ECE 3084
QUIZ 2
SCHOOL OF ECE
GEORGIA INSTITUTE OF TECHNOLOGY

NOVEMBER 12, 2020

NAME: ______________________________________

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

"My signature below attests that I have neither given nor received help during the taking of this exam, and that I am in complete compliance with the Georgia Tech honor code:"

____
(your signature)

- This is a timed quiz: After you start you have 120 minutes to click the "Submit Quiz" button.
- The nominal duration of the quiz itself is 75 minutes; the extra 45 minutes is generously budgeted for scanning and uploading.
- The countdown starts once you begin. It will not pause if you disconnect from the internet, and it will not reset if you log in again.
- It is your responsibility to observe the time.
- You must begin the exam prior to the cutoff time specified.
- Open book, open calculator, open MATLAB. Google and other internet search engines are not recommended.
- No communications (electronic or otherwise) to anyone regarding the quiz, except for one exception:
- Please use Piazza for any questions, or to correct a typo.
- A correct answer without justification will not receive full credit.
- Full credit requires not only a correct answer but also a clear justification, so show your work and explain your thinking.
- Partial credit will be available but limited.
- Before you click "Submit Quiz," be sure to upload a single PDF document that shows your work and answers.
- If you have a printer: Print this PDF, work on it directly, then scan and upload it.
- If you do not have timely access to a printer, you have three other options:
  - You can print the template and work on it,
  - You can work directly on this PDF electronically, using a tablet and stylus.
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- Do not attach extra pages. Only the first 7 pages will be graded.
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PROBLEM 1.

Consider the signal $x(t) = 3084\cos(200\pi t + \frac{\pi}{3})$, a sinusoid with frequency $f_0 = 100$ Hz.

Let $y_I(t)$ be the in-phase component of $y(t) = x(t - t_0)$ (a delayed version of the original) with respect to f_0.

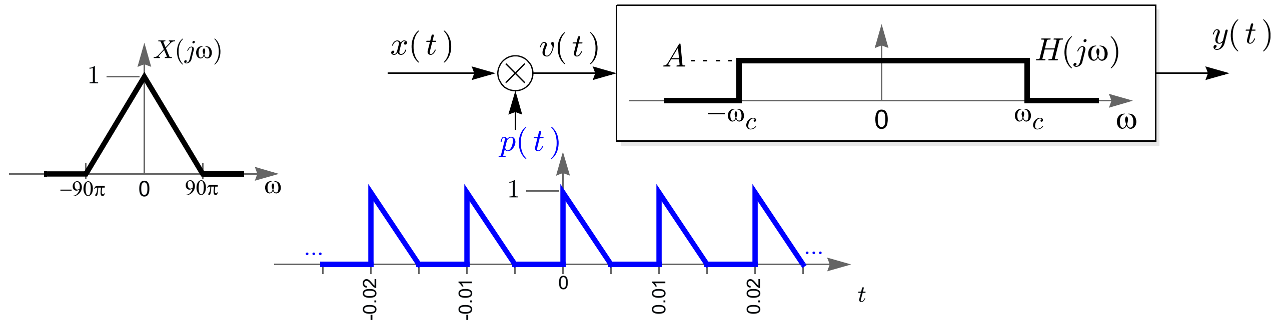
(a) Find $y_I(t)$ when the delay is zero ($t_0 = 0$):

$y_I(t) =$

(b) Find the *smallest positive* value of the delay $t_0 > 0$ that results in $y_I(t) = 0$ (for all t).

smallest $t_0 =$ > 0 seconds

PROBLEM 2. Consider the following system:



where $x(t)$ has the triangular Fourier transform shown in the figure, where $p(t)$ is the periodic signal with fundamental period 0.01 seconds shown in the figure, and where $v(t) = x(t)p(t)$ is passed through an ideal low-pass filter with gain A and cutoff frequency ω_c rad/s, producing $y(t)$.

(a) Is it possible to choose A and ω_c so that $y(t) = x(t)$? YES ☐ NO ☐

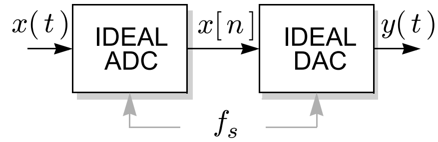
(b) If YES, specify values for A and ω_c so that $y(t) = x(t)$:

$A =$ ☐

$\omega_c =$ ☐ rad/s

If NO, explain: Why not?

PROBLEM 3.



