Announce

- HW1 posted, not to be turned in

Lecture:

- sampling theorem
- combining complex envelopes and sampling: canonical processing
Pop Quiz

Find the Hilbert transform of $x(t) = g(t)\cos(5\pi t)$, where $g(t) = \frac{\sin(\pi t)}{\pi t}$.
Pop Quiz

The Hilbert transform is \( \hat{x}(t) = g(t)\sin(5\pi t) \):
Pop Quiz

(a) If $x(t) = g(t) \cos(5\pi t)$, find $\hat{x}(t)$

(b) Answer in one word, as best you can:
What does the Hilbert transform do to a signals I & Q components?
Pop Quiz: True or False

The real envelope $e(t)$ is independent of $f_0$. 
Pop Quiz: True or False

The real envelope $e(t)$ is independent of $f_0$.

True: \[ \tilde{s}(t) = \left( s(t) + j\hat{s}(t) \right) e^{-j2\pi f_0 t} \]

\[ \Rightarrow e(t) = |\tilde{s}(t)| = \sqrt{s^2(t) + \hat{s}^2(t)} \]
Example: (Real) Envelope of Sinc?

Find the real envelope of \( s(t) = \frac{\sin(\pi t)}{\pi t} \):
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Solution:
\[
e(t) = \left| \frac{\sin(\pi t/2)}{\pi t/2} \right|
\]
The FT of an Impulse Train?

\[ p(t) = \sum_k \delta(t - kT) \]

\[ P(f) = \frac{1}{T} \sum_k \delta(f - \frac{k}{T}) \]
Sampling in 2 Steps

1. Multiply by $\sum_k \delta(t - kT)$

2. Convert train of continuous-time impulses to train of discrete-time impulses:

$$x_k = x(kT)$$

Compare the Fourier transform of these two signals. They are identical!

Why?
The Impact of Sampling on the F.T.

If the Fourier transform of $x(t)$ looks like this:

Then the F.T. of $x_k = x(kt)$ is:

$$X(e^{j2\pi ft}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} X(f - \frac{m}{T})$$
Nyquist Sampling Theorem:

Perfect reconstruction is possible when $1/T > 2W$.

How?

Connects dots:

Linear interpolation $\Rightarrow g(t) = \text{triangle}$.

Bandlimited interpolation $\Rightarrow g(t) = \text{sinc function}$.
Combine Sampling Theorem with Complex Envelope:

Any passband signal \( s(t) \) that is bandlimited to bandwidth \( W \):
Combine Sampling Theorem with Complex Envelope:

Any passband signal $s(t)$ that is bandlimited to bandwidth $W$:

In other words, any such signal is uniquely specified by:

- a carrier frequency $f_0$
- a symbol rate $f_s = W$
- a complex-valued discrete-time sequence $\{s_k\}$. (How to find it?)
Example: Design Comm System

\[ s(t) \]

\[ r(t) \]

FCC LIMIT

2.39 GHz

2.41 GHz
Example: Design Comm System

\[ a_k \rightarrow \text{DAC} \rightarrow \tilde{s}(t) \rightarrow \text{UP} \rightarrow s(t) \rightarrow \text{Antenna} \rightarrow r(t) \rightarrow \text{DOWN} \rightarrow \tilde{r}(t) \rightarrow \text{ADC} \rightarrow r_k \]

- \( a_k \) input
- DAC: Digital-to-Analog Converter, 20 MHz
- \( \tilde{s}(t) \) output of DAC
- UP: Upconverter, 2.4 GHz
- FCC LIMIT: 2.39 GHz to 2.41 GHz
- \( s(t) \) output of UP
- Antenna
- \( r(t) \) output of Antenna
- DOWN: Downconverter, 2.4 GHz to 20 MHz
- \( \tilde{r}(t) \) output of DOWN
- ADC: Analog-to-Digital Converter, 20 MHz
- \( r_k \) output of ADC
Quad Mod & Demod OK

\[ s(t) = a_k(I) \cdot \cos(2\pi(2.4 \text{GHz})t) + a_k(Q) \cdot \sin(2\pi(2.4 \text{GHz})t) \]

\[ r(t) = \cos(2\pi(2.4 \text{GHz})t) \cdot s(t) + \sin(2\pi(2.4 \text{GHz})t) \cdot s(t) \]

\[ x_k = \text{ADC} \cdot \text{LPF} \cdot 10 \text{ MHz} \]

\[ y_k = \text{ADC} \cdot \text{LPF} \cdot 10 \text{ MHz} \]
Alphabet

• Define $a_k = a_k(I) + j a_k(Q)$ as $k$-th complex-valued “symbol”
• Assume $a_k$ drawn independent/uniform from finite alphabet $A$ of size $|A|$  
  $\Rightarrow R_b = W \log_2 |A|$

• Common choices
  - $A = \{\pm 1\}$  “2-PSK = BPSK”
  - $A = \{1, j, -1, -j\}$  “4-PSK = QPSK”
  - $A = \{\pm 1 \pm j\}$  “4-QAM”
PSK Alphabets

$M = 2$

$M = 4$

$M = 8$

$M = 16$

$M = 32$