ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
MARCH 31, 2017

Name: ____

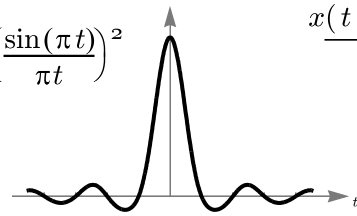
1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

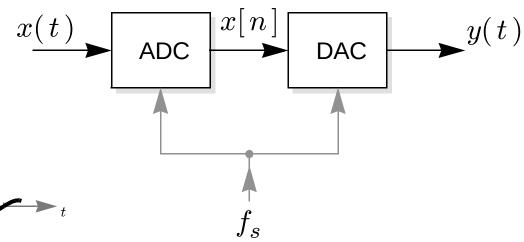
Problem	Points	Score
1	30	
2	40	
3	30	
TOTAL:	100	

PROBLEM 1. (30 points)

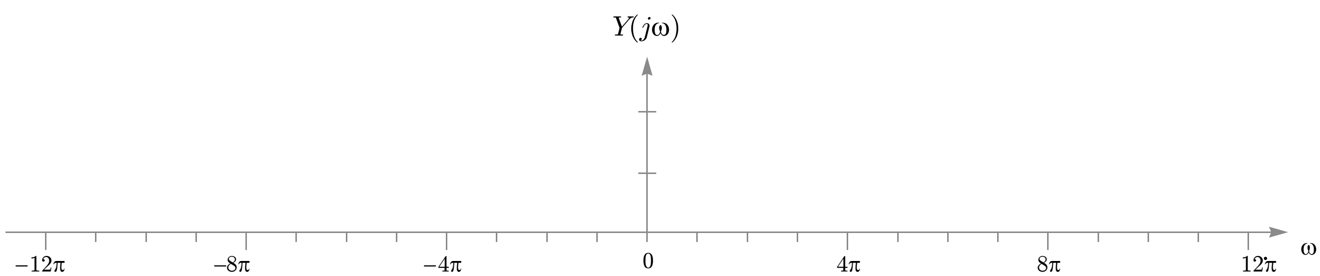
Consider the following system, where a signal $x(t)$ (consisting of the sum of a sinc and a sinc-squared function) is passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter:

$$x(t) = \frac{\sin(\pi t)}{\pi t} + \left(\frac{\sin(\pi t)}{\pi t}\right)^2$$

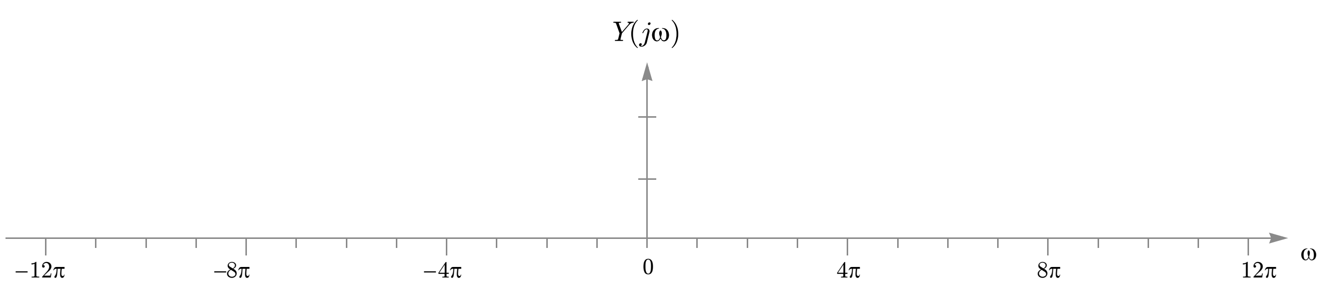




(a) In the space below, sketch the Fourier transform of $y(t)$ when the sampling rate is $f_s = 4$ Hz:

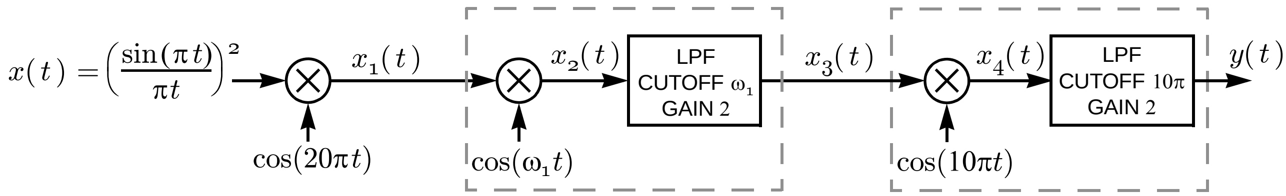


(b) In the space below, sketch the Fourier transform of $y(t)$ when the sampling rate is $f_s = 1$ Hz:



