ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
APRIL 7, 2016

Name: ____

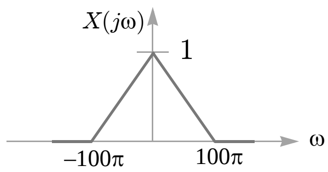
1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.

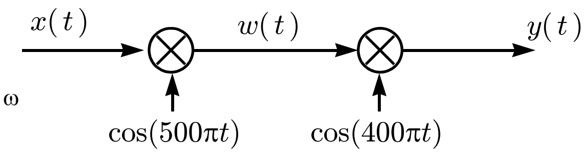
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

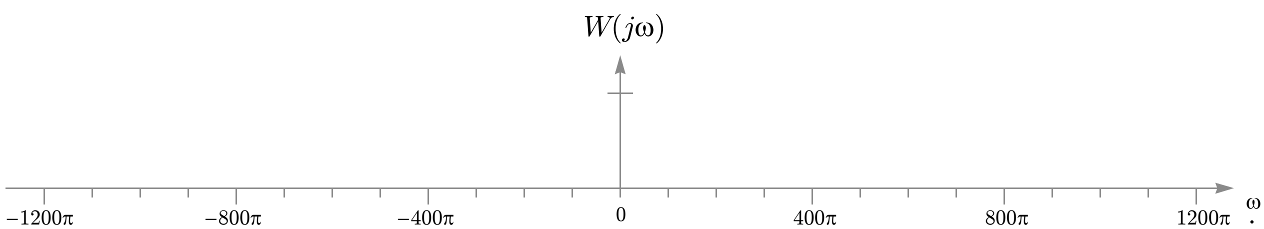
PROBLEM 1. (20 points) *Label amplitudes to receive full credit!*

Consider the system shown below, where the input $x(t)$ is modulated twice.
Assume that $x(t)$ has the bandlimited triangular spectrum shown on the left.

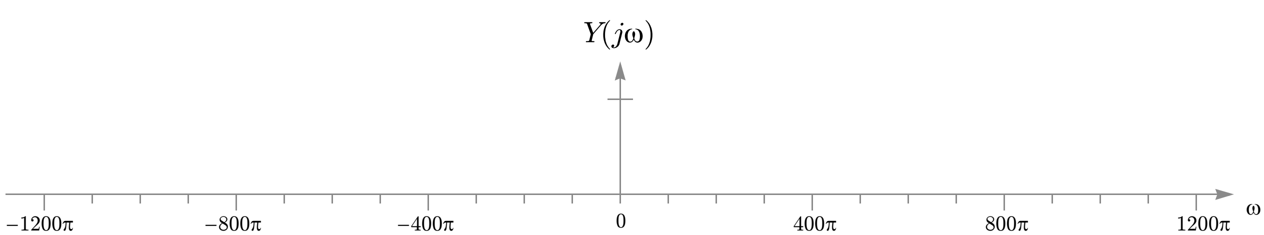




(a) Sketch $W(j\omega)$, the Fourier transform of the signal $w(t)$ after the first modulator.

