But the decision region for $a = 1$ is defined by

$$\Re\{z\} > \ln |z| \quad \text{and} \quad \Re\{z\} > -\ln |z|.$$ 

And with $z = 1 + X + jY$, this leads to

$$\Pr[\text{corr} \mid 1] = \Pr[Y < 1 + X \quad \text{and} \quad -Y < 1 + X]$$

$$= \Pr[X - Y > 1] \quad \text{and} \quad X + Y > -1.$$ 

Fact: if $X, Y \text{ iid } N(0, \frac{N_0}{2g})$ then $U = X - Y$ and $V = X + Y$ are iid $N(0, 2\frac{N_0}{2g})$.


$$= 2E[X^2] = 2\frac{N_0}{2g} = E[V^2].$$


But $E[UV] = E[U]E[V]$ implies uncorrelated, which because of Gaussianity implies independence.

$$\Rightarrow \Pr[\text{corr} \mid 1] = \Pr[U > -1] \Pr[V > -1]$$

$$= \Pr\left[\frac{U}{\sqrt{2N_0/2g}} > -1\right] \Pr\left[\frac{V}{\sqrt{2N_0/2g}} > -1\right]$$

$$= Q\left(-\frac{\sqrt{2N_0/2g}}{\sqrt{\frac{N_0}{2g}}}\right) \cdot Q\left(-\sqrt{\frac{N_0}{2g}}\right)$$

$$= (1 - Q\left(-\frac{\sqrt{2N_0/2g}}{\sqrt{\frac{N_0}{2g}}}\right))^2.$$ 

But $(Q(-x) = 1 - Q(x)$