LDPC Coded OFDM with Alamouti/SVD Diversity Technique *

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Abstract

Two transmit two receive space-time processing with LDPC coding is evaluated for OFDM transmission. The two methods for space-time processing are Alamouti's combining and the SVD technique. The channel estimates are calculated and provided to the diversity combiner, the SVD filters and LDPC decoder. Noise variance estimates are provided to the LDPC decoder. Using the proposed scheme we can obtain a BER of 10^{-5} *at an SNR of* 2.6 d*B with spectral efficiency of* 0.4 bits/sec/Hz *and* 14.5 dB *with a spectral efficiency of* 4.2 bits/sec/Hz.

1. Introduction

Recently, many space-time techniques have been proposed for array-to-array communication systems when channel information is available at the receiver but not the transmitter. Such techniques provide transmit diversity in a flat-fading channel. Alamouti suggested a space-time code for two transmit antennas, which provides a diversity gain and has a very simple decoder [2]. If the transmitter knows the channel, this knowledge can be exploited to further improve the performance. In this case, it is known that a procedure based on singular-value decomposition (SVD) is optimal in information-theoretical sense [3]. The SVD scheme employs a transmit prefilter and a receive filter to diagonalize the array-to-array channel into a bank of independent scalar channels, where some of scalar channels have much larger channel gains than the fading channel. This advantage in channel gain can be interpreted as a diversity gain.

Space-time techniques can be integrated with orthogonal frequency division multiplexing (OFDM) for frequency-selective channels by applying either Alamouti's scheme (Alamouti-OFDM) or SVD scheme (SVD-OFDM) to each OFDM subcarrier.

In order to provide the channel coding gain, low-density parity-check (LDPC) codes are used for both Alamouti-OFDM and SVD-OFDM schemes. LDPC coding was first introduced by Gallager in the 1960's [4]. Several recent research results show that turbo codes can be expressed as LDPC codes, and well-structured irregular LDPC codes outperform turbo codes at high code rates. Aside from the superior performance, message-passing decoders of LDPC codes have a fully parallelized structure which can be realized with parallel connections of simple basic elements. Thus, we can achieve a good coding gain with relatively small complexity. LDPC codes inherently have random interleavers which can mitigate performance degradation due to deep attenuation of a data symbol on a subcarrier.

Parameter estimation for such a scheme is important. The Alamouti and SVD schemes require accurate channel estimates for their functioning whereas the LDPC codes require accurate noise variance estimates to calculate the log-likelihood ratios. This paper addresses this issue as well.

Throughout this paper, we only consider a 2×2 system with two transmit and two receive antennas. The SVD scheme is more flexible to extend to any number of antennas, since SVD exists for any size of matrix channel. For more than two transmit antennas, space-time block codes [5] replace Alamouti's scheme sacrificing the code rate.

2 Two Space-Time Strategies

First, consider a single-carrier 2×2 communication system in a flat-fading environment. Let h_{ij} denote the channel response at the i-th receive antenna from the j-th transmit antenna. At the receiver, the sampled base-band signal $r_{i,n}$ at the i-th receive antenna during the n-th symbol interval

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is represented in discrete time by

$$\begin{bmatrix} r_{1,n} \\ r_{2,n} \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_{1,n} \\ x_{2,n} \end{bmatrix} + \begin{bmatrix} w_{1,n} \\ w_{2,n} \end{bmatrix}, \qquad (1)$$

where $x_{j,n}$ is the transmitted sequence from the j-th transmit antenna, and $w_{i,n}$ is the complex additive white Gaussian noise. The 2×2 matrix $\mathbf{H}$ is called matrix channel, which is given by $\mathbf{H} = [h_{ij}]$.

2.1 Alamouti's Transmit Diversity Technique

In Alamouti's scheme, the transmitted sequences $x_{1,n}$ and $x_{2,n}$ are generated from the sequences s_1 and s_2 during two symbol intervals according to $\underline{x}_{2n} = [s_1, s_2]^T$ and $\underline{x}_{2n+1} = [-s_2^*, s_1^*]^T$, where the asterisk denotes complex conjugate.

