distributions as follows:

$$p(L_i) = \frac{\left(\exp\left(-\frac{(L_i-\mu_L)^2}{2\sigma_L^2}\right) + \exp\left(-\frac{(L_i+\mu_L)^2}{2\sigma_L^2}\right)\right)}{2\sqrt{2\pi\sigma_L^2}}. \tag{5}$$

If a mix of hard and soft output is generated by the detector, the transmission model can be formulated as depicted in Figure 2. Depending on the state of the switch, the received signal y_i is either fed to the logAPP detector (multiplication with $4E_s/N_0$) or quantized to $\pm\Lambda$, by using a 1-bit quantizer followed by a scaling by the LLR clipping level Λ. The decoder is unaware of the state of the switch and consequently treats all input information as soft input.

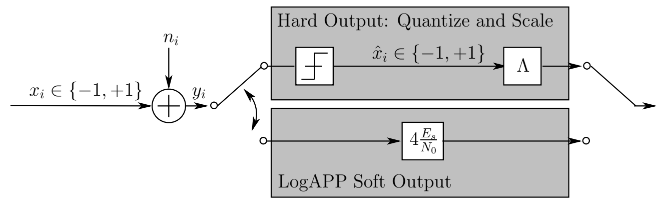

Figure 2. System model for a mixed hard/soft output detector.

III. Optimal LLR Clipping Level for Repetition Codes

Consider the case of a repetition code of rate $R_c = 1/r$ being used as channel code. Assuming i.i.d. information bits with $\Pr(u_l = 0) = \Pr(u_l = 1) = 1/2$, the optimal decoder decides in favor of $\hat{u}_l = 0$ for the lth information bit, iff

$$\Pr(y_j, \ldots, y_k|u_l = 0) \geq \Pr(y_j, \ldots, y_k|u_l = 1) \tag{6}$$

and $\hat{u}_l = 1$ otherwise, where $j = (l-1)r + 1$ and $k = l \cdot r$. Under the assumption of i.i.d. channel noise, the joint density of the received symbols factorizes, such that this decision problem can be reformulated as follows:

$$\hat{u}_l := \begin{cases} 0 & \text{iff} \quad \sum_{i=j}^{k} L_i \geq 0 \\ 1 & \text{iff} \quad \sum_{i=j}^{k} L_i < 0 \end{cases}, \tag{7}$$

with j and k as defined above. The decoder's decision is thus solely based on the sign of the sum of the LLRs corresponding to the code bits c_i which resulted from the encoding of information bit u_l. For brevity of exposition, the following proofs are based on the all-zero codeword ($\forall_{l=1,\ldots,K} : u_l = 0$). Since we assume linear coding and a (symmetric) binary AWGN channel, this does not imply any loss of generality.

A. Rate 1/2 Repetition Code

In order to provide an outline for the general case, we start by considering the simplest repetition code, the rate $R_c = 1/2$ code. We seek to minimize the error probability at the decoder output by appropriate selection of Λ. For the considered case of $r = 2$, there are but three possible cases for the type of LLRs involved in the decoding equation given by (7):

1) Both LLRs have been obtained by the logAPP detector, i.e., are based on soft output.
2) Exactly one of the LLRs is based on the detector's hard output, i.e., there is a mix of soft and hard output.
3) Both LLRs are based on the detector's hard output.

It is easily seen that for the first and the last of the three cases, the choice of Λ has no impact on the decoding result for the information bit in question. It is thus sufficient to concern ourselves with the second case. In the following, we omit the bit index for the information bit u, label the hard detected code bit as c_1 with associated LLR $\pm\Lambda$ and the soft demodulated bit as c_2 with associated LLR L_2.

Using (7) and the fact that $\forall l : u_l = 0$, the probability of decoding error is easily seen to be of the form:

$$\begin{aligned} P_f(\Lambda) &= \Pr(\hat{u} = 1|\hat{c}_1 = 0)\Pr(\hat{c}_1 = 0) \\ &\quad + \Pr(\hat{u} = 1|\hat{c}_1 = 1)\Pr(\hat{c}_1 = 1) \\ &= \Pr(L_2 + \Lambda < 0)(1 - P_b) + \Pr(L_2 - \Lambda < 0)P_b, \end{aligned} \quad (8)$$

where P_b is the bit error probability at the detector output. Based on this error probability expression, we are now ready to state the first result:

Lemma 1: The LLR clipping level Λ^* which minimizes the probability of decoding error $P_f(\Lambda)$ of (8) is given by:

$$\Lambda^* = \ln \frac{1 - P_b}{P_b}. \quad (9)$$

Proof: Using the fact that the soft output LLR L_2 generated by the detector is Gaussian distributed, the above error rate expression may be reformulated as:

$$\begin{aligned} P_f &= (1 - P_b) \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma_L^2}} \exp\left(-\frac{(y - (\mu_L + \Lambda))^2}{2\sigma_L^2}\right) \mathrm{d}\, y \\ &\quad + P_b \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma_L^2}} \exp\left(-\frac{(y - (\mu_L - \Lambda))^2}{2\sigma_L^2}\right) \mathrm{d}\, y \\ &= (1 - P_b)Q\left(\frac{\mu_L + \Lambda}{\sigma_L}\right) + P_b Q\left(\frac{\mu_L - \Lambda}{\sigma_L}\right). \end{aligned} \quad (10)$$

In order to find the minimum of this expression, we first take the derivative with respect to Λ:

$$\frac{dP_f}{d\Lambda} = \frac{P_b \exp\left(-\frac{(\mu_L - \Lambda)^2}{2\sigma_L^2}\right) - (1 - P_b)\exp\left(-\frac{(\mu_L + \Lambda)^2}{2\sigma_L^2}\right)}{\sqrt{2\pi\sigma_L^2}}.$$

Setting to zero and solving for Λ yields:

$$\ln \frac{1 - P_b}{P_b} = -\frac{(\mu_L - \Lambda)^2 - (\mu_L + \Lambda)^2}{2\sigma_L^2} = \frac{4\mu_L\Lambda}{2\sigma_L^2}. \quad (11)$$

Using the fact that $\sigma_L^2 = 2\mu_L$, this can be solved for:

$$\Lambda^* = \ln \frac{1 - P_b}{P_b}.$$

■

B. Repetition Codes of Arbitrary Rate

For a repetition code of rate $R_c = 1/r$, assume that a certain number n of the code bits involved in decoding have been detected hard. Following the argument given above, it is sufficient to restrict n to the range $1 \leq n \leq r - 1$. We again assume the all-zero codeword to have been transmitted.

For a given n, the probability of decoding error is given by:

$$P_f(\Lambda) = \sum_{k=0}^{n} \Pr(\hat{u} = 1|d = k)\Pr(d = k), \quad (12)$$

where d denotes the number of decision errors which have occurred in the hard detected bits. This expression can be expanded as follows:

$$\begin{aligned} P_f(\Lambda) &= \sum_{k=0}^{n} \binom{n}{k} (1 - P_b)^{n-k} P_b^k \quad \cdots \\ &\quad \cdot \int_{-\infty}^{0} \frac{\exp\left(-\frac{(y - (\mu_\Sigma + (n-2k)\Lambda))^2}{2\sigma_\Sigma^2}\right)}{\sqrt{2\pi\sigma_\Sigma^2}} \mathrm{d}\, y, \end{aligned} \quad (13)$$

where $\sigma_\Sigma^2 = (r - n)\sigma_L^2$ and $\mu_\Sigma = (r - n)\mu_L$. This formulation takes the fact into account that $r - n$ bits have been detected soft, whose LLRs are summed up to obtain a joint LLR of appropriate mean and variance. The term in front of Λ in (13) results from the fact that if there are k decision errors in the n hard detected bits, the sum of the corresponding hard output LLRs becomes $(n - k)\Lambda + k(-\Lambda) = (n - 2k)\Lambda$.

In order to solve this new optimization problem, we first extend the solution for the rate $1/2$ case:

Lemma 2: The value of Λ which minimizes an error probability of the form

$$\begin{aligned} P_f(\Lambda) &= \binom{n}{k} (1 - P_b)^{n-k} P_b^k P_k(n - 2k) \\ &\quad + \binom{n}{n-k} (1 - P_b)^k P_b^{n-k} P_k(2k - n) \end{aligned} \quad (14)$$

with $P_k(t)$ given by

$$P_k(t) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma_\Sigma^2}} \exp\left(-\frac{(y - (\mu_\Sigma + t\Lambda))^2)}{2\sigma_\Sigma^2}\right) \mathrm{d}\, y \quad (15)$$

is given by

$$\Lambda^* = \ln \frac{1 - P_b}{P_b}. \quad (16)$$

Proof: Taking the derivative of the error rate expression with respect to Λ yields:

$$\begin{aligned} \frac{dP_f}{d\Lambda} &= \binom{n}{n-k} (1 - P_b)^k P_b^{n-k} \frac{\exp\left(-\frac{(\mu_\Sigma - (n-2k)\Lambda)^2)}{2\sigma_\Sigma^2}\right)}{\sqrt{2\pi\sigma_\Sigma^2}} \\ &\quad - \binom{n}{k} (1 - P_b)^{n-k} P_b^k \frac{\exp\left(-\frac{(\mu_\Sigma + (n-2k)\Lambda)^2)}{2\sigma_\Sigma^2}\right)}{\sqrt{2\pi\sigma_\Sigma^2}}. \end{aligned}$$