PROBLEM 2. (40 points)

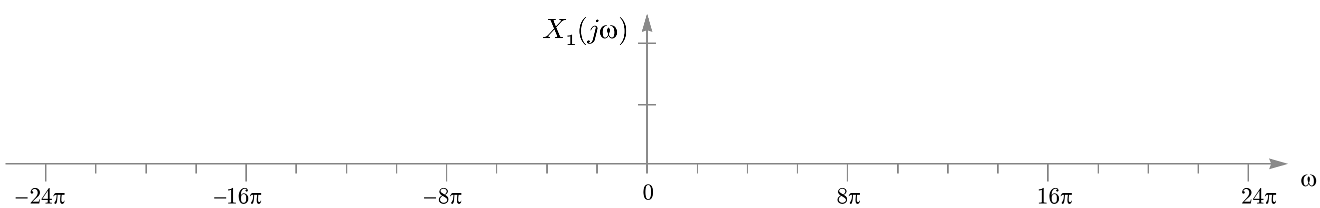
Suppose $x(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$ is AM modulated onto a 20π carrier, producing $x_1(t)$:

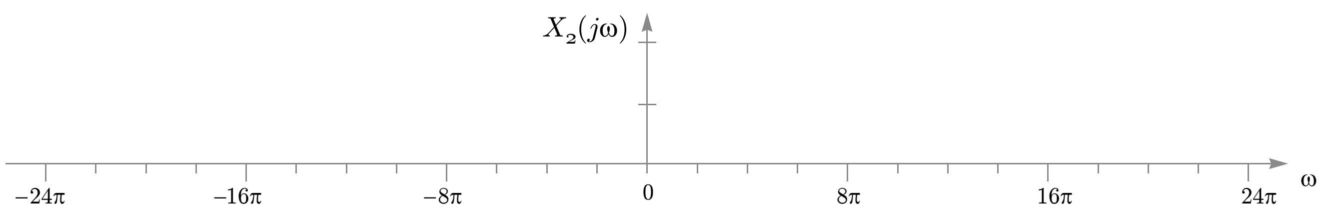


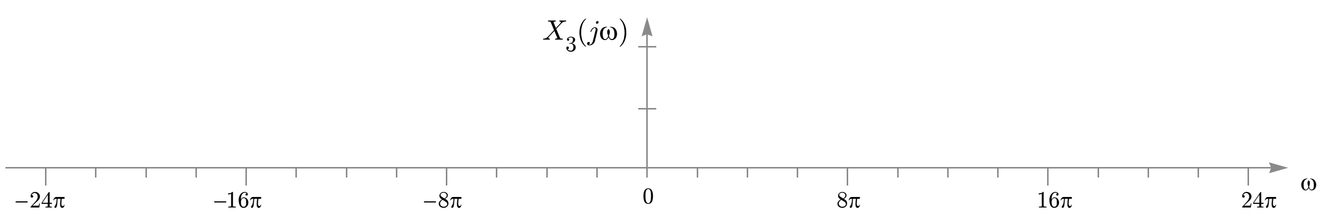
(a) Each of the dashed boxes above is an AM demodulator: The first one (by itself) would demodulate when the carrier is ω_1 rad/s, while the second one (by itself) would demodulate when the carrier is 10π rad/s. Together, with a proper choice for ω_1, the two can be used to demodulate when the carrier is 20π rad/s. Find a value for ω_1 so that the overall output recovers the input, $y(t) = x(t)$:

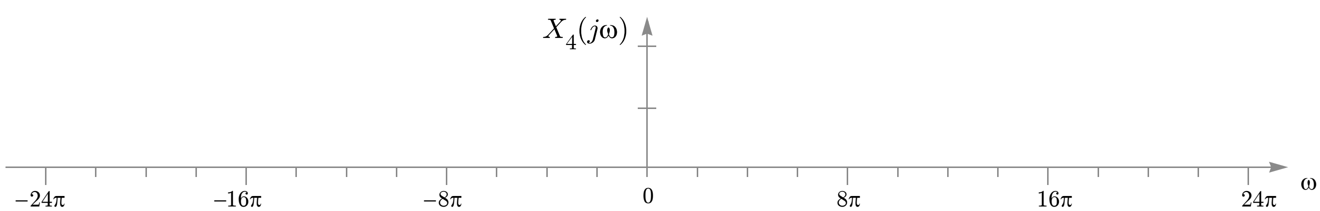
$\omega_1 =$ ☐ rad/s.

