ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
NOVEMBER 20, 2018

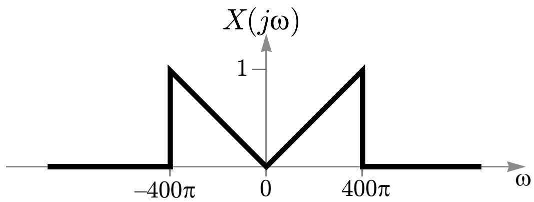
Name: ____

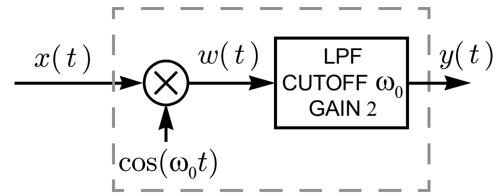
1. The quiz is closed book, closed notes, except for two 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

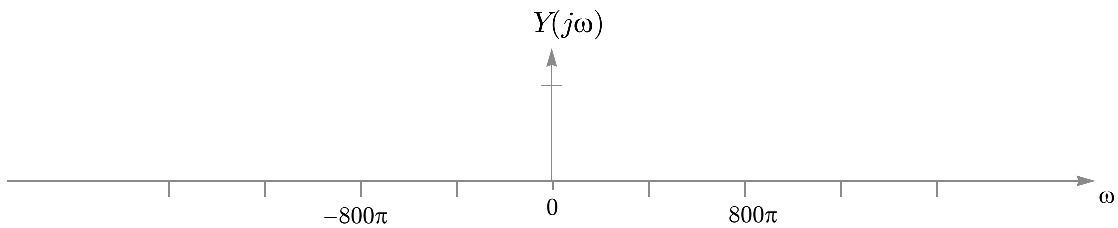
PROBLEM 1.

Suppose a signal $x(t)$ with the FT $X(j\omega)$ shown below is passed as an input into the illustrated AM demodulator (dashed box), producing an output $y(t)$:





(a) Sketch the Fourier transform $Y(j\omega)$ of the demodulator output for the special case when the demodulator carrier frequency is $\omega_0 = 800\pi$ (labeling amplitudes carefully!):



(b) In order for the demodulator output $y(t)$ to be a sinc function,

it must be that $\omega_0 =$ ☐ rad/s.

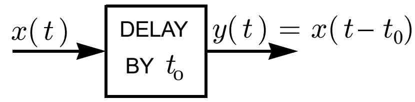
(c) Using the value of ω_0 from part (b), the demodulator output has the form $y(t) = \dfrac{B\sin(Ct)}{t}$, where:

$B =$ ☐

$C =$ ☐

PROBLEM 2.

Suppose the signal $x(t) = 30\cos(15\pi t) - 84\sin(15\pi t)$ is fed as an input into a delay-by-t_0 system, producing the output $y(t) = x(t - t_0)$, as shown in the figure.
Let $\omega_0 = 15\pi$.



(a) The complex envelope of the input $x(t)$, with respect to ω_0, is

$\tilde{x}(t) =$ ________ .

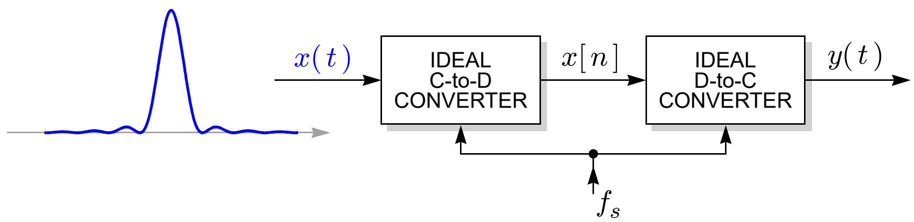
(b) Find *the smallest** positive value for the delay $t_0 > 0$ so that the I & Q components of the delay output (with respect to ω_0) are $y_I(t) = 84$ and $y_Q(t) = -30$, respectively.

*there are multiple positive values of t_0 that do the job, but only one of them is the smallest

$t_0 =$ ________

PROBLEM 3.

