Lecture 1: Thu Jan 11, 2018

Lecture

• context of 6602, modulation vs coding vs compression, etc.
• SNR
• The Shannon limit
• Power vs. Bandwidth
PREREQUISITE: ECE 6601. Strong background in probability is a must.

COURSE OBJECTIVE:
To study the design and implementation of digital communications systems.

OFFICIAL TEXT:


SUPPLEMENTAL READING:

INSTRUCTOR:

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E-Mail: barry@ece.gatech.edu  
Office Hours: after class, or by appointment.

WEBSITE: http://barry.ece.gatech.edu/6602/

HONOR CODE:

You are expected to uphold the honor code (http://www.honor.gatech.edu).

HOMEWORK:

Homework is assigned roughly once a week. May or may not be graded, depending on whether we are assigned a GTA.  
It is due at the beginning of class one week after it is assigned.  
Collaboration on homeworks is encouraged.  
*Copying* homework is an honor code violation.
GRADING:

Homework 10%
Quiz 1 (Thu Feb 15) 20%
Quiz 2 (Thu Mar 15) 20%
Quiz 3 (Thu Apr 19) 20%
Final Exam (Mon Apr 30) 30%
The "PHY" layer

Interface to the physical world

PHY tools:
- modulation, line coding, scrambling, precoding, error-control coding, equalization, synchronization, channel estimation, interference cancellation, space-time coding, multiuser detection, MIMO detection, turbo processing ...

Tangible outcomes
- modems, baseband processors
- read channels
- 100G transceivers
Coding + Deep Space

— “A Marriage Made in Heaven”

• plenty of bandwidth
• weak signal (path loss, transmit constraints)
  ⇒ power-limited linear AWGN channel model
  ⇒ each dB is incredibly valuable
    (range, launch costs, scientific)
• detector complexity nearly unlimited
  ⇒ sophisticated algorithms OK
Trading Complexity for Performance

1958 Explorer (uncoded)

1969 Mariner (rate-6/32 Reed-Muller/biorthogonal)

1977 Voyager (rate-1/3, $\mu = 6$, conv. code)

1968 Pioneer (rate-1/2, $\mu = 20$, sequential)

1981 Voyager: e-rate-1/2, $\mu = 6$, + RS

1989 Galileo rate-1/4, $\mu = 14$ + RS

1995 Galileo-S rate-1/4, $\mu = 13$ + RS$\times 4$

1997 Cassini rate-1/6, $\mu = 14$ + RS

512 b/s

[F. Pollara, Descanso Workshop, 1998.]
Pathfinder
Photo from Pathfinder
Pathfinder Modulation

BPSK ±60°

\[ f_0 = 8.43 \text{ GHz} \]
Trading Complexity for Performance

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1968 Pioneer (rate-1/2, μ = 20, sequential)
1981 Voyager: e-rate-1/2, μ = 6, + RS
1989 Galileo rate-1/4, μ = 14 + RS
1997 Cassini rate-1/6, μ = 14 + RS
2004 (rate-1/4 turbo, 2×16-state cc)

[F. Pollara, Descanso Workshop, 1998.]
Figure 1

- SOURCE CODER
- ERROR-CONTROL CODER
- MOD
- DEMOD
- NOISY DISPERSIVE CHANNEL

Symbols and Waveforms:
- Error-Control Code
- Bits
- Symbols
- Modulation
- Demodulation
Source Coding = Data Compression

Associated with each discrete source is an entropy $H$ bits/symbol.

The compression limit:

$\exists$ a compressor to $B$ bits/symbol with $\Pr[\text{err}] \to 0$ as $N \to \infty \iff B > H$. 
Separation Results

The Source-Channel Coding Separation Theorem

- compress source independent of channel
- channel encoder independent of source
A Communications System

Equivalent model:

A typical frequency response:

\[ |H(f)|^2 \text{ (dB)} \]

\[ W = 2000 \text{ Hz} \]
How to Communicate Across This Channel?

\[ s(t) \xrightarrow[-W]{-W} f \xrightarrow{+} r(t) \]
How to Communicate Across This Channel?

DAC → $s(t)$ → $-W$ → $W$ → $f$ → $+$ → $n(t)$ → $r(t)$

rate = $2W$
How to Communicate Across This Channel?

$a_k \in \mathcal{A}$

DAC

$s(t)$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

$\rightarrow$

rate = $2W$

$\mathcal{A}$

$W$

$-W$

$n(t)$

$r(t)$
Before Shannon

Nyquist [1924]: The maximum symbol rate is $2W$ symbols/second. If the symbol alphabet is $A$, then each symbol conveys $\log_2 |A|$ bits.

$$ \Rightarrow R_b = 2W \cdot \log_2 |A|. $$

Hartley [1928]: With amplitude constraint $\pm A$, and noise margin $\pm \Delta$, the maximum alphabet size is $|A| = 1 + A/\Delta$:

$$ \Rightarrow R_b = 2W \cdot \log_2 \left(1 + \frac{A}{\Delta}\right). $$

Modulation was instantaneous.

To fight random noise:
Increase BW, increase power, live with “inevitable” errors.
Turning Point: Claude Shannon, 1948

The father of info theory & modern communication theory
1948 paper — *A (The) Mathematical Theory of Communication*

- quantified information; separation theorem; noisy channel theorem

The “capacity” of an AWGN channel with bandwidth $W$ is:

$$C = W \cdot \log_2 \left(1 + \frac{P}{N_0 W}\right) \text{ bits/second.}$$

Capacity is a speed limit:

$$\exists \text{ a modem achieving bit rate } R_b \text{ with no errors } \iff R_b < C.$$

How to get close?
The Data Rate is Limited by 3 Key Parameters

- signal power $P$ [W]
- one-sided noise density $N_0$ [W/Hz]
- bandwidth $W$ [Hz]

The 3 combine to determine:

- **signal-to-noise ratio:** $\text{SNR} = \frac{P}{N_0 W}$
- **capacity:** $C = W \cdot \log_2 \left(1 + \frac{P}{N_0 W}\right)$
Pop Quiz

Q1: How much SNR does Shannon need to communicate at $R_b = 2 \text{ Gb/s}$ across a 200-MHz channel?
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A1: Solve capacity equation $\Rightarrow SNR = 2^{R_b/W} - 1$.

\[
= 2^{10} - 1 \\
= 1023 \\
= 30.1 \text{ dB}.
\]
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Q2: What happens to capacity if we remove bandwidth constraint?
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A1: Solve capacity equation $\Rightarrow SNR = 2^{R_b/W} - 1.$

$$= 2^{10} - 1$$

$$= 1023$$

$$= 30.1 \text{ dB.}$$

Q2: What happens to capacity if we remove bandwidth constraint?

A2: As $W \rightarrow \infty$, capacity saturates to $\frac{P/N_0}{\ln 2}$ bits/s.