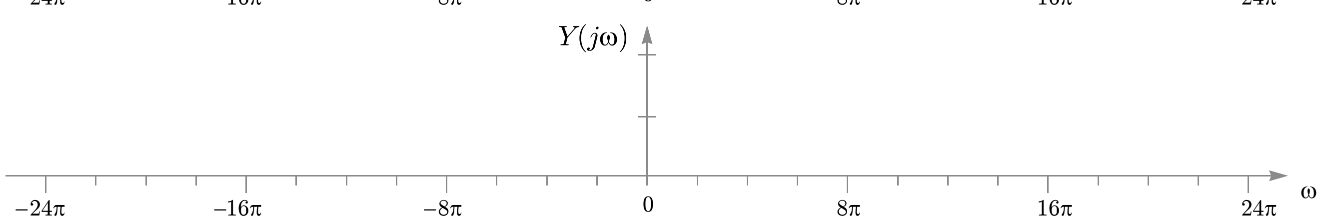
(b) Using your answer from part (a), sketch in the space below the Fourier transform of each of the signals $x_1(t)$ through $x_4(t)$ and $y(t)$:











PROBLEM 3.

(a) The Laplace transform of $x(t) = (t - 1)^2 u(t - 2)$ is $X(s) =$ [ ] .

(b) Subject to the initial condition $y(0^-) = 4$, the differential equation $\frac{1}{2}\frac{d}{dt}y(t) = 3u(t) - y(t)$ (where $u(t)$ is the unit step) has the following solution:

$$y(t) = (A + Be^{-t} + Ce^{-2t} + De^{-3t})u(t),$$

where:

$A =$ [ ] $B =$ [ ] $C =$ [ ] $D =$ [ ]

ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

GEORGIA INSTITUTE OF TECHNOLOGY

MARCH 31, 2017

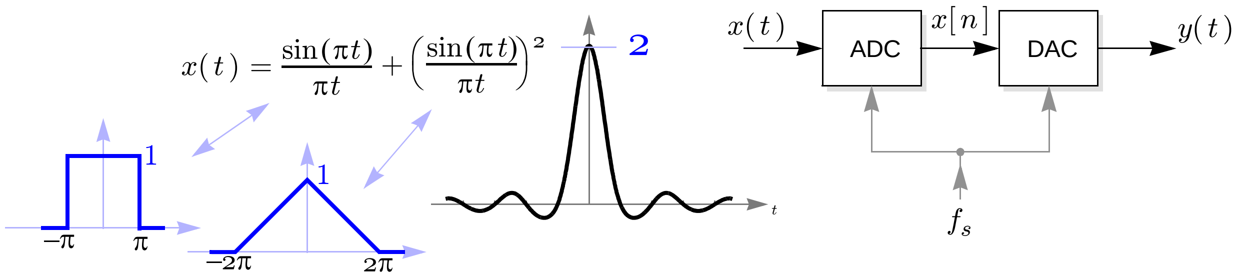
Name: SOLUTIONS

1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

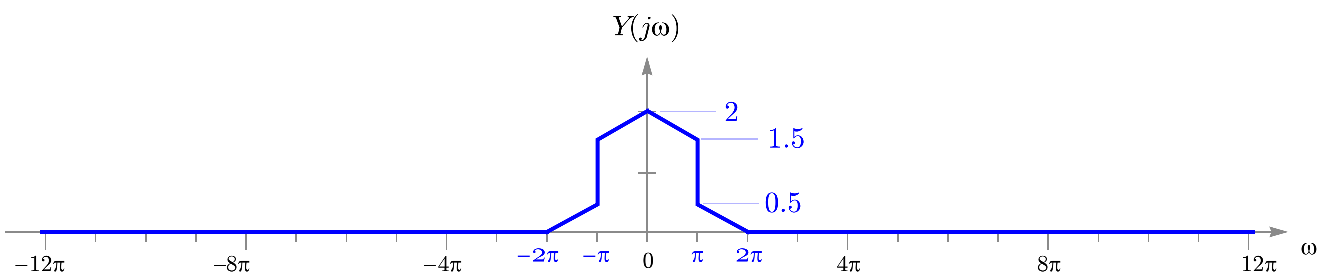
Problem	Points	Score
1	30	
2	40	
3	30	
TOTAL:	100	

