# Exploiting Error-Control Coding in Blind Channel Estimation

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***Abstract* — Despite the widespread use of forward-error control (FEC) coding, most channel estimation techniques ignore its presence, and instead make the simplifying assumption that the transmitted symbols are uncoded. However, FEC induces structure in the transmitted sequence that can be exploited to improve channel estimates. Furthermore, soft-output decoding can improve decision-driven techniques. In this work we propose a technique for exploiting FEC in channel estimation that combines iterative channel estimation with turbo equalization. We present one example showing that an estimator that exploits FEC can attain the same accuracy as one that ignores FEC, but with an SNR that is 6 dB lower.**

## I. INTRODUCTION

Practical communications systems use forward-error control (FEC) coding, which restricts the possible transmitted sequences so as to increase their minimum distance, thus reducing the signal-to-noise ratio (SNR) required to attain a given bit-error rate. Nevertheless, the presence of FEC codes is seldom exploited in decision-directed and blind estimation techniques. Rather, most estimators assume the channel inputs are independent, identically distributed (i.i.d.) over a finite alphabet. A likely explanation for this assumption is that, at sufficiently high SNR, even decision-directed blind iterative estimation techniques that ignore FEC can perform well [1,2]. There is then little incentive to incur the extra complexity required to exploit FEC. However, the last decade has seen the discovery of powerful FEC techniques that, with reasonable complexity, allow reliable transmission at an SNR only fractions of a dB from channel capacity [3-5]. When powerful codes are used at low SNR, estimation techniques that ignore FEC are doomed to fail.

There is little prior work that relates FEC to channel estimation. In [6] it was shown that FEC, though violating the i.i.d. assumption, does not hurt the performance of some blind equalizers that rely on this assumption. However, practical blind estimators that benefit from FEC were unknown until the estimators of [7-10] were proposed. These techniques combine the good performance of blind iterative channel estimation [1,2], shown in Fig. 1(a), and turbo equalization [11–13], shown in Fig. 1(b). Iterative channel estimation ignores FEC, and turbo equalization ignores channel estimation. As illustrated in Fig. 1(a) and (b), both are based on an iterative exchange of information between blocks.

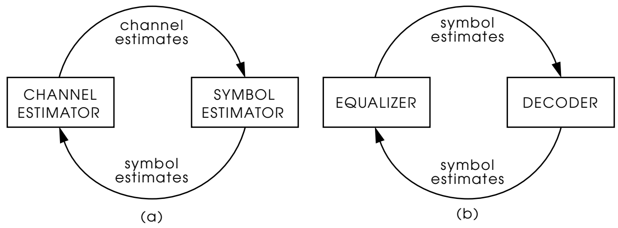


**Fig. 1.** A blind iterative channel estimator (a) iterates between channel estimation and equalization; a turbo equalizer (b) iterates between equalization and decoding.

In Fig. 1(a), the symbol estimator uses the channel estimates to compute soft estimates of the transmitted sequence. The channel estimator then uses these soft symbol estimates to improve the channel estimates, which in turn produce better symbol estimates, and so on. Likewise, in the turbo equalizer of Fig. 1(b), soft symbol information from the FEC decoder is used as *a priori* information to improve the soft symbol estimates produced by the equalizer. These estimates in turn are then passed again to the decoder, and so on. In this work, the two techniques of Fig. 1(a) and (b) will be combined into a single, practical method for exploiting FEC in blind channel estimation.

The FEC-aware schemes of [7-10] were based on the channel estimator of [1]. In contrast, we propose a blind iterative FEC-aware channel estimator based on the channel estimator of [2], which has lower complexity and improved convergence performance, being less likely to become trapped in a undesirable stationary point of the iterative scheme. Furthermore, the soft-output equalizer in [7-10] is based on the BCJR algorithm [8], which has complexity exponential in the channel memory. In contrast, we propose a low-complexity soft-output equalizer based on a modified decision-feedback equalizer (DFE) [14] which has complexity linear in the number of equalizer coefficients, making it feasible to apply the proposed techniques to channels with severe ISI.