At the receiver, assuming that $\mathbf{H}$ does not change during the $(2n)$-th and $(2n+1)$-st symbol intervals, the received signals

$$\begin{aligned} \underline{r}_{2n} &= \mathbf{H}\underline{x}_{2n} + \underline{n}_{2n} \\ \underline{r}_{2n+1} &= \mathbf{H}\underline{x}_{2n+1} + \underline{n}_{2n+1} \end{aligned} \qquad (2)$$

are combined by a matched filter:

$$\begin{aligned} &[h_{11}, h_{21}]^* \, \underline{r}_{2n} + [h_{12}, h_{22}]\underline{r}_{2n+1}^* \text{ for } s_1 \\ &[h_{12}, h_{22}]^* \, \underline{r}_{2n} + [h_{11}, h_{21}]\underline{r}_{2n+1}^* \text{ for } s_2. \end{aligned} \qquad (3)$$

This combining results in a separable decoding of s_1 and s_2 owing to the orthogonality of $\underline{x}_{2n}$ and $\underline{x}_{2n+1}$.

2.2 Space-Time Processing Based on SVD

A transmitter with knowledge of $\mathbf{H}$ can exploit this knowledge in order to approach Shannon capacity. In particular, it is known that a capacity-achieving transmitter bases its space-time processing on a channel SVD, $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^\dagger$, where † denotes the Hermitian transpose. A capacity-achieving transmitter will then preprocess the transmitted symbols with a unitary prefilter $\mathbf{V}$, and the receiver will postprocess with a unitary filter $\mathbf{U}^\dagger$ such that the overall system is diagonal: $\mathbf{D} = \mathrm{diag}[d_1, d_2] = \mathbf{U}^\dagger\mathbf{H}\mathbf{V}$, as shown in Fig. 1-a. The problem has thus been reduced to one of communication across two independent parallel scalar channels in Fig. 1-b, where the channel gains are singular values d_1 and d_2 ($d_1 \geq d_2$).

Once the matrix channel is diagonalized, there remains the problem of allocating bits and power to each of the scalar channels. In this paper, we use a fixed allocation instead of dynamic allocation to reduce the complexity with a marginal performance loss [6]. The fixed allocation will distribute all information bits to the first singular channel

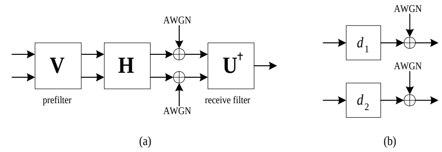

Figure 1. SVD scheme for a matrix channel.

d_1, and nothing to the second singular channel d_2, exploiting the fact $d_1 \geq d_2$. If we use the second singular channel, its error probability will bound the overall error probability, since d_2 becomes too small for reliable communcation.

Although the SVD scheme requires the channel information at the transmitter, it will significantly outperform Alamouti's scheme with very small increase in complexity. For a 2×2 system, the SVD scheme has an advantage of approximately 2.43 dB in SNR over Alamouti's scheme, when only the first singular channel is used [6]. This SNR gap increases up to approximately 3.6 dB when dynamic allocation is adopted.

3 Integration with OFDM

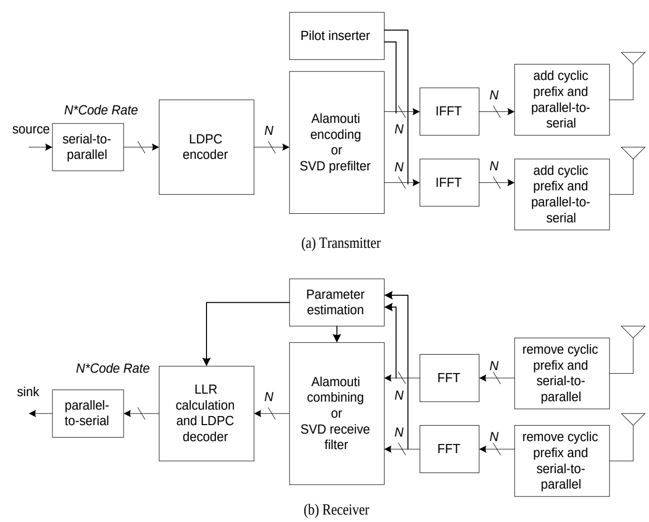

Figure 2. Block diagram of the proposed system

OFDM has become popular for wide-band wireless communications. It can be efficiently implemented in discrete time using inverse fast Fourier transform (IFFT) as a modulator and fast Fourier transform (FFT) as a demodulator. Here, a single-antenna OFDM is extended to the array-to-array antenna system [1]. An example of 2×2 OFDM system is illustrated in Fig. 2.

