GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 31-Mar-14 COURSE: ECE 3084A (Prof. J. Michaels)

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

- Write your name on the front page ONLY. Do not unstaple the test. You should have an additional sheet with Fourier transform tables on one side and Laplace transform tables on the other.
- Calculators are permitted, but are to be used only for calculations, not to store notes, etc. No laptops, phones, or other electronic devices are allowed. Keep the desks clear of all backpacks, books, etc.
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- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.
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Signature: ____

Problem	*Value*	*Score*
1	25	
2	25	
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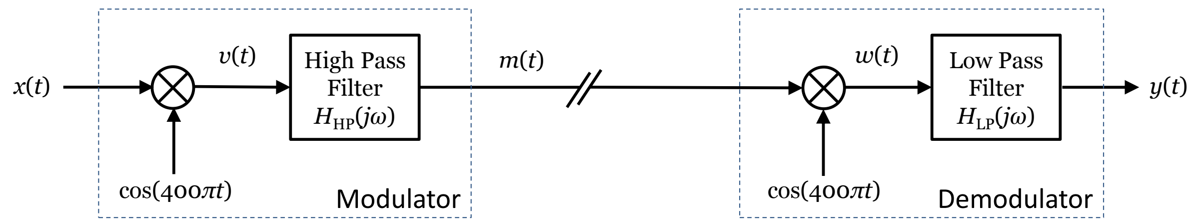
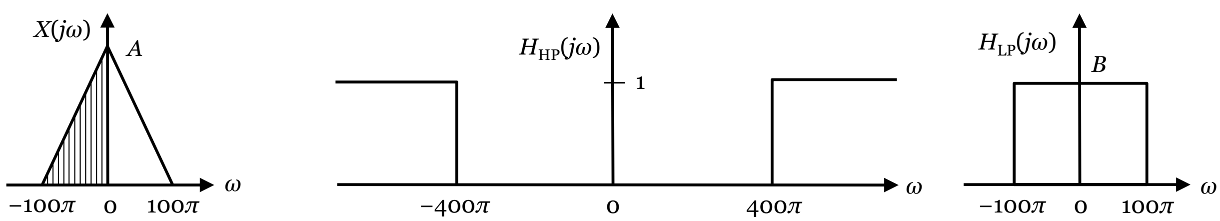
Problem Q2.1:

In lecture and lab, we explored a communication strategy called double-sideband amplitude modulation (DSBAM). In this problem, we will consider a very similar strategy called single-sideband amplitude modulation (SSBAM) as shown in the following block diagram:

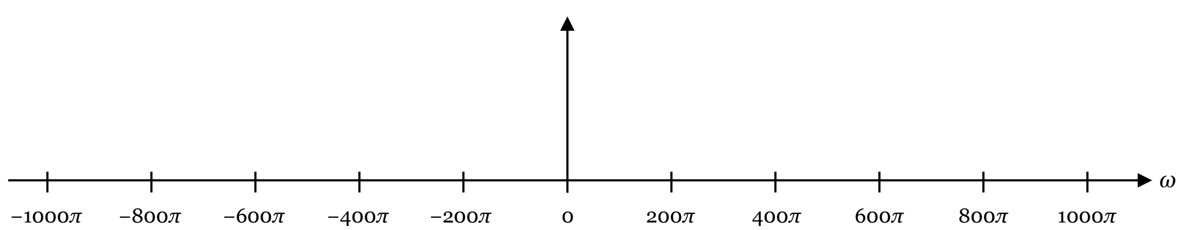


The only difference between SSBAM and DSBAM is the addition of a highpass filter to the modulator. We will consider the specific input $X(j\omega)$, high pass filter $H_{\mathrm{HP}}(j\omega)$ and lowpass filter $H_{\mathrm{LP}}(j\omega)$ shown below (note that the highpass filter is not shown on the same frequency scale as the lowpass filter).