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## II. Channel Model and Problem Statement

Consider the system model shown in Fig. 2, where a binary message $\boldsymbol{m} = [m_0, \ldots, m_{K-1}]$ of length $K$ is encoded by a rate $K/N$ encoder, producing a codeword $\boldsymbol{c} = [c_0, \ldots, c_{N-1}]$ of length $N$, with each symbol drawn from the binary alphabet $\{\pm 1\}$. The codeword is then permuted according to an interleaver $\pi$, resulting in the transmitted sequence $\boldsymbol{a} = [a_0, \ldots, a_{N-1}]$, with $a_k = c_{\pi(k)}$. Let $\boldsymbol{r} = [r_0, \ldots, r_{L-1}]$ denote the received sequence, where $L = N + \mu$, and where:

$$r_k = \boldsymbol{h}^T \boldsymbol{a}_k + n_k , \tag{1}$$

where the channel impulse response is $\boldsymbol{h} = [h_0, \ldots, h_\mu]^T$, $\boldsymbol{a}_k = [a_k, \ldots, a_{k-\mu}]^T$ is a vector of channel inputs, and $n_k \sim \mathcal{N}(0, \sigma^2)$ represents the white Gaussian noise.

The *blind estimation problem* is to estimate $\boldsymbol{h}$ and $\sigma^2$ from $\boldsymbol{r}$, without the assistance of training (i.e., without knowledge of $\boldsymbol{m}$). Instead, the estimator must rely solely on its knowledge of the probability distribution function (pdf) of $\boldsymbol{m}$, assumed here to be uniform, knowledge of the encoder and interleaver, and knowledge of the channel model (1). The *joint maximum-likelihood estimator* [15] would choose $\boldsymbol{h}$, $\sigma^2$, and $\boldsymbol{m}$ so as to jointly maximize $p(\boldsymbol{r} \mid \boldsymbol{m}; \boldsymbol{h}, \sigma^2)$, the conditional pdf of $\boldsymbol{r}$ given $\boldsymbol{h}$, $\sigma^2$, and $\boldsymbol{m}$. Its complexity is prohibitive, however, and thus we seek lower complexity approximations.

## III. State of the Art

Unlike the joint ML estimator, a conventional receiver separates the tasks of channel estimation, equalization and FEC decoding: first the channel is estimated, then the transmitted sequence is equalized, and finally the equalized sequence is decoded. This approach is suboptimal, and performance can be improved with iterative techniques such as turbo equalizers and iterative channel estimators. In this section we briefly review these two techniques.

A key ingredient of iterative algorithms is the use of soft information, which, for a BPSK system, takes the form of the log-likelihood ratio (LLR):

$$\lambda_k = \log \frac{\Pr(a_k = 1 \mid \boldsymbol{r})}{\Pr(a_k = -1 \mid \boldsymbol{r})} . \tag{2}$$

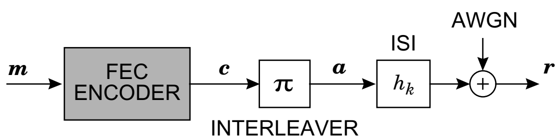


**Fig. 2.** Transmitter and channel model.

The LLR fully describes the *a posteriori* probability (APP) $\Pr(a_k \mid \boldsymbol{r})$ of $a_k$. The BCJR algorithm [8] computes $\lambda_k$ exactly, while the decision-aided equalizer [16] and the soft-output DFE [2] provide reduced-complexity approximations. The LLR can be used to compute two important quantities:

- Maximum *a posteriori* (MAP) decision: $\hat{a}_k = \text{sign}(\lambda_k)$ ,
- *A posteriori* expectation: $\tilde{a}_k = \text{E}[a_k \mid \boldsymbol{r}] = \tanh(\lambda_k / 2)$ .