Let $\{S_{j,k}\}_{k=0}^{N-1}$ be the input symbols to the N-point IFFT for j-th transmit antenna. *Capital* letter in $S_{j,k}$ is used to

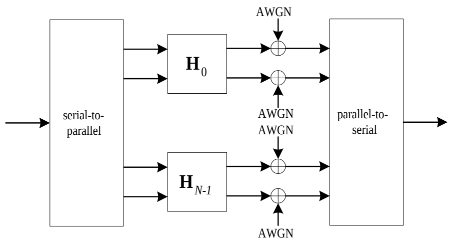

Figure 3. An equivalent matrix-channel model of 2×2 OFDM.

emphasize that input symbols are in the frequency domain. The output sequence of the IFFT is

$$s_{j,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_{j,k} \mathrm{exp}\left(j\frac{2\pi nk}{N}\right) \quad 0 \le n \le N-1. \tag{4}$$

A cyclic prefix is inserted in front of the IFFT output sequence. The time length of the cyclic prefix should be greater than the maximum delay spread of the channel. The main function of the cyclic prefix is to guard the OFDM symbol against inter-symbol interference. Hence, this cyclic prefix is called the guard interval of the OFDM symbol and has a time duration $T_g = GT$. The guard-inserted sequence is applied to a pair of balanced D/A converters, unconverted to radio frequency, and transmitted over the channel. The received sequence for the (vT_s)-th time instant after the removal of the guard interval given by

$$r_{i,n} = \sum_{j=1}^{2} \sum_{m=0}^{M-1} h_{ij,m,v(N+G)+n} s_{j,(n-m)_N} + w_{i,n}, \tag{5}$$

where $h_{ij,m,v(N+G)+n}$ is the channel impulse response at lag m and instant $v(N+G)+n$, from the j-th transmit antenna to the i-th receive antenna and T_s is the OFDM symbol period including the guard interval. The $w_{i,n}$ are complex additive white Gaussian noise samples with $\mathrm{E}[|w_{i,n}|^2] = 2\sigma^2$. The received sample sequence $\{r_{i,n}\}_{n=0}^{N-1}$ is demodulated as

$$\begin{aligned} R_{i,k} &= \mathrm{FFT}\{r_i\}(k) \\ &= \sum_{j=1}^{2} \eta_{ij,k} S_{j,k} + W_{i,k} \end{aligned} \tag{6}$$

where [7]

$$\eta_{ij,k} = \sum_{m=0}^{M-1} H_{ij,m}^{v}(0) \mathrm{exp}\left(\frac{-j2\pi km}{N}\right) \tag{7}$$

and where

$$H_{ij,m}^{v}(0) = \frac{1}{N} \sum_{n=0}^{N-1} h_{ij,m,v(N+G)+k}. \tag{8}$$

The 2×2 OFDM system is equivalently described by a bank of matrix channels, as shown in Fig. 3. The received samples at the k-th subcarrier in (6) can be rewritten as

$$\begin{bmatrix} R_{1,k} \\ R_{2,k} \end{bmatrix} = \mathbf{H}_k \begin{bmatrix} S_{1,k} \\ S_{2,k} \end{bmatrix} + \begin{bmatrix} W_{1,k} \\ W_{2,k} \end{bmatrix}, \tag{9}$$

where the matrix channel is

$$\mathbf{H}_k = \begin{bmatrix} \eta_{11,k} & \eta_{12,k} \\ \eta_{21,k} & \eta_{22,k} \end{bmatrix}. \tag{10}$$

Consequently, we can use either Alamouti's scheme or the SVD scheme to provide the diversity of $\mathbf{H}_k$.

By the same argument for a channel that is flat over each subcarrier, in SVD-OFDM, each matrix channel is diagonalized by SVD: $\mathbf{D}_k = \mathrm{diag}[d_{1,k}, d_{2,k}] = \mathbf{U}_k^{\dagger}\mathbf{H}_k\mathbf{V}_k$, with $d_{1,k} \ge d_{2,k}$. Then, the matrix-channel model in Fig. 3 further reduces to a bank of $2N$ parallel scalar channels.