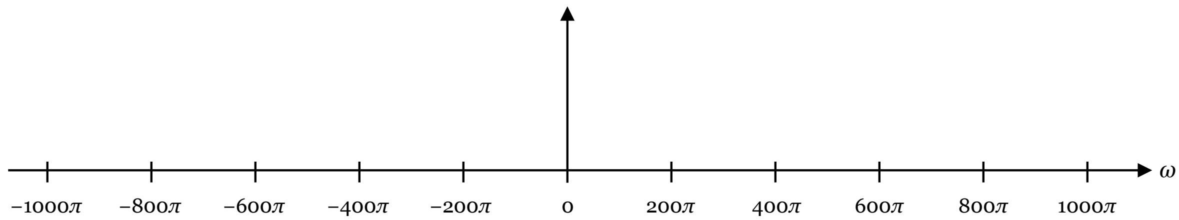
For parts (a), (b) and (c), be sure to properly identify the two halves (sidebands) of $X(j\omega)$ as it is shifted, filtered, etc. by shading the negative sideband as is done in the graph above.

(a) (5 pts) Using the axes below, sketch $V(j\omega)$, the Fourier transform of $v(t)$.

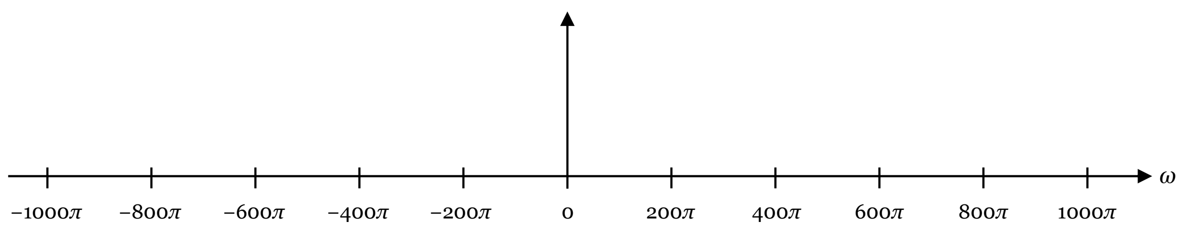


Problem Q2.1 (continued):

(b) (5 pts) Using the axes below, sketch $M(j\omega)$, the Fourier transform of $m(t)$.



(c) (5 pts) Using the axes below, sketch $W(j\omega)$, the Fourier transform of $w(t)$.

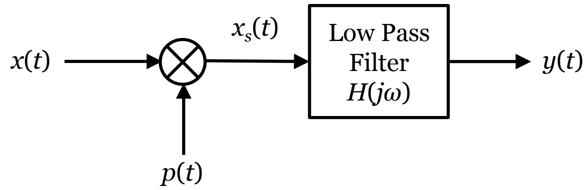


(d) (5 pts) What value of B, the gain of the lowpass filter in the demodulator, will make the output $y(t)$ exactly equal to the input $x(t)$? This should be straightforward if you have properly kept track of the amplitudes in parts (a), (b) and (c).

(e) (5 pts) SSBAM looks like it is more complicated to implement than DSBAM since it has an additional filter. What aspect of SSBAM may make it preferable to DSBAM? (This is a thinking question.)

Problem Q2.2:

Consider sampling and reconstruction as shown in the system below where $x(t)$ is the signal being sampled, $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ is an impulse train, $H(j\omega)$ is an ideal low pass filter with cutoff frequency $\omega_s/2$, and $y(t)$ is the reconstructed signal. For all parts of this problem, $x(t) = 2e^{-2t}u(t)$.



(a) (5 pts) Find $|X(j\omega)|$, where $X(j\omega)$ is the Fourier transform of $x(t)$. Is $x(t)$ bandlimited? (That is, is there a frequency ω_b such that $|X(j\omega)| = 0$ for $|\omega| > |\omega_b|$?)