### A. Blind Iterative Channel Estimators [1-2]

The blind channel estimation problem can be simplified by ignoring the presence of the code, and instead assuming that the transmit symbols are i.i.d. uniform, and choosing $\boldsymbol{h}$, $\sigma^2$, and $\boldsymbol{a}$ so as to jointly maximize the likelihood $p(\boldsymbol{r} \mid \boldsymbol{a}; \boldsymbol{h}, \sigma^2)$. Even this simplified ML estimator is infeasible when the codeword length $N$ is large, however, since it would require that all $2^N$ possible sequences $\boldsymbol{a}$ be tested. Blind iterative channel estimators based on the expectation-maximization (EM) algorithm [1] compute a sequence of estimates $\hat{\boldsymbol{h}}^{(i)}$ and $\hat{\sigma}^{(i)}$ with non-decreasing likelihood, and hence can converge to the simplified ML solution with proper initialization.

The EM algorithm has high complexity and is prone to being caught in a local maximum of the likelihood function. To mitigate these problems, the extended-window EM algorithm was proposed [2]. Let $\tilde{a}_k^{(i)} = \text{E}[a_k \mid \boldsymbol{r}; \hat{\boldsymbol{h}}^{(i)}, \hat{\sigma}^{(i)}]$ be an estimate of the *a posteriori* expected value of $a_k$, computed by the symbol estimator at iteration $i$ assuming that the channel and noise standard deviation are given by $\hat{\boldsymbol{h}}^{(i)}$ and $\hat{\sigma}^{(i)}$. Given $\tilde{a}_k^{(i)}$, the extended-window channel estimator computes

$$g_n = \frac{1}{L} \sum_{k=0}^{L-1} r_k \tilde{a}_{k-n}^{(i)} , \quad \text{for } n \in \{-\mu, \ldots, 2\mu\} . \tag{3}$$

The new channel estimate for iteration $i + 1$ is then taken to be $\hat{\boldsymbol{h}}^{(i+1)} = [g_\delta, \ldots, g_{\delta+\mu}]^T$, where $\delta \in \{-\mu, \ldots, \mu\}$ is chosen so as to maximize the energy of $\hat{\boldsymbol{h}}^{(i+1)}$. The noise variance is computed as:

$$\hat{\sigma}^2_{(i+1)} = \frac{1}{L} \sum_{k=0}^{N-1} \left| r_k - (\hat{\boldsymbol{h}}^{(i)})^T \hat{\boldsymbol{a}}_k^{(i)} \right|^2 , \tag{4}$$

where $\hat{\boldsymbol{a}}_k^{(i)} = [\text{sign}(\tilde{a}_{k-\delta}^{(i)}), \ldots, \text{sign}(\tilde{a}_{k-\delta-\mu}^{(i)})]^T$.

### B. Turbo Equalizers [11-13]

As seen in Fig. 1(b), a turbo equalizer is based on the exchange of information between the equalizer and the decoder. Key to its success is the fact that only extrinsic information is exchanged. The extrinsic information provided by the equalizer can be seen as the information on the transmitted bits gained at the equalizing stage by exploiting only the structure of the channel. Similarly, the extrinsic information provided by the decoder contains the information about the transmitted bits that was not apparent to the FEC-ignorant equalizer. The equalizer and decoder use this extrinsic information as *a priori* information to compute new values for $\lambda_k$. The extrinsic information, denoted by $\lambda'_k$, is computed as the difference between this new value of $\lambda_k$ and the extrinsic information used in its computation.

An ideal receiver that jointly equalizes and decodes would find the information sequence $\boldsymbol{m}$ that maximizes $p(\boldsymbol{r} \mid \boldsymbol{m})$. Solving this problem exactly is computationally hard, since the presence of the interleaver implies that the number of states in the joint encoder/channel super-trellis would be large. Turbo equalization provides a low-complexity approximation.

