PROBLEM 1.1. Let \( s(t) = \sum_{k = -\infty}^{\infty} g(t - kT) \) be a periodic pulse train with period \( T \) and the following pulse shape:
\[ g(t) = 4 \frac{\sin(10\pi t/T)}{10\pi t/T}. \]

(a) Sketch \( s(t) \).
(b) Find the energy of \( g(t) \).
(c) Find the power of \( g(t) \).
(d) Find the energy of \( s(t) \).
(e) Find the power of \( s(t) \).

PROBLEM 1.2.

(a) Find the Hilbert transform of \( s(t) = \cos(440\pi t + \pi/8) \).
(b) Find the in-phase and quadrature components of \( s(t) = -\sqrt{2} \cos(90\pi t + \pi/3) \) with respect to the carrier frequency \( f_0 = 45 \) Hz.
(c) Find the in-phase and quadrature components of \( s(t) = -\sqrt{2} \cos(90\pi t + \pi/3) \) with respect to the carrier frequency \( f_0 = 40 \) Hz.
(d) Find the in-phase and quadrature components of \( s(t) = \sqrt{2} \cos(500\pi t)(\sin(\pi t)/\pi t) \) with respect to the carrier frequency \( f_0 = 250 \) Hz.
(e) Find the in-phase and quadrature components of \( s(t) = \sqrt{2} \cos(500\pi t)(\sin(\pi t)/\pi t) \) with respect to the carrier frequency \( f_0 = 251 \) Hz.

PROBLEM 1.3. Let \( s(t) = \sin(1024\pi t)/(1024\pi t) \). Specify a particular carrier frequency \( f_0 \) (in Hz) such that the quadrature component of \( s(t) \) with respect to \( f_0 \) is zero.

PROBLEM 1.4. Find the in-phase component \( s_I(t) \) of the sinc-squared signal \( s(t) = (\frac{\sin(\pi t)}{\pi t})^2 \) with respect to a carrier frequency \( f_0 = 0.5 \) Hz.

PROBLEM 1.5. The following statement is almost true: The Hilbert transformer swaps the I & Q components of the input signal. (I becomes Q, and Q becomes I). Find the error and fix the statement so that it is true.
PROBLEM 1.6.

(a) Prove Parseval’s relationship: the energy of a signal \( s(t) \) as measured in the time domain is identical to the energy as measured in the frequency domain:

\[ \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df. \]

(Hint: in left-hand integral,
1. replace \( |s(t)|^2 \) by \( s(t)s(t)^* \);
2. replace \( s(t)^* \) by the conjugate of the inverse FT integral \( s(t) = \int_{-\infty}^{\infty} S(f)e^{j2\pi ft} df \);
3. swap order of integrals.)

(b) Prove that a real signal \( s(t) \) has the same energy as its Hilbert transform \( \hat{s}(t) \).

(c) Prove that the energy of a real \( s(t) \) is equal to the energy of its complex envelope \( \tilde{s}(t) \).

PROBLEM 1.7. Let \( s(t) = g(t)\cos(20\pi t) \), where \( g(t) = \left( \frac{\sin(\pi t)}{\pi t} \right)^2 \) is the sinc² pulse shown below:

(a) Sketch the signal \( s(t) \).

(b) Is it possible to choose the parameter \( f_0 \) so that the complex envelope of \( s(t) \) w.r.t. \( f_0 \) can be written as \( \tilde{s}(t) = Ag(t) \)?
   If yes, specify the parameters \( f_0 \) and \( A \).
   If no, explain why it is not possible.

(c) Specify all values of \( f_0 \) for which the real envelope \( e(t) \) is the same sinc-squared pulse shown above, i.e., for which \( e(t) = g(t) \).

PROBLEM 1.8. Let \( s(t) \) be a signal whose in-phase and quadrature components with respect to a 5-Hz carrier frequency are

\[ s_I(t) = (1 + \frac{\sin(\pi t)}{\pi t})g(t) \quad \text{and} \quad s_Q(t) = \left( \frac{\sin(\pi t)}{\pi t} \right)^2 g(t), \]

respectively, where \( g(t) = \frac{\sin(0.2\pi t)}{0.2\pi t} \).

(a) Using MATLAB, generate a plot of the signal \( s(t) \).

(b) Using MATLAB, generate a plot of the signal’s real envelope \( e(t) \).

(c) Verify that the real envelope tracks the “peaks” of the original signal in a smooth way.