We have the same problem of bit and power allocation as a single-carrier SVD scheme. In SVD-OFDM, however, the dynamic allocation requires much more complexity than the single-carrier system, since the number of scalar channels increases to $2N$. Thus, a fixed allocation algorithm is more attractive for SVD-OFDM. In this paper, we allocate the same number of bits to each $\mathbf{D}_k$. Then, the fixed allocation for single-carrier system is used for the allocation for $\mathbf{D}_k$. Power is distributed equally to all used channels, which is called on-off power allocation.

3.1 Parameter Estimation for the Proposed Scheme

Parameter estimation for the proposed scheme is carried out using the method described in [7]. Two consecutive blocks of a known sequence of samples $\{S_{j,k}\}_{k=0}^{N-1}$ which form the pilot symbols are transmitted. The N-point FFT coefficients of a chirp sequence are used as the pilot symbol. Chirp sequences are directly modulatable and are optimal for channel estimation. A chirp sequence can be represented as follows

$$s_n = \frac{1}{\sqrt{2}} \left\{ \cos\left(\frac{\pi}{N}n^2\right) + j\sin\left(\frac{\pi}{N}n^2\right) \right\}, \tag{11}$$

where $0 \le n \le N-1$. Since only two antennas are used at the transmitter, the training symbols transmitted from both the antennas can be identical thus simplifying the channel estimation problem. The channel estimates are utilized by the combiner in Alamouti's scheme, by the transmit and receive filters in the SVD scheme and for the log-likelihood ratio computation in the LDPC decoder. Noise variance estimates are provided to the LDPC decoder. The proposed technique gives sufficiently accurate channel estimates and almost perfect noise variance estimates.

3.2 Low-Density Parity-Check Codes

LDPC codes are specified by a sparse parity-check matrix and can be categorized into regular and irregular LDPC codes. The regular LDPC codes have parity-check matrices whose columns have the same number of ones. In this paper, we only consider regular LDPC codes.

A parity-check matrix, $\mathbf{P}$ of a (c, t, r) LDPC code has c columns, t ones in each column and r ones in a row. A (c, t, r) LDPC code has a code rate of $1 - t/r$. Gallager [4] showed that there is at least one LPDC code whose minimum distance, d_{min} grows linearly with block length c when $t > 2$. Therefore, we can expect a better coding gain with a longer code length, although the coding length is limited by practical considerations like decoding latency, decoder complexity etc.. The rate of growth of $d_{\min}$ is bounded by a nonzero number, which is determined by the selection of t and r.

The belief propagation algorithm has been widely adopted for decoding LDPC codes. MacKay [9] gives a good description of the iterative message passing decoder based on the belief propagation algorithm which can be implemented in either probability or log-probability domain. The decoder in this paper works in the log-probability domain. For the message-passing decoder, we need a log-likelihood ratio (LLR) of each bit. A general form of LLR computing formula is given by

$$\mathrm{LLR}(b_j) = \log \left[\frac{\sum_{i=1}^{2^{k-1}} \mathrm{P}(\mathbf{R}|b_j = 1, \mathbf{m}_j = m_i)}{\sum_{i=1}^{2^{k-1}} \mathrm{P}(\mathbf{R}|b_j = 0, \mathbf{m}_j = m_i)} \right] \tag{12}$$

where $\mathbf{R}$ is a received signal vector, b_j is j-th bit of a transmitted message, $\mathbf{m}_j$ is a message less the jth bit, m_i is one of 2^{k-1} possible symbols of $\mathbf{m}_j$ and each symbol carries k bits .

On an AWGN channel and flat fading, (12) can be expressed as

$$\mathrm{LLR}(b_j) = \log \left[\frac{\sum_{i=1}^{2^{k-1}} e^{-\frac{d(\mathbf{R}, \mathbf{c}_i^{b_j=1})^2}{2\sigma^2}}}{\sum_{i=1}^{2^{k-1}} e^{-\frac{d(\mathbf{R}, \mathbf{c}_i^{b_j=0})^2}{2\sigma^2}}} \right] \tag{13}$$

where $\mathbf{c}_i^{b_j=b}$ is a signal constellation for a message defined by m_i and b_j and $d(\mathbf{R}, \mathbf{c}_i^{b_j=b})$ is a distance between a received signal vector $\mathbf{R}$ and $\mathbf{c}_i^{b_j=b}$.