### C. FEC-Aware Blind Channel Estimation

One important aspect of turbo equalizers is that they provide soft estimates of the transmitted sequence that benefit from the FEC code structure, and are much more reliable than the estimates provided by an equalizer alone. Its seems natural that using this information for channel estimation should provide better results than using FEC-ignorant symbol estimates, as is done in Fig. 1(a). Thus, we propose the channel estimator of Fig. 3, in which the symbol estimator in Fig. 1(a) is replaced by the turbo equalizer of Fig. 1(b).

The proposed estimator of Fig. 3 iterates between three blocks: a channel estimator, a soft-output equalizer, and a soft-output FEC decoder. A receiver would have to perform these functions anyway, so their presence alone does not imply any added complexity; the only added complexity is due to fact that these functions are performed multiple times as the algorithm iterates. The proposed estimator is a straightforward

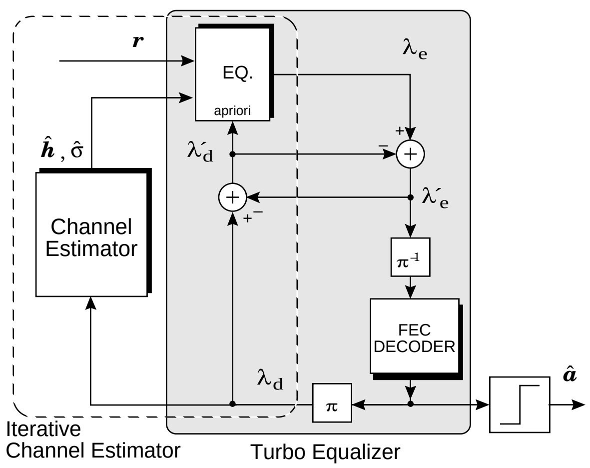


**Fig. 3.** Integrating channel estimation with turbo equalization.

combination of the FEC-ignorant estimator of [2] with a conventional turbo equalizer, but with two distinctions. First, each instance of the turbo equalizer may involve only one iteration. Second, the a priori information for the soft-output equalizer is not initialized to zero for each instance of the turbo equalizer, but instead, extrinsic information from the last instance is used as the initial a priori information in the next.

It is instructive to compare the proposed estimator with a conventional receiver that performs channel estimation just once, then uses these estimates in a turbo equalizer. The proposed estimator can be derived from this conventional receiver by making just one modification: Rather than using the initial channel estimates for every turbo iteration, the proposed receiver occasionally improves the channel estimates based on tentative soft decisions. Specifically, every $J$-th iteration of the turbo equalizer, the soft-symbol estimates produced by the FEC decoder are used in (3) and (4) to produce better channel estimates, which are then used for the next $J$ iterations. The choice of $J$ is a design parameter that can affect convergence speed, steady-state behavior, and overall complexity. Because of the low complexity of the channel estimator relative to the complexity of the equalizer and FEC decoder, we have found empirically that $J = 1$ is a reasonable choice. With this choice, each time the FEC decoder passes extrinsic information to the equalizer, the channel estimates are simultaneously improved. This is only marginally more complex than a conventional receiver that uses turbo equalization, but the performance improvement that results can abe significant.

## IV. Soft DFE with *a priori* Information

In [2] we showed how a DFE can be used to approximate a soft-output BCJR equalizer, but with significantly lower complexity. We now expand that idea to incorporate *a priori* information, making it useful as an inner equalizer in a turbo equalization system.