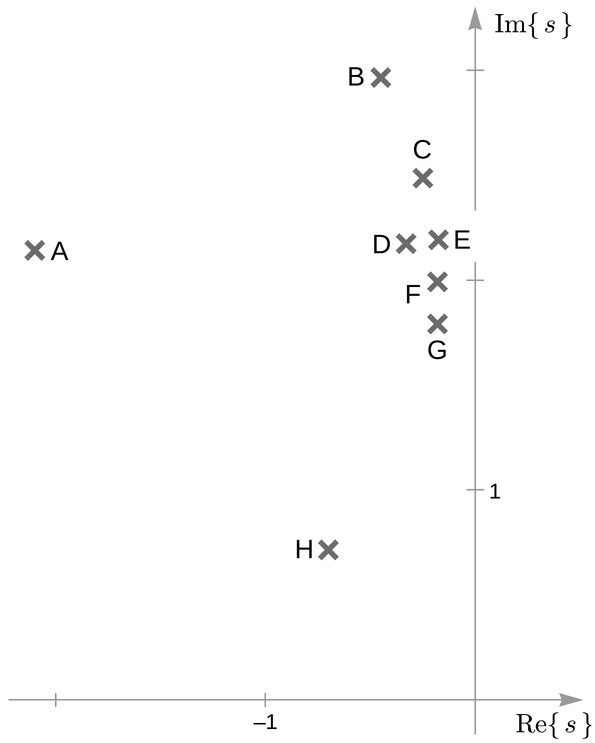


(b) Sketch $Y(j\omega)$, the Fourier transform of the signal $y(t)$ after the second modulator.

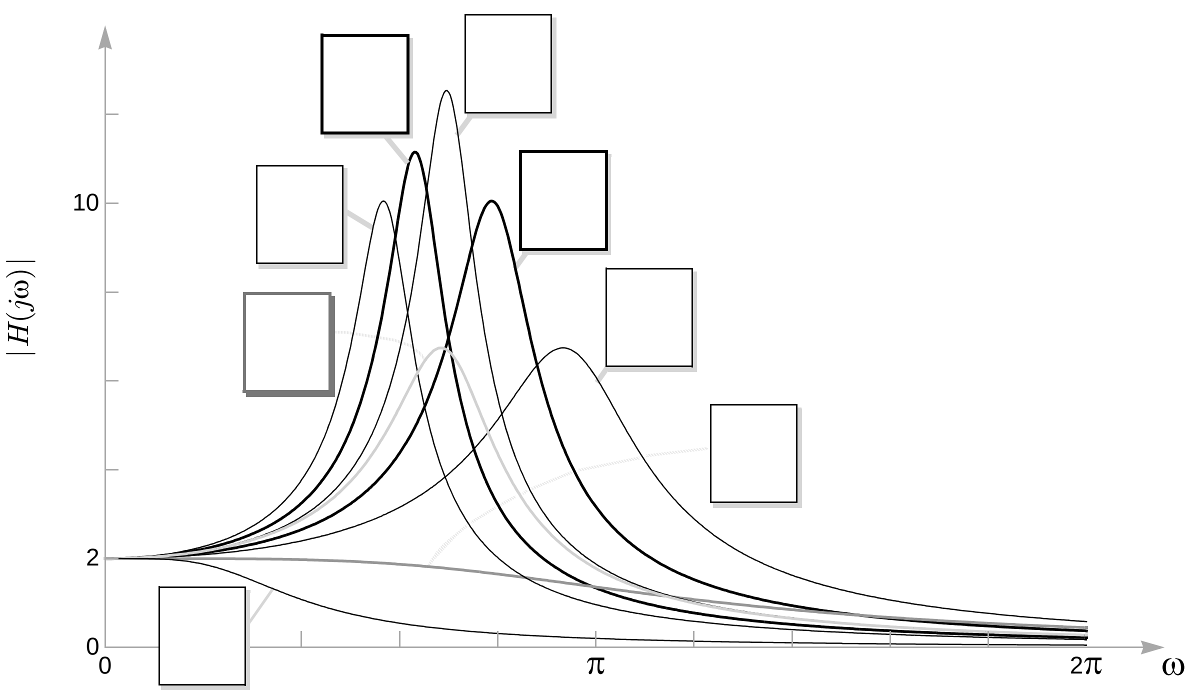


PROBLEM 2. (20 points)

Shown to the right are eight possible locations for one of the two poles of a second-order LTI system in the s-plane, labeled A through H. (Each pole has a companion pole in the complex conjugate location that is not shown.)