PROBLEM 1. (30 points)

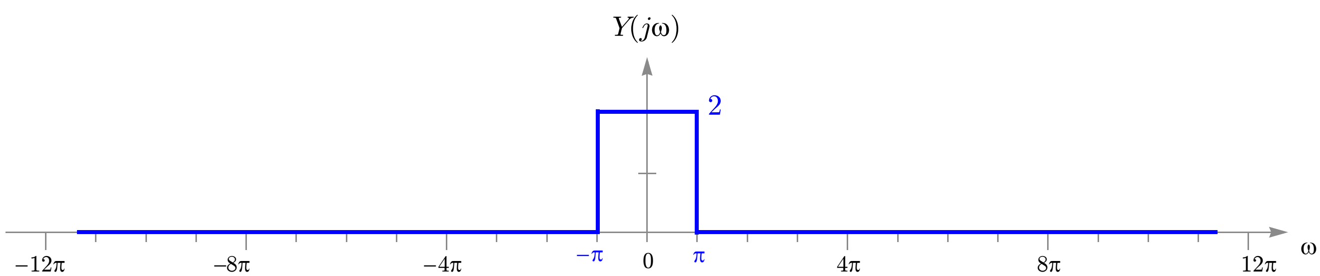
Consider the following system, where a signal $x(t)$ (consisting of the sum of a sinc and a sinc-squared function) is passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter:



(a) In the space below, sketch the Fourier transform of $y(t)$ when the sampling rate is $f_s = 3$ Hz:

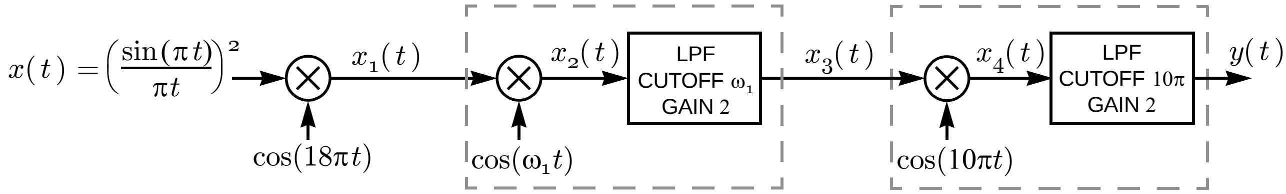


(b) In the space below, sketch the Fourier transform of $y(t)$ when the sampling rate is $f_s = 1$ Hz:



PROBLEM 2. (40 points)