Let $z_k$ be the output of an MMSE-DFE with $r_k$ as its input, $N_f$ forward taps and $N_b$ backward taps. This relationship is illustrated in Fig. 4. Roughly speaking, the equalizer eliminates ISI from its output, so that we may write $z_k \approx Aa_k + v_k$, where $A$ is the amplitude of the equivalent memoryless channel between $a_k$ and $z_k$, and $v_k$ is the equivalent noise with variance $\sigma_v^2$. This noise includes residual ISI, but we may approximate it as AWGN. We may thus approximate the extrinsic LLR by $\lambda'_k \approx 2Az_k/\sigma_v^2$. This extrinsic information depends only on the structure of the ISI channel. Any a priori information from the FEC decoder should be added to $\lambda'_k$ to produce the full LLR $\lambda_k$. Since the full LLR will provide much more reliable decisions than the extrinsic information alone, $\lambda_k$ should be used to compute symbol estimates in the feedback loop of the DFE, as illustrated in Fig. 4.

Computation of the coefficient vectors $\boldsymbol{f}$ and $\boldsymbol{b}$ is easy if the channel is known [14, p. 542]. Obviously, the channel information is not available, but in keeping with the iterative paradigm of Fig. 3 we may compute $\boldsymbol{f}$ and $\boldsymbol{b}$ using the current estimates $\hat{\boldsymbol{h}}$ and $\hat{\sigma}$. We also propose to use the soft information $\tilde{a}_k = \tanh(\lambda_k/2)$, as opposed to the traditional hard information coming from a slicer, in the feedback loop. As in [2], we estimate $A$ and $\sigma_v^2$ using a scalar channel version of EM.

## V. Simulation Results

In this section we present simulation results that illustrate the performance of the proposed estimation algorithm. For all the experiments, the channel estimates were initialized by measuring the energy of the received signal, and assigning half to signal and half to noise, yielding $\hat{\sigma}^2_{(0)} = \sum_{k=0}^{L-1} r_k^2/(2L)$ and $\hat{\boldsymbol{h}}^{(0)} = [\hat{\sigma}_{(0)}, 0, \ldots, 0]$.

We begin by showing how exploiting FEC can improve channel estimates. Consider the system of Fig. 2, and assume that $K = 2048$ bits are encoded by a rate $1/2$ recursive systematic convolutional (RSC) code with parity generator polynomial $(1 + \mathrm{D}^2)/(1 + \mathrm{D} + \mathrm{D}^2)$. The resulting 4096 coded bits are interleaved with a random interleaver and transmitted through an ISI channel with $\boldsymbol{h} = [0.5, 0.7, 0.5]$. We use the DFE-based soft-output equalizer of the previous section, with $N_f = 15$ forward and $N_b = 2$ feedback coefficients.

In Fig. 5 we plot the mean-square estimation error $\mathrm{MSE} = \mathrm{E}[\|\hat{\boldsymbol{h}} - \boldsymbol{h}\|^2]$ as a function of the per-bit SNR $E_b/N_0$ after five iterations, for both the proposed estimator that exploits FEC as well as an iterative blind estimator [2] that ignores FEC. Also shown are the same two curves when the DFE is replaced by BCJR. We see that the DFE-based estimator that exploits FEC can attain the same level of accuracy as the one that ignores FEC, but with an SNR that is 6 dB lower. Furthermore, the DFE-based estimator that exploits FEC requires only 2 dB more SNR than the a BCJR-based estimator that exploits FEC, to achieve the same MSE.

It is also interesting to note that the bit-error rate (BER) performance of the proposed blind schemes are comparable to that of a receiver with full channel knowledge. This can be seen in Fig. 6, where we plot the BER *versus* $E_b/N_0$ for several iterations of the DFE-based and BCJR-based turbo equalizers with channel knowledge, as well as the channel estimators based on these two algorithms. We see that as the number of iterations increases, the gap between the blind and the non-blind equalizers decreases, until it is almost closed. The blind scheme is seen to converge in about the same

