PROBLEM 3.1.

(a) Find the in-phase and quadrature components of \( s(t) = \cos(90\pi t - \pi/2) \) with respect to the carrier frequency \( f_0 = 55 \) Hz.

(b) Find the in-phase and quadrature components of \( s(t) = \cos(500\pi t)(\sin\pi t/(\pi t)) \) with respect to the carrier frequency \( f_0 = 250 \) Hz.

(c) Find the in-phase and quadrature components of \( s(t) = \cos(500\pi t)(\sin\pi t/(\pi t)) \) with respect to the carrier frequency \( f_0 = 251 \) Hz.

PROBLEM 3.2. Let \( r_I(t) \) and \( r_Q(t) \) denote the in-phase and quadrature components of a real signal \( r(t) \) with respect to a carrier frequency \( f_0 \), so that \( r(t) = r_I(t)\cos(2\pi f_0 t) - r_Q(t)\sin(2\pi f_0 t) \).

A quadrature demodulator with input \( r(t) \) is shown below:

Define \( x(t) \) as the output of the top LPF, and define \( y(t) \) as the output of the bottom LPF. Under certain conditions on the frequency content of \( r(t) \), the quadrature demodulator produces the I & Q components of \( r(t) \), so that \( x(t) = r_I(t) \) and \( y(t) = r_Q(t) \). However, if \( r(t) \) violates the conditions, then the above outputs are not the I & Q components.

(a) Find the conditions on \( r(t) \) such that \( x(t) = r_I(t) \) and \( y(t) = r_Q(t) \).

(b) Give an example of an \( r(t) \) such that \( x(t) \neq r_I(t) \) and \( y(t) \neq r_Q(t) \).

PROBLEM 3.3.

(a) True or False: If \( p(t) \) is a Nyquist pulse for symbol rate 2400 baud but not for symbol rate 4800 baud, then \( p(t) \) cannot be a Nyquist pulse for symbol rate 7200 baud. If true, explain why. If false, give a counter-example.

(b) Are the following three propositions true Always, Sometimes, or Never? Explain.

- The sum of two Nyquist pulses is a Nyquist pulse.
- The product of two Nyquist pulses is a Nyquist pulse.
- The convolution of two Nyquist pulses is a Nyquist pulse.

(c) Consider the pulse \( p(t) \) with Fourier transform \( P(f) \) shown below:

\[
P(f) = \begin{cases} 
1 & \text{if } |f| \leq 3000 \text{ Hz} \\
0 & \text{otherwise}
\end{cases}
\]
For what symbol rates is $p(t)$ a Nyquist pulse? Hint: There are many — specify them all.

**Problem 3.4.** Consider the 16-ary PSK signal set $s(t) = \text{Re}(ag(t)e^{j2\pi f_0 t})$, where the complex PSK symbol is $a \in \{1, e^{j\pi/8}, e^{j2\pi/8}, e^{j3\pi/8}, e^{j4\pi/8}, e^{j5\pi/8}, \ldots, e^{j14\pi/8}, e^{j15\pi/8}\}$, and the pulse shape $g(t)$ given by Fig. 2.2(a) of the textbook. The carrier frequency is $f_0 = 900$ MHz.

(a) Specify the complex envelope $\tilde{s}(t)$ of $s(t)$, the in-phase component $s_I(t)$ of $s(t)$, and the quadrature component $s_Q(t)$ of $s(t)$.

(b) Plot the in-phase $s_I(t)$ and quadrature $s_Q(t)$ components of $s(t)$ vs. time when $a = e^{j3\pi/8}$.

(c) Plot the constellation for this modulation scheme.

(d) Clearly illustrate the minimum-distance decision regions for this modulation scheme.

(e) What decision $\hat{a}$ is made by the minimum-distance receiver when it receives the waveform $r(t) = \sin(2\pi f_0 t) + 2\cos(2\pi f_0 t)$?

**Problem 3.5.** A 64-QAM signal set is described by $s(t) = \text{Re}(ag(t)e^{j2000\pi t})$, where $a = a_c + ja_s$ and $a_c$, $a_s \in \{\pm 1, \pm 3, \pm 5, \pm 7\}$, and $g(t) = 1$ for $t \in [0, 1)$ and zero elsewhere. What decision $\hat{a}$ is made by a minimum-distance receiver when it receives $r(t) = 6\cos(2000\pi t + \pi/6)$?

**Problem 3.6.** Consider the 5-ary modulation scheme $s(t) = \text{Re}(ag(t)e^{j2\pi f_0 t})$, where the complex symbol $a$ is chosen from the alphabet $\{-1, 0, 1, 1+j, 1+2j\}$, the pulse shape $g(t)$ is given by Fig. 2.2(a) of the textbook, and $f_0 = 1800$ MHz.

(a) Plot the constellation for this modulation scheme.

(b) Clearly illustrate the minimum-distance decision regions for this modulation scheme.

(c) What decision $\hat{a}$ is made by the minimum-distance receiver when it receives the waveform $r(t) = \frac{-1}{\sqrt{2}}\sin(2\pi f_0 t) - \frac{1}{3}\cos(2\pi f_0 t)$?

**Problem 3.7.** Consider the $M^2$-QAM signal set $s(t) = \text{Re}\{e^{j2\pi f_0 t} \sum_{k=-\infty}^\infty \alpha_k g(t - kT)\}$ with baud period $T = 1$, where $\text{Re}\{\alpha_k\}$ and $\text{Im}\{\alpha_k\}$ are chosen from the alphabet $\{\pm 1, \pm 3, \ldots, \pm (M-1)\}$, and where $g(t) = 1$ for $0 \leq t \leq 1$ and zero elsewhere. The following is an illustration of $s(t)$ for $0 \leq t \leq 20T$.

By carefully inspecting this picture we can make educated guesses as to what the carrier frequency is, and what the alphabet size is.

(a) Based on the above picture, what do you think $f_0$ is? Explain.

(b) Based on the above picture, what do you think $M$ is? Explain.