(b) (5 pts) Accurately sketch $|X(j\omega)|$ for $\omega = -20$ to $+20$ rad/s. Calculate and label amplitudes for $\omega = 0$, $\omega = \pm 2$ rad/s and $\omega = \pm 20$ rad/s.

Problem Q2.2 (continued):

(c) (5 pts) After sampling, the sampled signal is $x_s(t) = x(t)p(t)$. Find $X_s(j\omega)$, the Fourier transform of $x_s(t)$, for the specific $x(t)$ of this problem and an arbitrary ω_s.

(d) (10 pts) Consider $\omega_s = 100$ rad/s. Approximately sketch $|X_s(j\omega)|$ for $\omega = -200$ to $+200$ rad/s. How well would you expect the reconstructed signal $y(t)$ to match the input $x(t)$? Justify your answer.

Problem Q2.3:

(5 points for each part) For all parts of this problem, consider the system with transfer function $H(s) = \dfrac{s-3}{s^2+4s+29}$, input $x(t)$ and output $y(t)$. Note that later parts of the problem do not necessarily depend upon earlier parts, so don't give up if you get stuck.

(a) Find the poles of this system. Is it stable?

(b) Find $h(t)$, the impulse response of the system.

(c) What is the corresponding differential equation associated with this transfer function that relates the input $x(t)$ and the output $y(t)$?

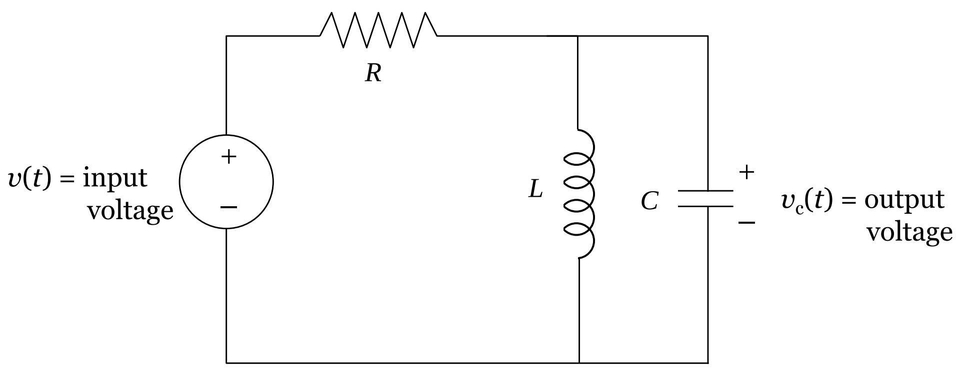
Problem Q2.3 (continued):

(d) Let $x(t) = te^{-3t}u(t)$. Find $Y(s)$ and set up its partial fraction expansion. Do not solve for the coefficients! Write an expression for $y(t)$ in terms of the coefficients of the partial fraction expansion. Make sure that your expression for $y(t)$ is real, and that it does not contain any complex exponentials.

(e) Find $y(0^+)$ and $\lim_{t \to \infty} y(t)$ for the input $x(t)$ of part (d). You do not have to do the full partial fraction expansion to find these, but be sure to justify your answers to receive credit.

Problem Q2.4:

Consider the circuit shown below where the input is the voltage $v(t)$ and the output is the voltage across the capacitor, $v_c(t)$.



(a) (5 pts) Draw this circuit in the s-domain assuming zero initial conditions.

(b) (10 pts) What is $H(s)$, the transfer function of this circuit relating the input $V(s)$ to the output $V_c(s)$? Express it as a ratio of polynomials in s.

Problem Q2.4 (continued):

(c) (10 pts) Suppose that $C = 10\ \mu\text{F}$ and $L = 10$ mH, and that you can select either $R = 10\ \Omega$ or $R = 1\ \text{k}\Omega$. Will either of these resistors result in a circuit that does not oscillate? (That is, the poles are purely real.) If so, which one(s)? Answers without any justification will not receive any credit.