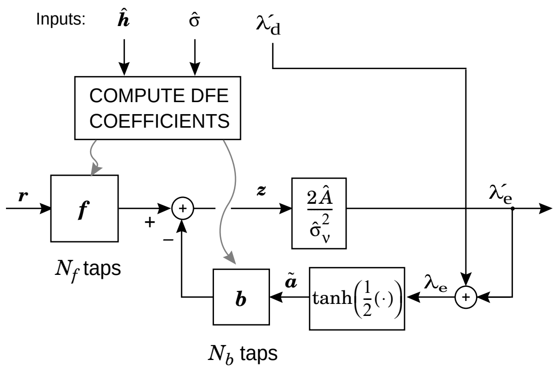


**Fig. 4.** A soft-output equalizer based on a MMSE-DFE.

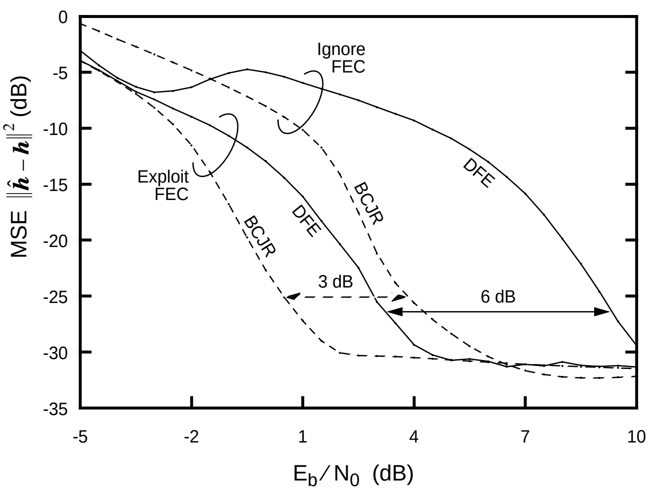


**Fig. 5.** .Mean-square estimation error versus SNR.

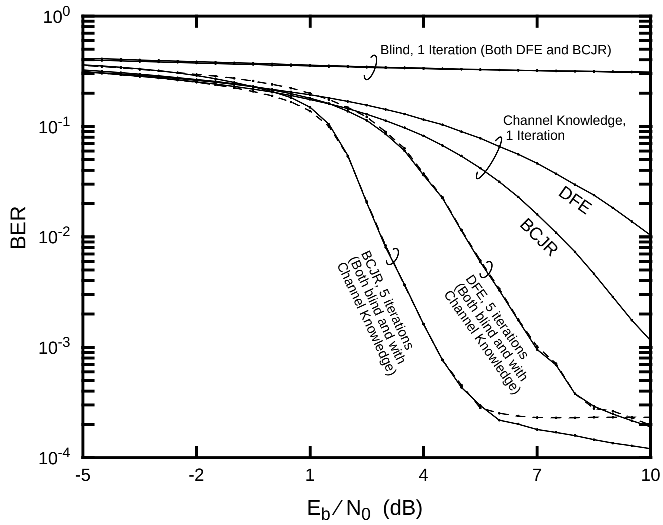


**Fig. 6.** Bit-error rate versus SNR for both DFE and BCR, both with channel knowledge and using blind estimates.

number of iterations as the scheme with channel knowledge. Furthermore, we again see that the DFE-based system requires only 2 dB more SNR than the BCJR-based one for a BER of $10^{-2}$. Another important feature made clear in this plot is the good use that the proposed DFE makes of *a priori* information, as evidenced by the 5 dB gap between the first and last iteration of the DFE-based system with channel knowledge.

To analyze the performance of the channel estimator across different channels, we tested its performance over an ensemble of 1000 randomly generated channels. In each case, $K = 400$ message bits were encoded by a rate $1/4$ serially concatenated turbo code using two identical rate $1/2$ RSC encoders, each with parity generator $(1 + D^2)/(1 + D + D^2)$. The channels were generated randomly according to $\boldsymbol{h} = \boldsymbol{u}/\|\boldsymbol{u}\|$, where $\boldsymbol{u} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ is a circularly symmetric Gaussian random vector of length five, and the noise variance was chosen so that $E_b/N_0 = 2$ dB. The receiver used the BCJR algorithm for both equalization and decoding. The turbo decoder went through only one iteration ($J = 1$) for each iteration of the overall scheme.

