ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
APRIL 2, 2015

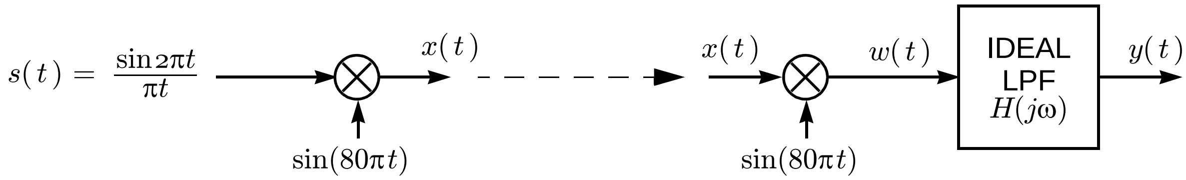
Name: ____

1. The quiz is closed book, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

PROBLEM 1. (20 points) — *Be sure to label both axes, i.e. label important frequencies and amplitudes!*

The left-hand side of the figure below shows a continuous-time sinc signal $s(t) = \sin(2\pi t)/(\pi t)$ being fed to an AM *modulator* with carrier frequency 40 Hz, resulting in a modulated signal $x(t)$:



[This AM modulator differs from the usual because it multiplies by $\sin(\,\cdot\,)$ instead of $\cos(\,\cdot\,)$.] The right-hand side shows a *demodulator* consisting of another multiplication by $\sin(\,\cdot\,)$, resulting in the signal $w(t)$, followed by an ideal LPF with an unspecified gain and cutoff frequency.

(a) Sketch $X(j\omega)$, the Fourier transform of the modulated signal $x(t)$.

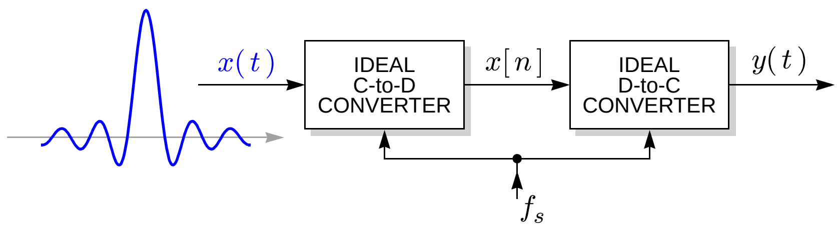
(b) Sketch $W(j\omega)$, the Fourier transform of the signal $w(t)$.

(c) YES ☐ NO ☐ Can the LPF be designed so that its output is the same as the modulator input, *i.e.* to achieve $y(t) = s(t)$?

(d) If YES, sketch below the frequency response $H(j\omega)$ of the LPF that results in $y(t) = s(t)$; If NO, explain below why this is impossible:

PROBLEM 2. (20 points)

Suppose that the continuous-time sinc signal $x(t) = \sin(202\pi t)/(202\pi t)$ is sampled at an unspecified sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:

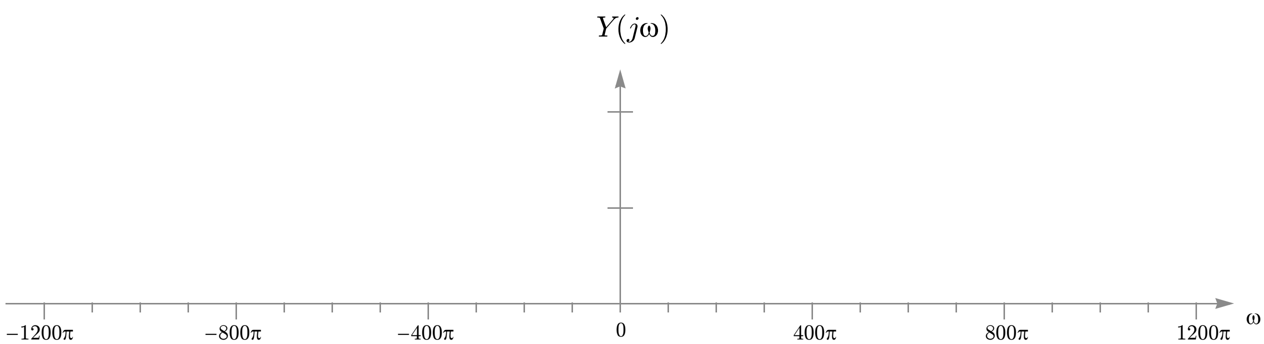


(a) The zero-th sample is $x[0] =$ ______ .

(b) In order for the D-to-C converter to reconstruct the original signal (i.e., to achieve $y(t) = x(t)$), the sampling frequency must satisfy:

$f_s >$ ______ Hz.

(c) In the space below, carefully sketch the output Fourier transform $Y(j\omega)$ when $f_s = 200$ Hz, carefully labeling important frequencies and amplitudes:



(d) There are an infinite number of sampling frequencies f_s such that the output signal will be $y(t) = \sin(\pi f_s t)/(\pi f_s t)$. Name any three:

$f_s =$ ______ Hz OR $f_s =$ ______ Hz OR $f_s =$ ______

PROBLEM 3. (20 points)

(a) The Laplace transform of $x(t) = (t^2 - 1)u(t - 2)$ is $X(s) =$ $\boxed{}$.