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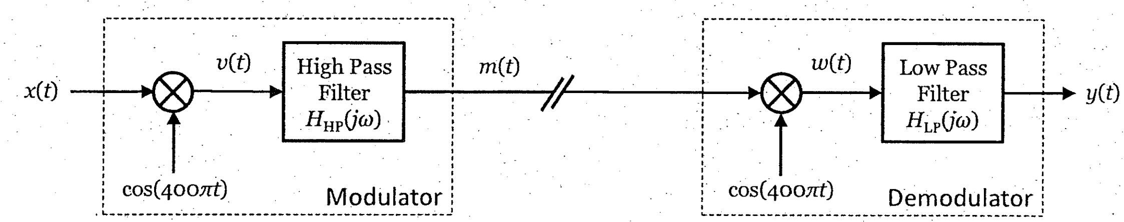

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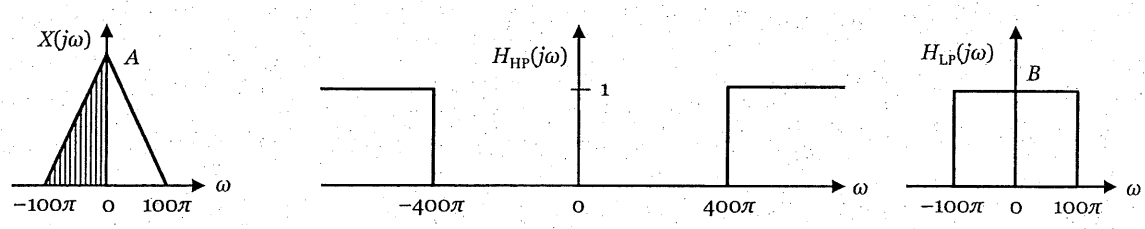

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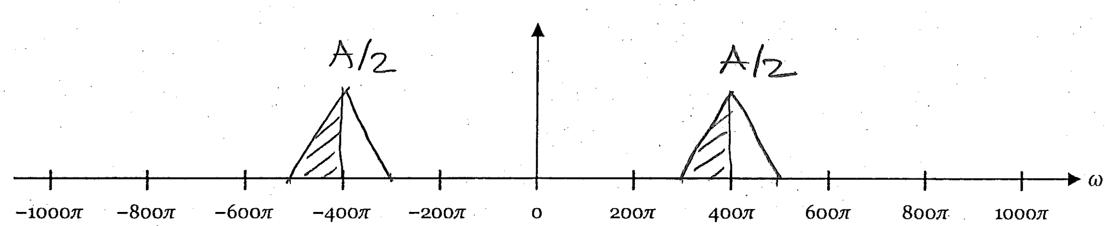
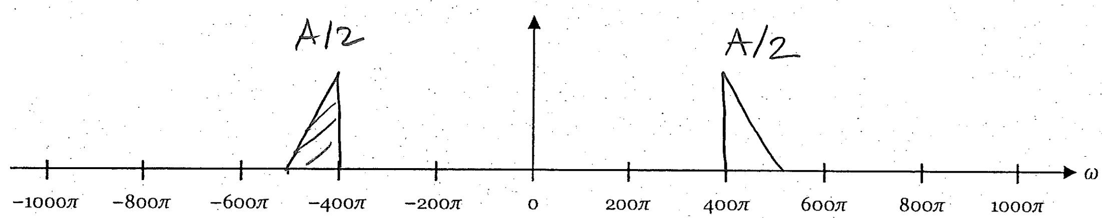
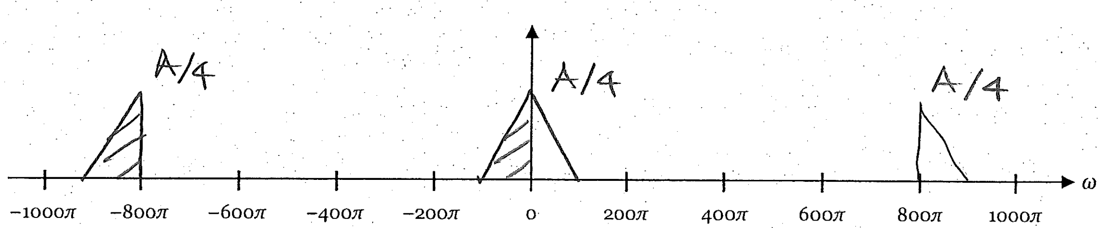