In Fig. 7, we plot the estimated probability density function $p_e(e)$ for the estimation error $e = \|\hat{\boldsymbol{h}} - \boldsymbol{h}\|$, produced after 60 iterations of the FEC-aware and the FEC-ignorant extended-window channel estimators. We observe that the FEC-aware estimator produced errors larger than –10 dB in only 2.7% of the experiments, while the errors produced by the FEC-ignorant estimator were larger than –10 dB in 82.9% of the experiments. To test the quality of these estimates, we used

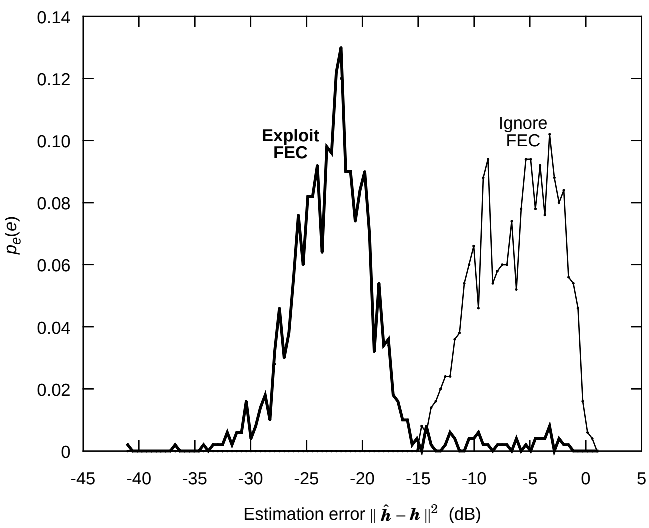


**Fig. 7.** Estimate of pdf of estimation error $e = \|\hat{\boldsymbol{h}} - \boldsymbol{h}\|^2$, estimated from histogram over 1000 random channels.

them to perform a turbo equalization for each trial. After 30 iterations of the turbo equalizer, we could recover the transmitted codeword without errors for 90.7% of the channels using the FEC-aware estimates, while this was possible for only 12.3% of the channels using the FEC-ignorant estimates. The benefit of using FEC information for channel estimation are thus clear.

It is well known that blind channel estimators cannot resolve delays. A blind equalizer that estimates $\hat{a}_k \approx a_{k-1}$ is often just as good as one that estimates $\hat{a}_k \approx a_k$. However, this delay is problematic when the transmitter includes a random interleaver. For example, if we deinterleave $a_{k-1}$ instead of $a_k$, the result will bear no resemblance whatsoever to $c_{k-1}$. The frame boundaries must be identified before FEC decoding is meaningful. In the random channel experiment, we assumed perfect frame synchronization. A truly blind frame synchronizer would be difficult to implement in practice. Fortunately, in practice there will exist side information from preambles and pilot symbols that can be used to synchronize the frame in a semiblind fashion.

## VI. Conclusions

We proposed a blind iterative channel estimator that benefits from the presence of forward-error correction coding. The benefits can be significant. In one example, compared to an estimator that ignores FEC, an estimator that exploits FEC can attain the same performance with 6 dB less SNR. In our simulations, the performance of the proposed blind schemes was as good as that of a turbo equalizer with channel knowledge, and it converged equally fast. We also proposed a soft-output equalizer based on a DFE that incorporates *a priori* information. We showed that, even though a blind FEC-aware scheme based on this equalizer performs slightly worse than the scheme based on the BCJR algorithm, the performance of the DFE-based system improves as the iterations progress, providing a gain of 5 dB over a non-iterative system with channel knowledge that employs a conventional DFE followed by a decoder.

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