Suppose that a signal $x(t) = \cos(120\pi t)\left(\frac{\sin(120\pi t)}{\pi t}\right)^2$

is sampled by an ideal analog-to-digital converter with sampling rate f_s Hz, and that the sampled sequence $x[n]$ is fed to an ideal digital-to-analog converter with the same f_s parameter, to produce an output $y(t)$.

(a) In order for $y(t) = x(t)$ (no aliasing), the sampling rate must satisfy $f_s >$ ☐ Hz.

(b) If $f_s = 120$ Hz then $y(t)$ has the form $y(t) = A\frac{\sin(2\pi Bt)}{\pi t}$, where:

$A =$ ☐

$B =$ ☐.

(c) Specify a sampling rate satisfying $f_s > 120$ Hz so that $y(t)$ has the form $y(t) = C\frac{\sin(2\pi Dt)}{\pi t}$, and further specify the constants C and D:

$f_s =$ ☐ Hz (> 120 Hz)

$C =$ ☐

$D =$ ☐.

PROBLEM 4. (The two parts are unrelated.)

(a) In terms of the unit ramp $r(t) = tu(t)$, the inverse Laplace transform of $X(s) = \frac{(1 - e^{-2s})^3}{s^2}$ can be written as: $x(t) = Ar(t) + Br(t - C) + Dr(t-E) + Fr(t-G)$, where (*hint:* all answers are integers!):

$A =$ ☐, $B =$ ☐, $C =$ ☐, $D =$ ☐, $E =$ ☐, $F =$ ☐, $G =$ ☐.

(b) The Laplace transform of $x(t) = 2te^{-t}\cos(t)u(t)$ can be written as $X(s) = \frac{As^2 + Bs}{(s^2 + Cs + D)^2}$, where:

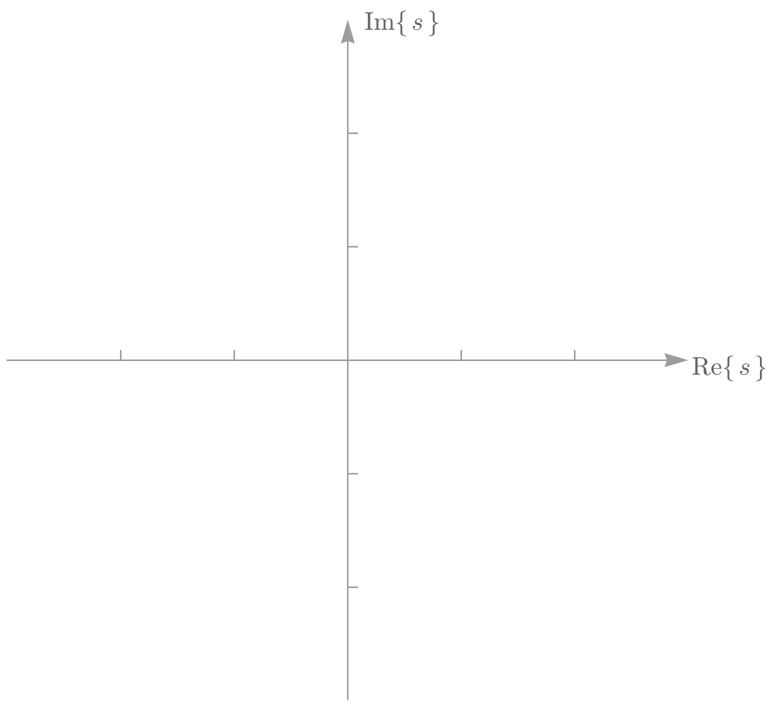
$A =$ ☐ $B =$ ☐

$C =$ ☐ $D =$ ☐.

PROBLEM 5.

In the space below, carefully sketch the *pole-zero plot* for the Laplace transform $H(s)$ of the following signal:

$$h(t) = (e^{-t} - \sin(t))u(t).$$



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PROBLEM 1.

Consider the signal $x(t) = 3084\cos(200\pi t + \frac{\pi}{3})$, a sinusoid with frequency $f_0 = 100$ Hz.

Let $y_I(t)$ be the in-phase component of $y(t) = x(t - t_0)$ (a delayed version of the original) with respect to f_0.