Problem Q2.1 (continued):

(b) (5 pts) Using the axes below, sketch $M(j\omega)$, the Fourier transform of $m(t)$.



(c) (5 pts) Using the axes below, sketch $W(j\omega)$, the Fourier transform of $w(t)$.



(d) (5 pts) What value of B, the gain of the lowpass filter in the demodulator, will make the output $y(t)$ exactly equal to the input $x(t)$? This should be straightforward if you have properly kept track of the amplitudes in parts (a), (b) and (c).

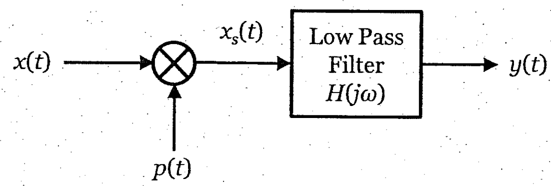
$$\frac{AB}{4} = A \Rightarrow B = 4$$

(e) (5 pts) SSBAM looks like it is more complicated to implement than DSBAM since it has an additional filter. What aspect of SSBAM may make it preferable to DSBAM? (This is a thinking question.)

Only half the bandwidth is required for the same amount of information $\Rightarrow$ more channels for the same total bandwidth

Problem Q2.2:

Consider sampling and reconstruction as shown in the system below where $x(t)$ is the signal being sampled, $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ is an impulse train, $H(j\omega)$ is an ideal low pass filter with cutoff frequency $\omega_s/2$, and $y(t)$ is the reconstructed signal. For all parts of this problem, $x(t) = 2e^{-2t}u(t)$.



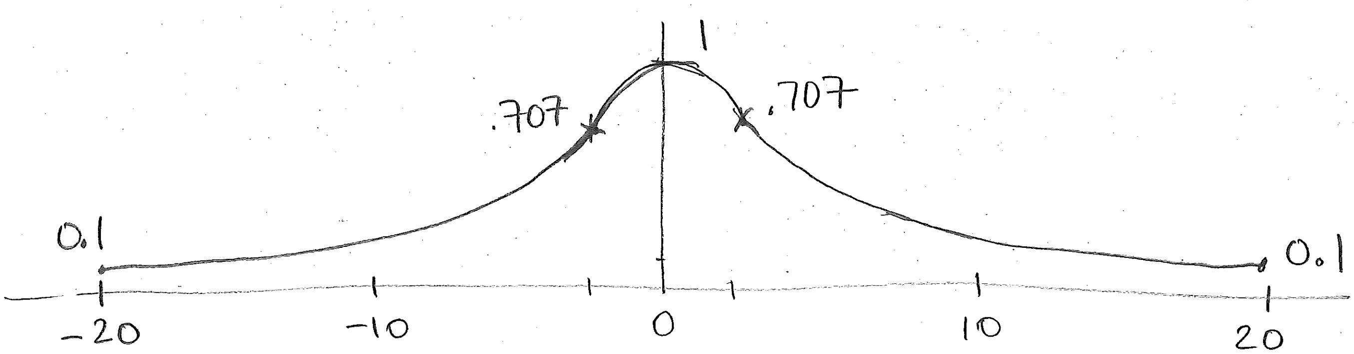
(a) (5 pts) Find $|X(j\omega)|$, where $X(j\omega)$ is the Fourier transform of $x(t)$. Is $x(t)$ bandlimited? (That is, is there a frequency ω_b such that $|X(j\omega)| = 0$ for $|\omega| > |\omega_b|$?)