Setting this expression to zero and using the fact that $\binom{n}{k} = \binom{n}{n-k}$, this can finally be reformulated into:

$$(1 - P_b)^{n-2k} P_b^{2k-n} = \exp\left((n - 2k)\frac{4\mu_\Sigma\Lambda}{2\sigma_\Sigma^2}\right).$$

Taking the logarithm on both sides yields:

$$\ln \frac{1 - P_b}{P_b} = \frac{4\mu_\Sigma\Lambda}{2\sigma_\Sigma^2} = \Lambda^*, \quad (17)$$

where we again exploited the fact that $\mu_L = 2\sigma_L^2$, from which $\mu_\Sigma = 2\sigma_\Sigma^2$ follows immediately. ■

We are now ready to formulate the main result of the paper:

Theorem 1: The optimal LLR clipping level for a rate $1/r$ repetition code transmitted over the AWGN channel using antipodal signaling is given by:

$$\Lambda^* = \ln \frac{1 - P_b}{P_b} \tag{18}$$

Proof: Consider the case where n is odd. Thus, (12) can be reformulated as:

$$P_f(\Lambda) = \sum_{k=0}^{(n-1)/2} \left(\Pr(\hat{u} = 1 | d = k) \Pr(d = k) + \Pr(\hat{u} = 1 | d = n-k) \Pr(d = n-k)\right). \tag{19}$$

There are thus $(n+1)/2$ terms of the form described by (14). Since the solution Λ^* to the optimization problem stated by Lemma 2 is independent of both n and k, repeated application of Lemma 2 yields the result stated above. Now assume n is even. In this case, (12) can be rewritten as:

$$\begin{aligned} P_f(\Lambda) &= \Pr(\hat{u} = 1 | d = n/2) \Pr(d = n/2) \\ &+ \sum_{k=0}^{n/2-1} \left(\Pr(\hat{u} = 1 | d = k) \Pr(d = k) + \Pr(\hat{u} = 1 | d = n-k) \Pr(d = n-k)\right). \end{aligned} \tag{20}$$

The first term in (20) corresponds to the case where a decision error occurred in exactly half of the hard detected bits. The sum of the corresponding hard output LLRs is thus $\frac{n}{2}\Lambda - \frac{n}{2}\Lambda = 0$, independent of Λ. Following the argument given above for the remaining terms in (20) again yields Λ^* as the optimum solution, thus completing the proof. ∎

IV. Numerical Results

For simulation we consider transmission over the AWGN and the temporally i.i.d. Rayleigh fading channel, using BPSK modulation. The transmission block size is 18432 bits. For the first two results, we use a repetition code as considered in the proofs provided in this paper. The last result is obtained for transmission over the fast Rayleigh fading channel, using a rate 1/2 PCCC based on $(7_R, 5)$ convolutional codes for channel coding, and 8 internal iterations of logMAP decoding. This setup is equivalent to the SISO case from [1].

In the following, δ denotes the fraction of bits in the codeword which have been hard detected. Thus, $\delta = 0$ corresponds to standard soft output logAPP detection while $\delta = 1$ corresponds to hard output detection. For more sophisticated system setups, such as MIMO detection, a larger fraction of hard detected bits would typically correspond to a lower complexity detector (e.g. smaller list size for a list detector).

For BPSK transmission over the AWGN channel, the error probability at the detector output is well known to be:

$$P_b = Q\left(\sqrt{2\frac{E_s}{N_0}}\right). \tag{21}$$

Figure 3 shows results for the transmission of repetition codes of rates $R_c = 1/2$, $R_c = 1/4$ and $R_c = 1/8$ over the AWGN channel, where the fractions of hard detected bits are $\delta \in \{0.25, 0.5, 1\}$, respectively. As was to be expected from the proofs provided in this work, the use of the optimal clipping level Λ^* consistently yields the best performance.