To prevent possible underflow or overflow, the equation can be modified to a more applicable form as

$$\begin{aligned} \mathrm{LLR}(b_j) = & \frac{\left(d_{\min}^0(j)\right)^2 - \left(d_{\min}^1(j)\right)^2}{2\sigma^2} \\ & + \log \left[1 + \sum_{\substack{i=1, \\ i \neq l_1}}^{2^{k-1}} e^{-\frac{d(\mathbf{R}, \mathbf{c}_i^{b_j=1})^2 - \left(d_{\min}^1(j)\right)^2}{2\sigma^2}} \right] \\ & - \log \left[1 + \sum_{\substack{i=1, \\ i \neq l_0}}^{2^{k-1}} e^{-\frac{d(\mathbf{R}, \mathbf{c}_i^{b_j=0})^2 - \left(d_{\min}^0(j)\right)^2}{2\sigma^2}} \right] \end{aligned} \tag{14}$$

where $d_{\min}^b(j) = d(\mathbf{R}, \mathbf{c}_{l_b}^{b_j=b}) = \min_{1 \le i \le 2^{k-1}} d(\mathbf{R}, \mathbf{c}_i^{b_j=b})$ and b is in $\{0, 1\}$.

Fig. 4 shows the BER performance of LDPC codes having code length, c = 1024 and code rates of 0.5, 0.75, 0.875 and 1.0 (uncoded) with 16 and 64-QAM modulation on an AWGN channel.

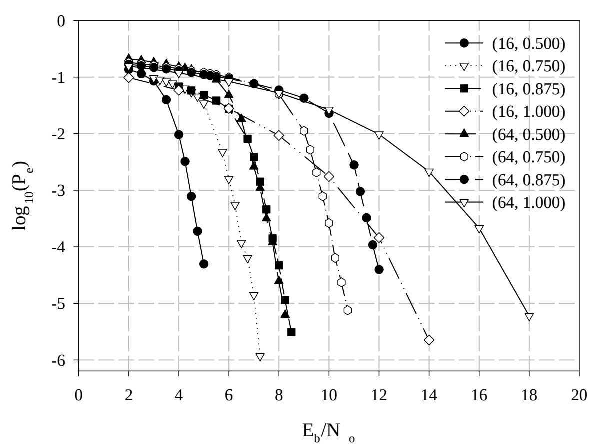

Figure 4. BER performances of LDPC codes having code length, c = 1024 and code rates of 0.5, 0.75, 0.875 and 1.0 (uncoded) with 16 and 64-QAM on an AWGN channel.

4 Simulation Results

Simulations are carried out in a frequency-selective faded quasi-static indoor environment. A 2×2 system is simulated. Each channel is assumed to be composed of 6 uncorrelated Rayleigh faded taps with the tap coefficients obtained from the modified Jakes simulator [8]. All the channels are uncorrelated and length of each channel impulse response is restricted to 100 ns. The Jakes simulator assumes uniformly distributed angle of arrival for the incoming incident waves. The complex low-pass channels are modeled as transversal filters with the sample-spaced taps. The symbol rate at the input of the OFDM modulator is 64 Mbaud. The performance is evaluated by sending $50,000$ OFDM symbols of block size $N = 1024$ and guard length $G = 64$. It is assumed that the maximum delay spread of the indoor channel (T_m) is less than the guard time (T_g). The carrier frequency is chosen to be 5.8 GHz. Simulations are carried out for LDPC code rates of $1/2$, $3/4$ and $7/8$ using BPSK, 16-QAM and 64-QAM constellations. The SNR per bit is defined as $E_b/N_o = 1/2\sigma^2 R\gamma$, where R is the transmission per symbol interval in bits/sec/Hz, and where γ denotes the code rate. We set $\mathrm{E}[|S_{1,k}|^2 + |S_{2,k}|^2] = 1$ and $\mathrm{E}[|\eta_{ij,k}|^2] = 1$.

The simulations are carried out for a Doppler frequency of 48.33 Hz corresponding to a velocity of 2.5 m/s. Two training symbols are sent for every 10 OFDM symbols for channel estimation. Quasi-static assumption implies that the time of arrival of rays can change from frame to frame but it remains constant for a particular frame. It is found that as symbol rate becomes higher, less frequent pilot transmissions are required for parameter estimation. We have assumed perfect time and frequency synchronization. The channel parameters are estimated using the technique described in [7]. The BER degradation due to imperfect channel estimation in the following simulations is around 1.1 dB. Once channel parameters are estimated, the same parameters are used for the entire frame until the transmission of the new training symbols.