$$X(j\omega) = \frac{2}{2+j\omega}$$

$$|X(j\omega)| = \sqrt{\frac{4}{4+\omega^2}}$$

This signal is <u>NOT</u> bandlimited

(b) (5 pts) Accurately sketch $|X(j\omega)|$ for $\omega = -20$ to $+20$ rad/s. Calculate and label amplitudes for $\omega = 0$, $\omega = \pm 2$ rad/s and $\omega = \pm 20$ rad/s.



$$|X(j0)| = 1$$

$$|X(j2)| = \sqrt{\tfrac{1}{2}} = 0.707$$

$$|X(j20)| = \sqrt{\frac{4}{404}} = 0.1$$

Problem Q2.2 (continued):

(c) (5 pts) After sampling, the sampled signal is $x_s(t) = x(t)p(t)$. Find $X_s(j\omega)$, the Fourier transform of $x_s(t)$, for the specific $x(t)$ of this problem and an arbitrary ω_s.

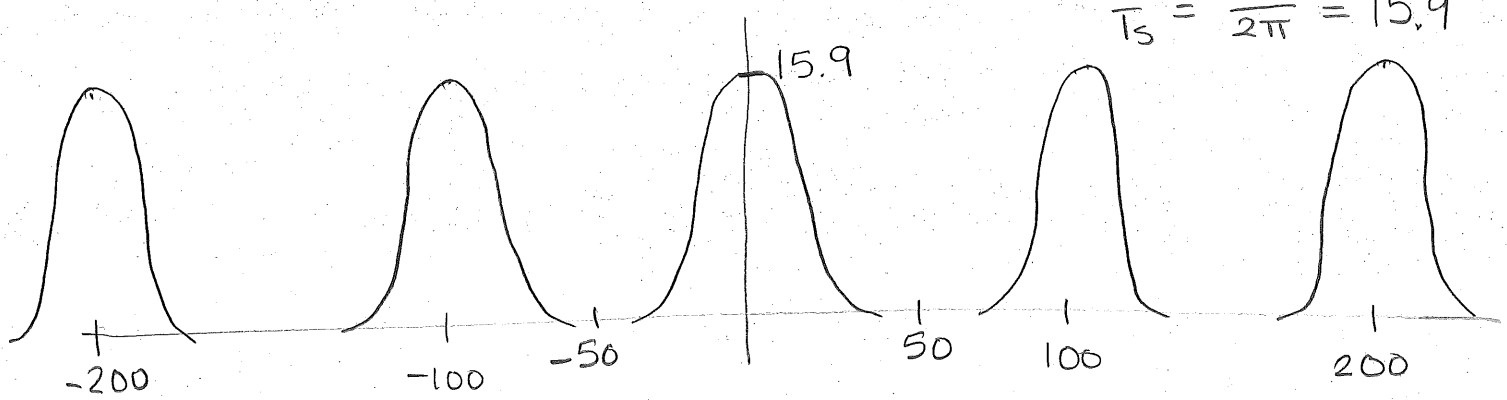
$$X_s(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$= \frac{1}{2\pi} \frac{2}{2+j\omega} * \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \frac{2}{2+j(\omega - k\omega_s)}$$

(d) (10 pts) Consider $\omega_s = 100$ rad/s. Approximately sketch $|X_s(j\omega)|$ for $\omega = -200$ to $+200$ rad/s. How well would you expect the reconstructed signal $y(t)$ to match the input $x(t)$? Justify your answer.

$$\frac{1}{T_s} = \frac{\omega_s}{2\pi} = 15.9$$



There is very little aliasing since $X(j\omega)$ is small at $\omega = 50$ ($|X(j50)| = 0.04$. So the reconstructed signal should match the input quite well.