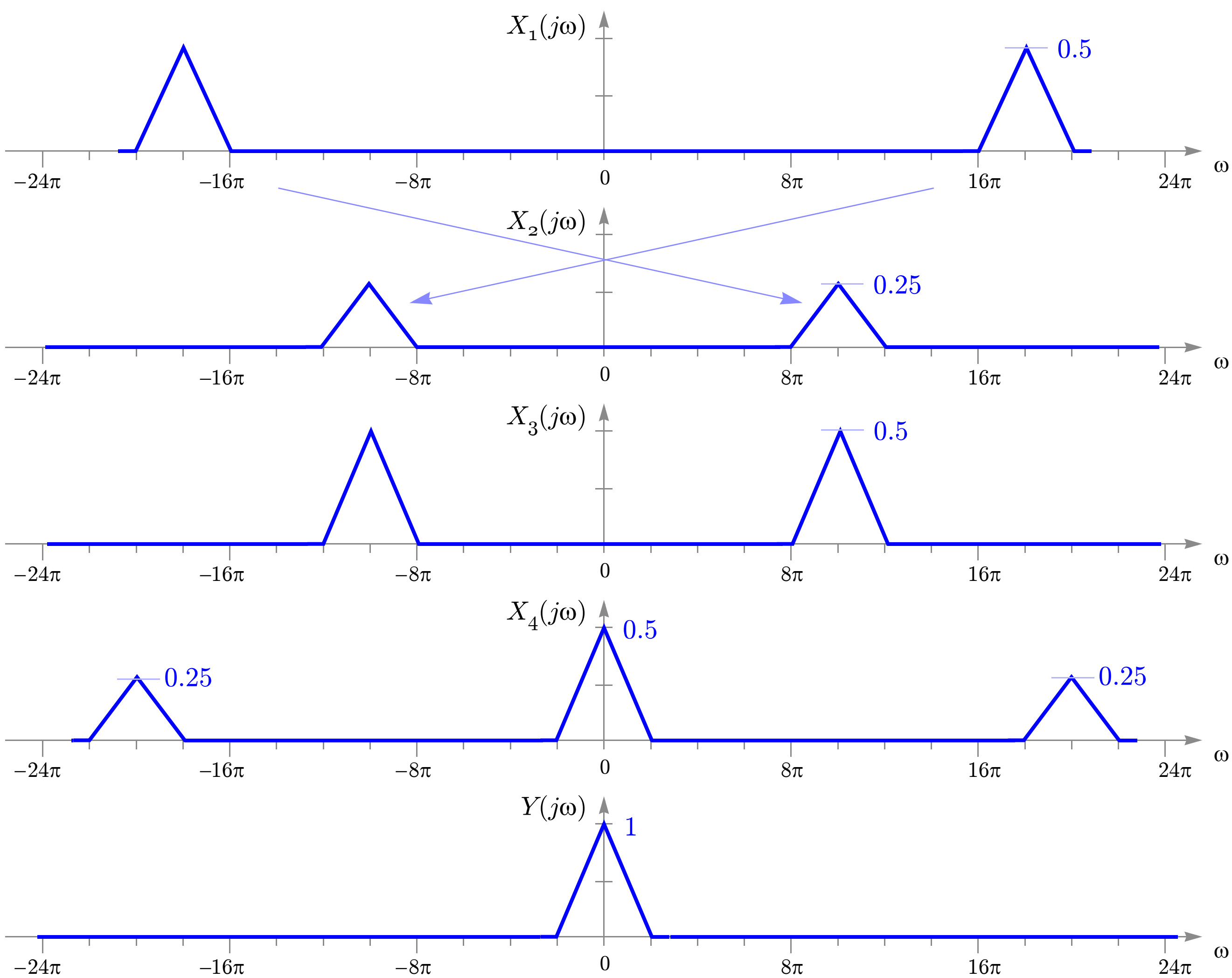
Suppose $x(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$ is AM modulated onto a 18π carrier, producing $x_1(t)$:



(a) Each of the dashed boxes above is an AM demodulator: The first one (by itself) would demodulate when the carrier is ω_1 rad/s, while the second one (by itself) would demodulate when the carrier is 10π rad/s. Together, with a proper choice for ω_1, the two can be used to demodulate when the carrier is 18π rad/s. Find a value for ω_1 so that the overall output recovers the input, $y(t) = x(t)$:

$\omega_1 =$ 28π rad/s.

(b) Using your answer from part (a), sketch in the space below the Fourier transform of each of the signals $x_1(t)$ through $x_4(t)$ and $y(t)$:



PROBLEM 3.

(a) The Laplace transform of $x(t) = (t-1)^2 u(t-4)$ is $X(s) = \boxed{\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)e^{-4s}}$.

$$x(t) = (t^2 - 2t + 1)u(t-4)$$

$$= ((t-4)^2 + 6t - 15)u(t-4)$$

$$= (t-4)^2 u(t-4) + 6(t-4)u(t-4) + 9u(t-4)$$

$$\Rightarrow \quad X(s) = \left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)e^{-4s}$$

(b) Subject to the initial condition $y(0^-) = 4$, the differential equation $\frac{1}{2}\frac{d}{dt}y(t) = 4u(t) - y(t)$ (where $u(t)$ is the unit step) has the following solution:

$$y(t) = (A + Be^{-t} + Ce^{-2t} + De^{-3t})u(t),$$

where:

$A = \boxed{4}$ $B = \boxed{0}$ $C = \boxed{0}$ $D = \boxed{0}$

$$\frac{d}{dt}y(t) = 8u(t) - 2y(t)$$

$$\Rightarrow \quad sY(s) - 4 = \frac{8}{s} - 2Y(s)$$

$$\Rightarrow \quad Y(s)(s+2) = \frac{4}{s}(s+2)$$

$$\Rightarrow \quad Y(s) = \frac{4}{s} \cdot \frac{\cancel{(s+2)}}{\cancel{s+2}} = \frac{4}{s}$$