(a) Find $y_I(t)$ when the delay is zero ($t_0 = 0$):

$y_I(t) =$ 1542

x(t) is in env-and-phase form

$\Rightarrow$ complex envelope is $\tilde{x}(t) = 3084 e^{j\pi/3}$

$\Rightarrow$ real part is $x_I(t) = 3084 \cos(\frac{\pi}{3}) = 1542$

(b) Find the *smallest positive* value of the delay $t_0 > 0$ that results in $y_I(t) = 0$ (for all t).

smallest $t_0 =$ $\frac{1}{240}$ > 0 seconds

with nonzero t_0, y(t) still in env-and-phase form

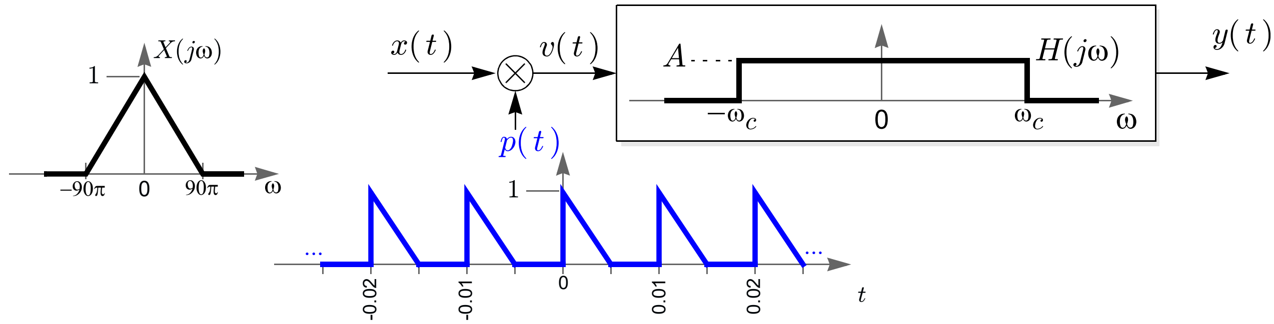
$\Rightarrow \tilde{y}(t) = 3084 e^{j(\frac{\pi}{3} - 200\pi t_0)}$

$\Rightarrow$ real part zero when this is $-\frac{\pi}{2}$:

$$\frac{\pi}{3} - 200\pi t_0 = -\frac{\pi}{2}$$

$$\Rightarrow t_0 = \frac{5\pi/6}{200\pi} = \frac{1}{240}$$

PROBLEM 2. Consider the following system:



where $x(t)$ has the triangular Fourier transform shown in the figure, where $p(t)$ is the periodic signal with fundamental period 0.01 seconds shown in the figure, and where $v(t) = x(t)p(t)$ is passed through an ideal low-pass filter with gain A and cutoff frequency ω_c rad/s, producing $y(t)$.

(a) Is it possible to choose A and ω_c so that $y(t) = x(t)$? YES [X] NO [ ]

(b) If YES, specify values for A and ω_c so that $y(t) = x(t)$:

$A =$ 4

If NO, explain: Why not?

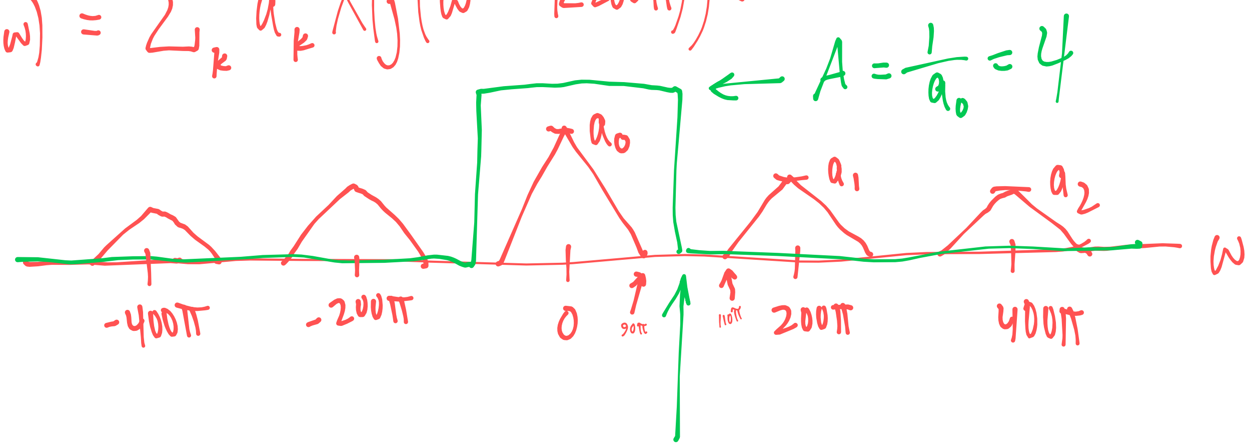
$\omega_c =$ 100π rad/s

$p(t) = \sum_k a_k e^{jk200\pi t}$ (where $a_0 = \frac{1}{0.01}(\frac{1}{2})(0.005) = 0.25$)