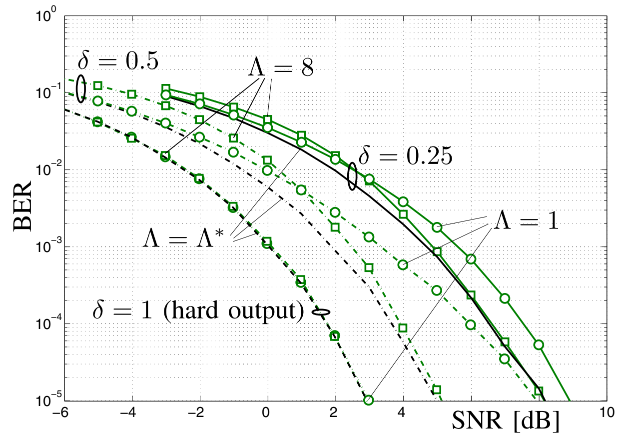

Figure 3. Performance results for BSPK transmission over the AWGN channel. The code rates corresponding to the different fractions of erased bits are $R_c = 1/2$ for $\delta = 0.25$, $R_c = 1/4$ for $\delta = 0.5$ and $R_c = 1/8$ for $\delta = 1$. Using the optimal clipping level leads to the best performance.

Only for the case $\delta = 1$ (hard output detection) do the schemes with fixed clipping level achieve the same performance. This is due to the fact that in this setup, the particular choice of Λ does not have any impact on the decisions made by the decoder. Note that the choice of a low fixed clipping level (e.g. $\Lambda = 1$) yields close-to-optimal performance for higher error rates while the choice of a higher clipping level (e.g. $\Lambda = 8$) is more suitable for lower error rates. Using the relation $P_b^* = 1/\left(e^{\Lambda^*} + 1\right)$ for obtaining the detector error probability corresponding to a specific choice of Λ^*, it becomes clear that $\Lambda = 1$ and $\Lambda = 8$ are the optimal choice for detector output error rates of $P_b \approx 0.27$ and $P_b \approx 3.4 \cdot 10^{-4}$. Taking the coding gain of the repetition code into account, this agrees very well with the simulation results for $\delta = 0.25$ and $\delta = 0.5$. It can also be observed that (unless all bits are hard detected), a larger fraction of hard detected bits results in larger performance losses if the clipping level is chosen inappropriately. This effect is well known from list MIMO detection [3], [4], [6]: the smaller the list size, the more sensitive is the detector to a correct choice of the clipping level.

Figure 4 shows results for the same system configuration, but transmission over the temporally i.i.d. fading Rayleigh channel (every transmitted symbol sees a new channel realization). Again, using the optimal LLR clipping level Λ^* results in superior performance over any scheme with a fixed clipping level, over the whole range of bit error rates and for all investigated code rates of the repetition code. In contrast to the AWGN case, there is a gap between the schemes using the optimal clipping level and those using a fixed clipping level, even for the case $\delta = 1$. This is due to the fact that the former includes the knowledge of the instantaneous channel realization, and thus the instantaneous channel signal-to-noise ratio, in the determination of the clipping level. Allowing the fixed clipping levels to vary as a linear function of the instantaneous channel SNR $E_s/N_0|h_i|^2$, where $|h_i|^2$ is the channel gain for the ith transmitted code bit, would remove this advantage in performance for the hard output case ($\delta = 1$).

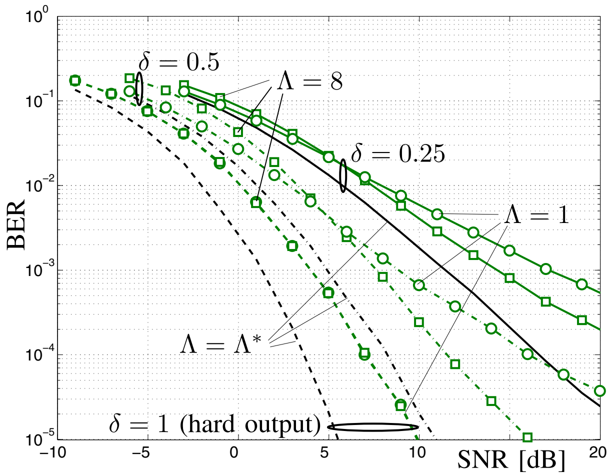

Figure 4. Performance results for BSPK transmission over the temporally i.i.d., flat fading Rayleigh channel. The code rates and corresponding fraction of hard output bits are the same as in Figure 3. Exploiting the knowledge of the instantaneous channel SNR enables the scheme using an optimal clipping level to significantly outperform schemes using a fixed clipping level.