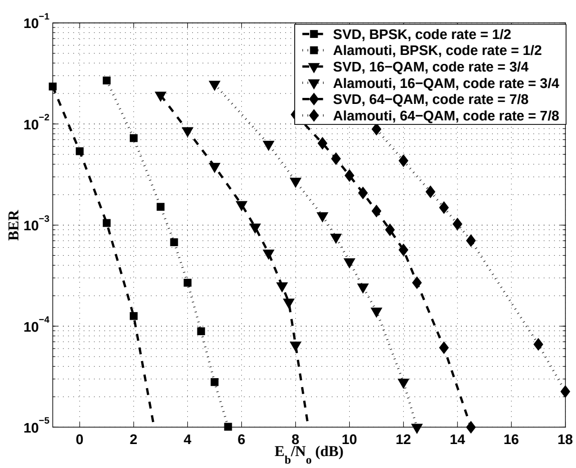

Figure 5. Performance of rate 1/2, 3/4 and 7/8 coded LDPC with BPSK, 16 and 64-QAM Alamouti/SVD diversity techniques over a frequency selective fading channel.

Fig. 5 illustrates the BER performance of Alamouti-OFDM and SVD-OFDM using BPSK. The performance improves tremendously when LDPC coding is introduced. Use of LDPC code along with the Alamouti's scheme provides a gain of around 5.8 dB as compared to the uncoded case at a BER of 10^{-3} and a $1/2$ rate code. Using the SVD scheme instead of Alamouti's scheme along with LDPC coding gives a further improvement of 2.3 dB.

Fig. 5 also shows the performance of the system using higher rate codes and higher order constellations. The SVD technique outperforms Alamouti' s scheme almost always by around 3 dB. The simulation results show that the proposed system can provide low BER at a high spectral efficiency and low SNR.

5 Concluding Remarks

Performance of a 2×2 space-time processing with LDPC coding is evaluated for OFDM transmission. The two methods for space-time processing are Alamouti' s scheme and the SVD technique. The channel estimates are calculated and provided to Alamouti' s combiner, the SVD filters and LDPC decoder. Noise variance estimates are provided to the LDPC decoder. By using the proposed scheme we can obtain a BER of 10^{-5} at an SNR of 2.6 dB with spectral efficiency of 0.4 bits/sec/Hz and 14.5 dB with a spectral efficiency of 4.2 bits/sec/Hz. Hence, the proposed system can provide low BER at a high spectral efficiency and low SNR.

References

[1] Y. (Geoffrey) Li , N. Seshadri and S. Ariyavisitakul "Channel Estimation For OFDM Systems With Diversity in Mobile Wireless Channels," *IEEE JSAC*, vol. 17, no. 3, March 1999.

[2] S. Alamouti, "A Simple Transmit Diversity Technique for Wireless Comminications," *IEEE JSAC*, Vol. 16, No. 8, October 1998.

[3] A. Reial, and S. G. Wilson, "Capacity-Maximizing Transmitter Processing for Fading Matrix Channels," *IEEE Commun. Theory Mini-Conference*, 1999.

[4] R. G. Gallager, *Low-Density Parity-Check Codes*, Cambridge, MA: MIT Press, 1963.

[5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans. on Information Theory*, vol. 45, no. 5, July 1999.

[6] J. H. Sung, and J. R. Barry, "Space-Time Processing with Channel Knowledge at the Transmitter," to appear, *IEEE EUROCON 2001*, Bratislave, July 2001.

[7] A. N. Mody and G. L. Stüber, “Parameter Estimation for OFDM with Transmit Receive Diversity,” *IEEE VTC* 2001, Rhodes, Greece.

[8] P. Dent, G. E. Bottomley and T. Croft, “Jakes Fading Model Revisited,” *Electronic Letters*, Vol. 7, pp. 1162-1163, June 1993.

[9] D. J. C. MacKay, “Good Error-Correcting Codes Based on Sparse Very Matrices,” *IEEE Trans. on Information Theory*, 45(2): 399-431, March 1999.