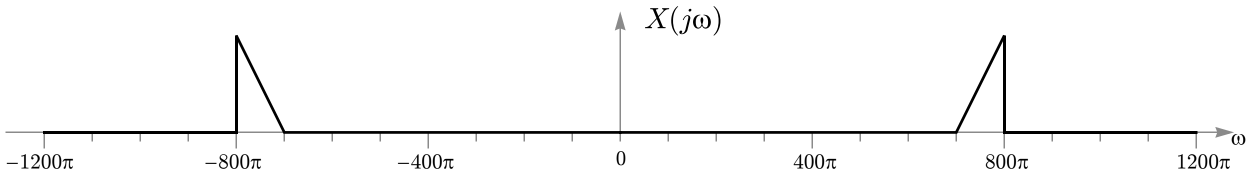


Shown below are the corresponding magnitude responses. Match each pole location to its corresponding magnitude response by writing a letter (A through H) in each answer box. *Justify your answers!*

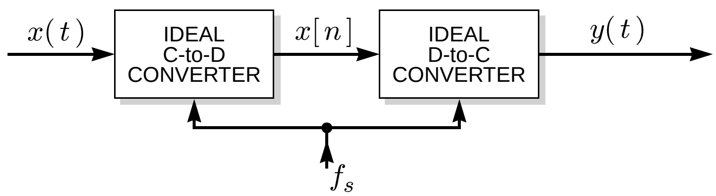


PROBLEM 3. (20 points)

Consider a continuous-time signal $x(t)$ whose Fourier transform is as sketched below:



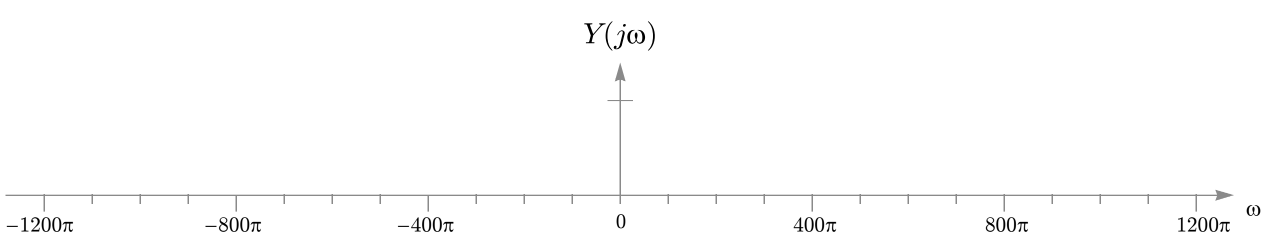
Suppose this signal is sampled at an unspecified sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:



(a) In order for the D-to-C converter to reconstruct the original signal (i.e., to achieve $y(t) = x(t)$), the sampling frequency must satisfy:

$f_s >$ ☐ Hz.

(b) In the space below, carefully sketch the Fourier transform $Y(j\omega)$ of the D-to-C output when the sampling frequency is $f_s = 400$ Hz, carefully labeling important frequencies and amplitudes:



PROBLEM 4. (20 points)

An LTI system (zero initial conditions) with input $x(t)$ and output $y(t)$ obeys the following differential equation:

$$6\frac{d^2}{dt^2}y(t) = 12x(t) - 24y(t) - 6\frac{d}{dt}y(t).$$

(a) Circle one: The system is [overdamped] [underdamped] [critically damped]?

(b) Its d.c. gain is H_0 = ______ .

(c) Its natural frequency is ω_n = ______ (in rad/s).

(d) Its damping ratio is ζ = ______ .

PROBLEM 5. (20 points)

Consider an LTI system with input $x(t)$, output $y(t)$, and transfer function $H(s) = \frac{2s}{s+5}$.

(a) YES NO ☐ ☐ The system is BIBO stable.

(b) The system acts as a [LPF][BPF][HPF]. (Circle one.)

(c) Write a differential equation relating the input $x(t)$ to the output $y(t)$ of this system:

(d) Find an equation for the "ramp response" of this system;
i.e., find the output $y(t)$ when the input is the unit ramp $x(t) = tu(t)$:

$y(t) =$ ☐

(e) Use the final value theorem to determine the steady-state value $y(\infty)$ of the ramp response:

$y(\infty) =$ ☐

(Sanity check: your answers to parts (d) and (e) should agree.)