Problem Q2.3:

(5 points for each part) For all parts of this problem, consider the system with transfer function $H(s) = \dfrac{s-3}{s^2+4s+29}$, input $x(t)$ and output $y(t)$. Note that later parts of the problem do not necessarily depend upon earlier parts, so don't give up if you get stuck.

(a) Find the poles of this system. Is it stable?

$$s^2+4s+29 = (s+2)^2+25$$

$$p_{1,2} = -2 \pm j5$$

The system is stable since both poles are in the LHP.

(b) Find $h(t)$, the impulse response of the system.

Approach #1

$$\frac{s-3}{(s+2)^2+25} = \frac{s+2}{(s+2)^2+25} - \frac{5}{(s+2)^2+25}$$

$$h(t) = e^{-2t}\cos(5t)u(t) - e^{-2t}\sin(5t)u(t)$$

$$= \sqrt{2}\cos(5t + \pi/4)u(t)$$

Approach #2

$$\frac{s-3}{s^2+4s+29} = \frac{c_1}{(s+2-5j)} + \frac{c_2^*}{s+2+5j}$$

$$c_1 = \left.\frac{s-3}{s+2+5j}\right|_{s=-2+5j}$$

$$c_1 = \frac{-5+5j}{10j} = \frac{1}{2} + \frac{1}{2}j$$

$$h(t) = 2|c_1|e^{-2t}\cos(5t+\angle c_1)u(t)$$

$$= \sqrt{2}e^{-2t}\cos(5t+\pi/4)u(t)$$

Either approach is fine.

(c) What is the corresponding differential equation associated with this transfer function that relates the input $x(t)$ and the output $y(t)$?

$$\frac{s-3}{s^2+4s+29} = \frac{Y(s)}{X(s)}$$

$$s^2Y(s) + 4sY(s) + 29Y(s) = sX(s) - 3X(s)$$

$$\boxed{\ddot{y} + 4\dot{y} + 29y = \dot{x} - 3x}$$

Problem Q2.3 (continued):

(d) Let $x(t) = te^{-3t}u(t)$. Find $Y(s)$ and set up its partial fraction expansion. Do not solve for the coefficients! Write an expression for $y(t)$ in terms of the coefficients of the partial fraction expansion. Make sure that your expression for $y(t)$ is real, and that it does not contain any complex exponentials.

$$X(s) = \frac{1}{(s+3)^2}$$

$$Y(s) = \frac{s-3}{(s+3)^2(s^2+4s+29)} = \frac{c_1}{s+3} + \frac{c_2}{(s+3)^2} + \frac{c_3}{s+2-5j} + \frac{c_3^*}{s+2+5j}$$

$$y(t) = c_1 e^{-3t}u(t) + c_2 t e^{-3t}u(t) + 2|c_3|e^{-2t}\cos(5t + \angle c_3)u(t)$$

You could also do, $Y(s) = \frac{c_1}{s+3} + \frac{c_2}{(s+3)^2} + \frac{c_3 s + c_4}{(s+2)^2+5^2}$,

but $y(t)$ is more complicated.

(e) Find $y(0^+)$ and $\lim_{t\to\infty} y(t)$ for the input $x(t)$ of part (d). You do not have to do the full partial fraction expansion to find these, but be sure to justify your answers to receive credit.

$y(0^+)$ Use the IVT

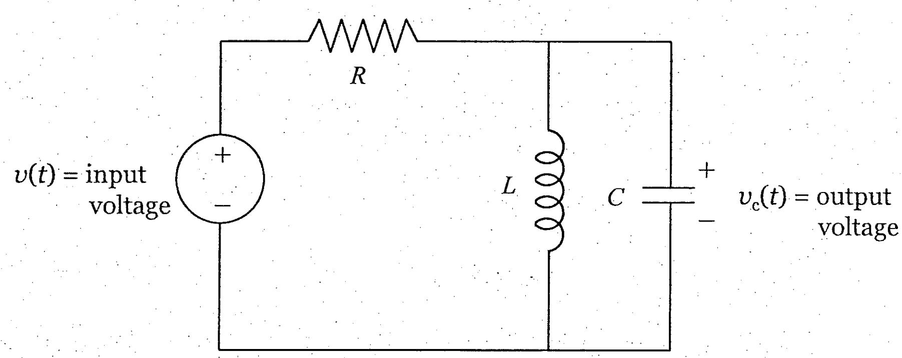
$$y(0^+) = \lim_{s\to\infty} sY(s) = 0$$