(b) The inverse Laplace transform of $X(s) = \dfrac{s + 3}{s(s + 0.5)}$ is $x(t) =$ $\boxed{()u(t)}$.

(c) Find the differential equation that relates the output $y(t)$ and input $x(t)$ of an LTI system whose transfer function is $H(s) = \dfrac{\pi + s}{s(s + 1)}$.

(d) The impulse response $h(t)$ of the system from part (c) approaches the constant $h(\infty) =$ $\boxed{}$ in the limit as time grows large.

PROBLEM 4. (20 points)

Subject to the initial conditions $x(0^-) = \dot{x}(0^-) = 1$, the following differential equation:

$$\frac{d^2}{dt^2}x(t) + 9x(t) = 0,$$

has the following solution:

$$x(t) = (A + B\cos(Ct) + D\sin(Et) + Ft)u(t),$$

where:

$A =$	$B =$	$C =$
$D =$	$E =$	$F =$

PROBLEM 5. (20 points)

An LTI system (zero initial conditions) with input $x(t)$ and output $y(t)$ obeys the following differential equation:

$$3\frac{d^2}{dt^2}y(t) = 2x(t) - 120y(t) - 12\frac{d}{dt}y(t).$$

(a) Circle one: The system is [overdamped] [underdamped] [critically damped]?

(b) Its d.c. gain is $H_0 =$ ☐ .

(c) Its natural frequency is $\omega_n =$ ☐ (in rad/s).

(d) Its damping ratio is $\zeta =$ ☐ .

(e) The *impulse* response of this system is $h(t) = Ae^{-Bt}\sin(Ct)$, where:

$A =$ ☐ $B =$ ☐ $C =$ ☐

Table of Fourier Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Frequency-Domain:* $X(j\omega)$
Right-sided exponential	$e^{-at}u(t) \quad (a>0)$	$\frac{1}{a+j\omega}$
Left-sided exponential	$e^{bt}u(-t) \quad (b>0)$	$\frac{1}{b-j\omega}$
Square pulse	$[u(t+T/2)-u(t-T/2)]$	$\frac{\sin(\omega T/2)}{\omega/2}$
"sinc" function	$\frac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega+\omega_0)-u(\omega-\omega_0)]$
Impulse	$\delta(t)$	1
Shifted impulse	$\delta(t-t_0)$	$e^{-j\omega t_0}$
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
General cosine	$A\cos(\omega_0 t+\phi)$	$\pi A e^{j\phi}\delta(\omega-\omega_0)+\pi A e^{-j\phi}\delta(\omega+\omega_0)$
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega-\omega_0)+j\pi\delta(\omega+\omega_0)$
General periodic signal	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega-k\omega_0)$
Impulse train	$\sum_{n=-\infty}^{\infty} \delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega-2\pi k/T)$

Table of Fourier Transform Properties		
Property Name	*Time-Domain* $x(t)$	*Frequency-Domain* $X(j\omega)$
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}X(j(\omega/a))$
Delay	$x(t-t_d)$	$e^{-j\omega t_d}X(j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega-\omega_0))$
Modulation	$x(t)\cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega-\omega_0))+\frac{1}{2}X(j(\omega+\omega_0))$
Differentiation	$\frac{d^k x(t)}{dt^k}$	$(j\omega)^k X(j\omega)$
Convolution	$x(t)*h(t)$	$X(j\omega)H(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega)*P(j\omega)$

Date 14-Sept-2013

Table of Laplace Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Laplace-Domain:* $X(s)$
Impulse	$\delta(t)$	1
Delayed Impulse	$\delta(t-t_0), \quad t_0 \geq 0$	e^{-st_0}
Step	$u(t)$	$\frac{1}{s}$
Rectangular Pulse	$u(t)-u(t-T), \quad T>0$	$\frac{1-e^{-sT}}{s}$
Ramp	$tu(t)$	$\frac{1}{s^2}$
Polynomial	$t^k u(t), \quad k \geq 0$	$\frac{k!}{s^{k+1}}$
Exponential	$e^{-at}u(t)$	$\frac{1}{s+a}$
Polynomial × Exponential	$t^k e^{-at}u(t)$	$\frac{k!}{(s+a)^{k+1}}$
Cosine	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$
Sine	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
Exponential × Cosine	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
Exponential × Sine	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

Table of Laplace Transform Properties		
Property Name	*Time-Domain:* $x(t)$	*Laplace-Domain:* $X(s)$
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Right-Shift	$x(t-t_0), \quad t_0 \geq 0$	$e^{-st_0}X(s)$
Time Scaling	$x(at), \quad a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
First Derivative	$\dot{x}(t)=\frac{d}{dt}x(t)$	$sX(s)-x(0)$
Second Derivative	$\ddot{x}(t)=\frac{d^2}{dt^2}x(t)$	$s^2X(s)-sx(0)-\dot{x}(0)$
Integration	$\int_0^t x(\tau)d\tau$	$\frac{X(s)}{s}$
Modulation	$x(t)e^{at}$	$X(s-a)$
Convolution	$x(t)*h(t)$	$X(s)H(s)$
Final Value Theorem	$\lim_{t\to\infty} x(t)$, (if limit exists)	$\lim_{s\to 0} sX(s)$
Initial Value Theorem	$\lim_{t\to 0} x(t)$, (if limit exists)	$\lim_{s\to\infty} sX(s)$