$\Rightarrow v(t) = \sum_k a_k e^{jk200\pi t} x(t)$

$\updownarrow$

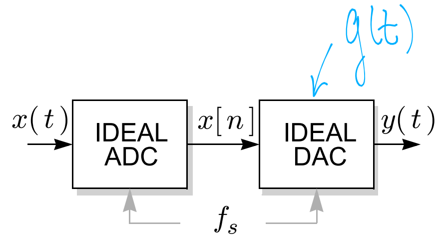
$V(j\omega) = \sum_k a_k X(j(\omega - k200\pi))$:



$\omega_c \in (90\pi, 110\pi)$

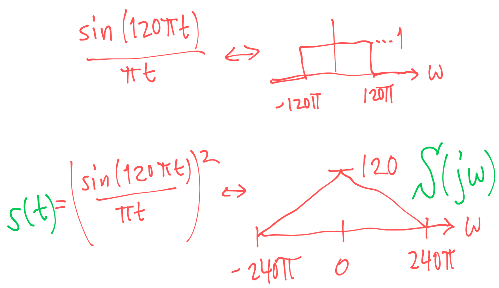
$X(j\omega) = \frac{1}{2} S(j(\omega \pm 120\pi))$

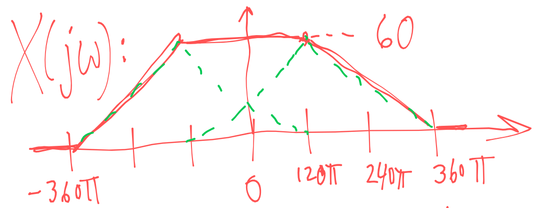
PROBLEM 3.



Suppose that a signal $x(t) = \cos(120\pi t)\left(\frac{\sin(120\pi t)}{\pi t}\right)^2$ $s(t)$ is sampled by an ideal analog-to-digital converter with sampling rate f_s Hz, and that the sampled sequence $x[n]$ is fed to an ideal digital-to-analog converter with the same f_s parameter, to produce an output $y(t)$.

(a) In order for $y(t) = x(t)$ (no aliasing), the sampling rate must satisfy $f_s >$ **360** Hz.





$f_{max} = 180$ Hz $\Rightarrow f_s > 2f_{max}$

(b) If $f_s = 120$ Hz then $y(t)$ has the form $y(t) = A\frac{\sin(2\pi Bt)}{\pi t}$, where:

$A =$ **120**

$B =$ **60**.

$x(t) = \cos(120\pi t)s(t)$

$\Rightarrow x[n] = x\left(\frac{n}{120}\right) = \cos(\pi n)\, s\left(\frac{n}{120}\right)$

$= \cos(\pi n)\, 120^2 \delta[n] = 120^2 \delta[n] \Rightarrow y(t) = 120^2 g(t) = 120\left(\frac{\sin(120\pi t)}{\pi t}\right)$

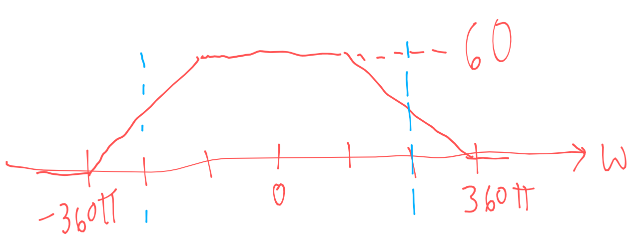
(c) Specify a sampling rate satisfying $f_s > 120$ Hz so that $y(t)$ has the form $y(t) = C\frac{\sin(2\pi Dt)}{\pi t}$, and further specify the constants C and D:

$f_s =$ **240** Hz (> 120 Hz)

$C =$ **60**

$D =$ **120**.

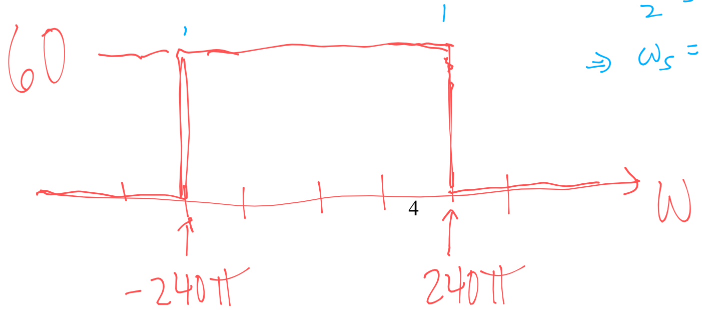
Same X(jω):



fold along this line to get rectangle

$\Rightarrow \frac{\omega_s}{2} = 240\pi$

$\Rightarrow \omega_s = 480\pi \Rightarrow f_s = 240$ Hz



PROBLEM 4. (The two parts are unrelated.)

