PROBLEM 4.1. Suppose a one-shot 64-PAM baseband transmitter sends \( s(t) = ag(t) \), where \( a \in \{-63, -61, -59, \ldots, 59, 61, 63\} \), and where \( g(t) \) is the following triangular pulse:

\[
g(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

Suppose the receiver observes the constant signal \( r(t) = -18 \) (for all \( t \)). Find the decision made by the minimum-distance receiver.

PROBLEM 4.2. Sketch the in-phase and quadrature components of a 16-QAM signal that conveys the bit sequence 1010001101111010, assuming a rectangular pulse of duration \( T = 1 \) second, and assuming a symbol rate of 1 Hz. Use a Gray mapping.

PROBLEM 4.3. Consider the \( M^2 \)-QAM signal \( s(t) = \sqrt{2} \text{Re}\{\exp(j2\pi f_0 t)\sum_{k=-\infty}^{\infty} a_k g(t-kT)\} \) with symbol period \( T = 1 \), where \( \text{Re}\{a_k\} \) and \( \text{Im}\{a_k\} \) are chosen independently from the same alphabet \( \{\pm 1, \pm 3, \ldots, \pm (M-1)\} \), and where \( g(t) = 1/\sqrt{2} \) for \( 0 \leq t \leq 1 \) and zero elsewhere. The following is an illustration of \( s(t) \) for \( 0 \leq t \leq 20T \):

By carefully inspecting this picture we can make educated guesses as to what the carrier frequency is, and what the alphabet size is.

(a) Based on the above picture, what do you think \( f_0 \) is? Explain.

(b) Based on the above picture, what do you think \( M \) is? Explain.

PROBLEM 4.4. What is the minimum value of \( M \) necessary for an \( M \)-ary PSK system to achieve a bit rate of 1.6 Mb/s (without ISI) over a channel with the ideal band-pass frequency response shown below?
PROBLEM 4.5. Suppose a one-shot $M$-ary complex-PAM transmitter sends the signal
\[ s(t) = \text{Re}\{a(t)e^{j2\pi f_0 t}\}, \]
where $f_0 = 440$ Hz, $\text{Re}\{a\} \in \{-5, -3, -1, 1, 3, 5\}$, and where
$\text{Im}\{a\} \in \{-1, 1\}$. Assume that $g(t)$ is rectangular, as shown below:

(a) Find $M$.

(b) Find the decision $\hat{a}$ made by the minimum-distance receiver when it observes the waveform $r(t) = \cos(2\pi t)$.

PROBLEM 4.6. Consider the 16-ary PSK signal set $s(t) = \sqrt{2} \text{Re}\{a(t)\exp(j2\pi f_0 t)\}$, where the complex
PSK symbol is $a \in \{1, e^{j\pi/8}, e^{j2\pi/8}, e^{j3\pi/8}, e^{j4\pi/8}, e^{j5\pi/8}, \ldots, e^{-j14\pi/8}, e^{j15\pi/8}\}$, and
where the pulse shape is $g(t) = 1$ for $t \in [0, 1)$ and zero elsewhere. The carrier frequency is
$f_0 = 900$ MHz.

(a) Plot the in-phase $s_I(t)$ and quadrature $s_Q(t)$ components of $s(t)$ vs. time when $a = e^{j3\pi/8}$.

(b) Plot the constellation for this modulation scheme.

(c) Clearly illustrate the minimum-distance decision regions for this modulation scheme.

(d) What decision $\hat{a}$ is made by the minimum-distance receiver when it receives the waveform $r(t) = \sin(2\pi f_0 t) - 2\cos(2\pi f_0 t)$?

PROBLEM 4.7. A 64-QAM signal set is described by $s(t) = \sqrt{2} \text{Re}\{a(t)e^{j2000\pi t}\}$, where $a = a_c + ja_s$ and
where $a_c, a_s \in \{\pm1, \pm3, \pm5, \pm7\}$. Assume that the pulse shape is rectangular, with $g(t) = 1$
for $t \in [0, 1)$ and zero elsewhere. What decision $\hat{a}$ is made by a minimum-distance
receiver when it receives $r(t) = 6\cos(2000\pi t + \pi/6)$?

PROBLEM 4.8. Consider the 16-ary complex-PAM signal set defined by a rectangular pulse shape $g(t)$, a
carrier frequency of $f_0 = 100$ Hz, and the symbol alphabet \{1 + 7j, 3 + 7j, 3 + 5j, 7 + 5j, 9 + 5j, 5 + 3j, 7 + 3j, 11 + 3j, 5 + j, 11 + j, 9 - j, 11 - j, 13 - j, 7 - 3j, 9 - 3j, 5 - 5j\}:

Suppose the received signal is a constant: $r(t) = 7\sqrt{2}$ for all $t$.
What decision $\hat{a}$ is made by the minimum-distance receiver?