Finally, Figure 5 shows results for transmission of a rate $1/2$ Turbo code over the fast fading Rayleigh channel. Although not proven to be optimal for this type of code, using the value of Λ^* for the clipping level again provides the best results. The raw error rate at the decoder input (and thus the output error rate of the detector) in the waterfall region is around 15% – corresponding to a clipping level of $\Lambda \approx 1.75$. This explains why the use of a rather low fixed clipping level, $\Lambda = 1$ provides consistently better performance than the use of a rather high clipping level, $\Lambda = 8$ used e.g. in [1].

V. Discussion and Conclusions

This paper considered the problem of finding the optimal soft representation (the "LLR clipping level") for hard detected bits in a setup where the detector generates a mix of hard and soft output. The optimal LLR clipping level was here defined to be the one minimizing the probability of decoding error for a soft input decoder. Specifically, transmission over the AWGN channel was considered using antipodal signaling and repetition codes of arbitrary rate. For this setup, it was proven that the optimal LLR clipping level Λ^* takes the form

$$\Lambda^* = \ln \frac{1 - P_b}{P_b},$$

where P_b is the bit error probability at the detector output. Simulation results illustrated the superior performance achievable by using this optimum clipping level.

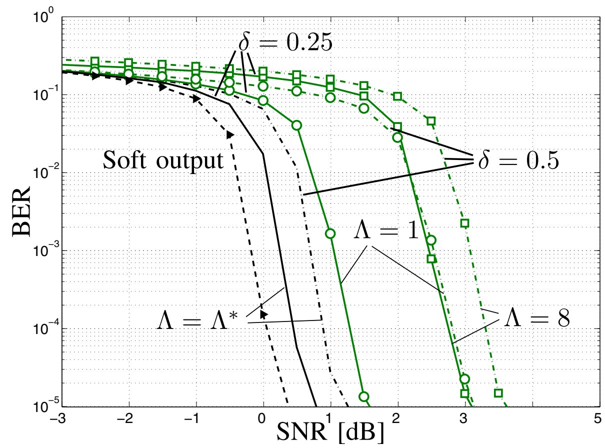

Figure 5. Performance results for BSPK transmission over the i.i.d. flat Rayleigh fading channel using a rate $R_c = 1/2$ Turbo code. Using the optimal clipping level enables a graceful performance degradation, depending on the fraction of hard detected bits. Using a fixed clipping level leads to poor performance, particularly for large clipping levels ($\Lambda = 8$).

A channel state information aware clipping level of a form similar to the one defined by Theorem 1 was used in the context of breadth-first list MIMO detection in [6]. Simulation results illustrated that choosing the clipping level in such a way maximized performance also in this rather complex system setup. Proving the optimality of the clipping level Λ^* for a wider range of communications channels and coding schemes thus constitutes an interesting path for further research.

For the case of low density parity check codes (LDPCC), a concatenation of repetition and single parity check codes, we conjecture that the proposed scheme also performs well. For single parity check codes, the soft output takes the form:

$$L_{spc} = \prod_{i=1}^{N-1} \text{sign}\,(L_i) \cdot \phi \left(\sum_{i=1}^{N-1} \phi\,(|L_i|) \right),$$

with $\phi(x) = \log\left((e^x + 1)/(e^x - 1)\right)$. Clearly, the decision on the considered bit only depends on the signs of the involved LLRs, but not their magnitude. The error rate would thus be independent of the chosen clipping level. Since we proved the optimality of Λ^* for repetition codes, it can thus be hoped that this choice is also appropriate for LDPCC.

References

[1] B. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Transactions on Communications*, vol. 51, pp. 389–399, Mar. 2003.

[2] J. Hagenauer and C. Kuhn, "The List-Sequential (LISS) algorithm and its application," *IEEE Transactions on Communications*, vol. 55, no. 5, May 2007.

[3] Y. de Jong and T. Willink, "Iterative tree search detection for MIMO wireless systems," *IEEE Transactions on Communications*, vol. 53, pp. 930–935, Jun. 2005.

[4] E. Zimmermann, "Complexity Aspects in Near-Capacity MIMO Detection-Decoding," Ph.D. dissertation, TU Dresden, Aug. 2007.

[5] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Transactions on Communications*, vol. 49, no. 10, pp. 1727–1737, Oct. 2001.

[6] D. L. Milliner, E. Zimmermann, G. Fettweis, and J. R. Barry, "Channel State Information Based LLR Clipping for List MIMO Detection," in *IEEE PIMRC'08*, Cannes, France, 15-18. Sep. 2008.