$R(s) = \frac{1}{s^2}$ ↕

(a) In terms of the unit ramp $r(t) = tu(t)$, the inverse Laplace transform of $X(s) = \frac{(1-e^{-2s})^3}{s^2}$ can be written as: $x(t) = Ar(t) + Br(t-C) + Dr(t-E) + Fr(t-G)$, where (*hint:* all answers are integers!):

$A =$ 1, $B =$ -3, $C =$ 2, $D =$ 3, $E =$ 4, $F =$ -1, $G =$ 6.

$$X(s) = \frac{(1-e^{-2s})^3}{s^2} = \frac{1}{s^2}\left(1 - 3e^{-2s} + 3e^{-4s} - e^{-6s}\right)$$

↕

$$x(t) = r(t) - 3r(t-2) + 3r(t-4) - r(t-6)$$

(b) The Laplace transform of $x(t) = 2te^{-t}\cos(t)u(t)$ can be written as $X(s) = \frac{As^2 + Bs}{(s^2 + Cs + D)^2}$, where:

$A =$ 2 $B =$ 4

$C =$ 2 $D =$ 2.

Let $g(t) = 2e^{-t}\cos(t)u(t)$, without factor of t

↕ modulation property of Laplace, with $a = 1$

$$G(s) = 2\frac{(s+a)}{(s+a)^2+1} = \frac{2s+2}{s^2+2s+2}$$

then $x(t) = tg(t)$

↕

$$X(s) = -\frac{d}{ds}G(s) = \frac{(s^2+2s+2)2 - (2s+2)(2s+2)}{(s^2+2s+2)^2}$$

$$= \frac{2s^2 + 4s}{(s^2+2s+2)^2}$$

PROBLEM 5.

In the space below, carefully sketch the *pole-zero plot* for the Laplace transform $H(s)$ of the following signal: $h(t) = (e^{-t} - \sin(t))u(t)$.

$$\Updownarrow$$

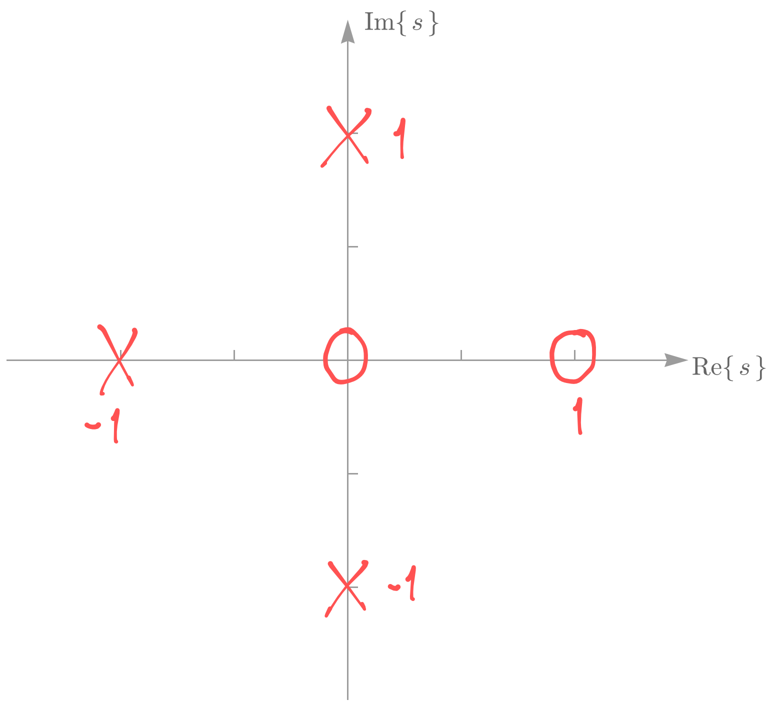
$$H(s) = \frac{1}{s+1} - \frac{1}{s^2+1}$$

$$= \frac{s^2+1-s-1}{(s+1)(s^2+1)}$$

$$= \frac{s(s-1)}{(s+1)(s^2+1)}$$

roots of numerator $\Rightarrow$ zeros at $s = 0, +1$

roots of denominator $\Rightarrow$ poles at $s = -1, \pm j$



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