For $\lim_{t\to\infty} y(t)$, Use the FVT (or part (d))

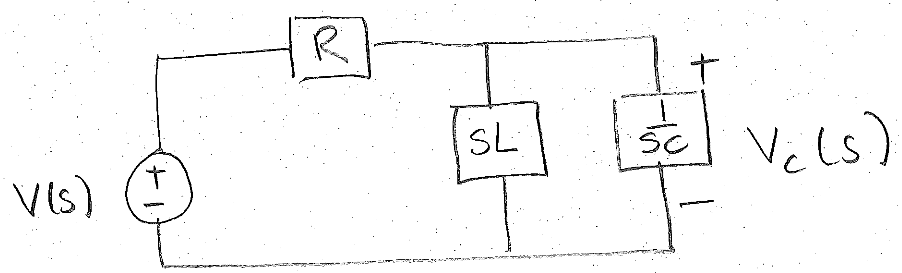
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0$$

Problem Q2.4:

Consider the circuit shown below where the input is the voltage $v(t)$ and the output is the voltage across the capacitor, $v_c(t)$.



(a) (5 pts) Draw this circuit in the s-domain assuming zero initial conditions.



(b) (10 pts) What is $H(s)$, the transfer function of this circuit relating the input $V(s)$ to the output $V_c(s)$? Express it as a ratio of polynomials in s.

Voltage divider

$$V_c(s) = V(s)\,\frac{Z}{R+Z}\ ; \quad Z = SL \,\|\, \frac{1}{SC} = \frac{\frac{SL}{SC}}{SL + \frac{1}{SC}} = \frac{SL}{S^2LC+1}$$

$$H(s) = \frac{V_c(s)}{V(s)} = \frac{\frac{SL}{\cancel{S^2LC+1}}}{\frac{R(S^2LC+1)+SL}{\cancel{S^2LC+1}}}$$

$$\boxed{H(s) = \frac{SL}{S^2RLC + SL + R}}$$

Problem Q2.4 (continued):

(c) (10 pts) Suppose that $C = 10\ \mu$F and $L = 10$ mH, and that you can select either $R = 10\ \Omega$ or $R = 1$ kΩ. Will either of these resistors result in a circuit that does not oscillate? (That is, the poles are purely real.) If so, which one(s)? Answers without any justification will not receive any credit.

$$H(s) = \frac{sL}{s^2 RLC + sL + R}$$

divide top & bottom by RLC

$$= \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Denominator:

$$s^2 + \frac{s}{RC} + \frac{1}{LC}$$

$R = 10\Omega$: $$s^2 + \frac{s}{10 \cdot 10 \cdot 10^{-6}} + \frac{1}{10 \cdot 10^{-3} \cdot 10 \cdot 10^{-6}} = 0$$

$$s^2 + 10^4 s + 10^7 = 0$$

$$P_{1,2} = \frac{-10^4 \pm \sqrt{10^8 - 4 \cdot 10^7}}{2} \Rightarrow \text{real}$$

$R = 1k\Omega$: $$s^2 + \frac{s}{10^3 \cdot 10 \cdot 10^{-6}} + 10^7 = 0$$

$$s^2 + 100s + 10^7 = 0$$

$$P_{1,2} = \frac{-100 \pm \sqrt{10^4 - 4 \cdot 10^7}}{2} \Rightarrow \text{complex roots}$$

The circuit won't oscillate for $R = 10\Omega$ but it will for $R = 1k\Omega$.