PROBLEM 1.1. Let \( s(t) = \sum_{k=-\infty}^{\infty} g(t-k) \) be a periodic pulse train with a one-second period, and let the pulse shape \( g(t) \) be the following sinc function:

\[
g(t) = 4 \frac{\sin(10\pi t)}{10\pi t}.
\]

(a) Sketch \( s(t) \).
(b) Find the energy of \( g(t) \).
(c) Find the power of \( g(t) \).
(d) Find the energy of \( s(t) \).
(e) Find the power of \( s(t) \).

PROBLEM 1.2.

(a) Find the Hilbert transform of \( s(t) = \cos(440\pi t + \pi/8) \).
(b) Find the in-phase and quadrature components of \( s(t) = \cos(440\pi t + \pi/8) \) with respect to the carrier frequency \( f_0 = 220 \) Hz.
(c) Find the in-phase and quadrature components of \( s(t) = \frac{\sin(20\pi t)}{\pi t} + \frac{\sin(40\pi t)}{\pi t} \) with respect to the carrier frequency \( f_0 = 10 \) Hz.
(d) Find the in-phase component \( s_I(t) \) of the sinc-squared signal \( s(t) = \left( \frac{\sin(\pi t)}{\pi t} \right)^2 \) with respect to the carrier frequency \( f_0 = 0.5 \) Hz.

PROBLEM 1.3. Let \( s(t) = \sin(1024\pi t)/(1024\pi t) \). Specify a particular carrier frequency \( f_0 \) (in Hz) such that the quadrature component of \( s(t) \) with respect to \( f_0 \) is zero.

PROBLEM 1.4. The following statement is almost true: “The Hilbert transformer swaps the I & Q components of the input signal (I becomes Q, and Q becomes I).” Find the error and fix the statement so that it is true.
PROBLEM 1.5.

(a) Prove Parseval’s relationship: the energy of a signal \( s(t) \) as measured in the time domain is identical to the energy as measured in the frequency domain:

\[
\int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df.
\]

(Hint: in left-hand integral,
1. replace \( |s(t)|^2 \) by \( s(t) s(t)^* \);
2. replace \( s(t)^* \) by the conjugate of the inverse FT integral \( s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df \);
3. swap order of integrals.)

(b) Prove that a real signal \( s(t) \) has the same energy as its Hilbert transform \( \hat{s}(t) \).

(c) Downconversion doubles energy — Prove that the energy of the complex envelope \( \tilde{s}(t) \) of a real signal \( s(t) \) is exactly twice the energy of the original:

\[
\int_{-\infty}^{\infty} |\tilde{s}(t)|^2 dt = 2 \int_{-\infty}^{\infty} s^2(t) dt.
\]

PROBLEM 1.6. Let \( s(t) \) be a signal whose in-phase and quadrature components with respect to a 5-Hz carrier frequency are

\[
s_I(t) = (1 + \frac{\sin(\pi t)}{\pi t}) g(t) \quad \text{and} \quad s_Q(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2 g(t),
\]

respectively, where \( g(t) = \frac{\sin(0.2\pi t)}{0.2\pi t} \).

(a) Using MATLAB, generate a plot of the signal \( s(t) \).

(b) Using MATLAB, generate a plot of the signal’s real envelope \( e(t) \).

(c) Verify that the real envelope tracks the “peaks” of the original signal in a smooth way.