Suppose that the continuous-time sinc-squared signal $x(t) = \left(\frac{\sin(80\pi t)}{\pi t}\right)^2$ is sampled at an unspecified sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (or ideal DAC, with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:

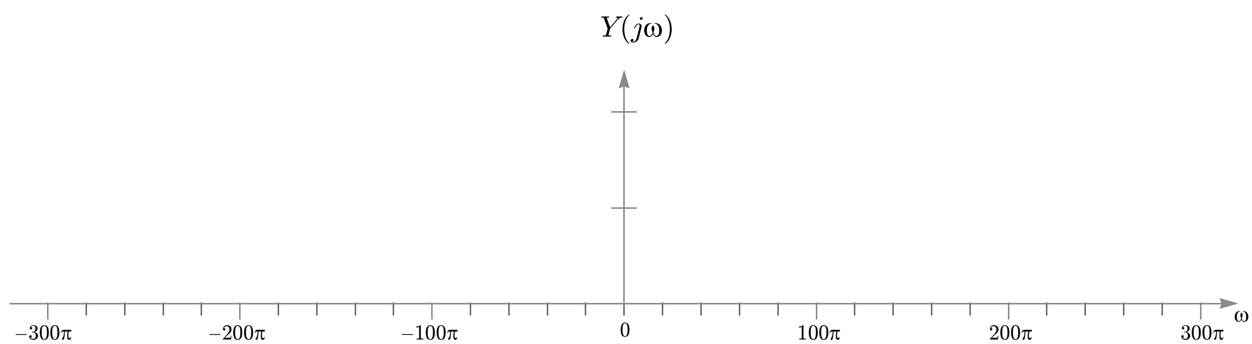


(a) The zero-th sample is $x[0] =$ ☐.

(b) In order for the D-to-C converter to reconstruct the original signal (i.e., to achieve $y(t) = x(t)$), the sampling frequency must satisfy:

$f_s >$ ☐ Hz.

(c) In the space below, carefully sketch the output Fourier transform $Y(j\omega)$ when $f_s = 140$ Hz, *carefully labeling important amplitudes*:



PROBLEM 4.

Consider a signal $x(t)$ that obeys the following differential equation:

$$\frac{d^2}{dt^2}x(t) - 9x(t) = 0,$$

where one initial condition is specified, namely $x(0) = 2$,
while the other initial condition $\dot{x}(0)$ is not specified and may be nonzero.

If the solution to this differential equation has the form $x(t) = Be^{-Ct}u(t)$, where $C > 0$, then it must be that:

$\dot{x}(0) =$ ☐

$B =$ ☐

$C =$ ☐

PROBLEM 5.

An LTI system (initially at rest, zero initial conditions) with input $x(t)$ and output $y(t)$ obeys the following differential equation:

$$\frac{d^2}{dt^2}y(t) = 2x(t) - 9y(t) - 4\frac{d}{dt}y(t).$$

(a) Circle one: The system is [undamped][overdamped][underdamped][critically damped]?

(b) Circle one: It most resembles a [LPF][HPF][BPF][notch] filter.

(c) Its d.c. gain is $H_0 =$ ______ .

(d) Its natural frequency is $\omega_n =$ ______ (in rad/s).

(e) Its damping ratio is $\zeta =$ ______ .

(f) The *impulse* response of this system is $h(t) = Ae^{-Bt}\sin(Ct)$, where:

$A =$ ______

$B =$ ______

$C =$ ______

ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

GEORGIA INSTITUTE OF TECHNOLOGY

NOVEMBER 20, 2018

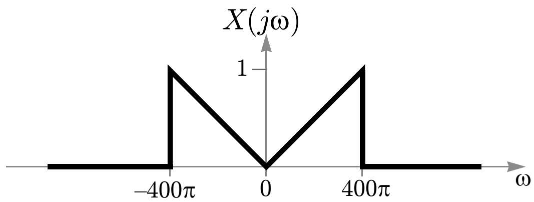
Name: ANSWER KEY

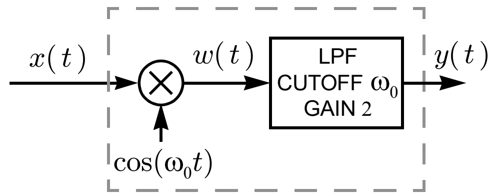
1. The quiz is closed book, closed notes, except for two 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

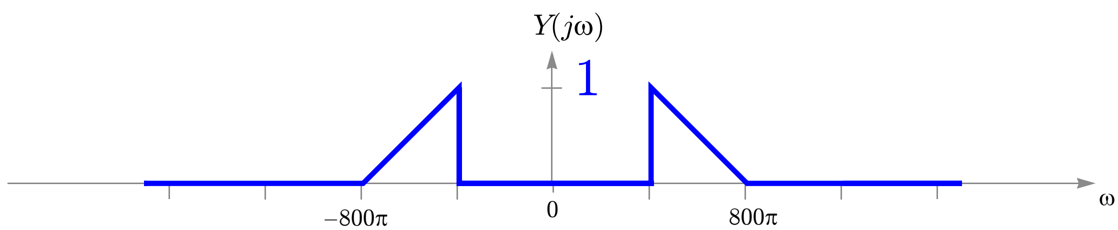
PROBLEM 1.

Suppose a signal $x(t)$ with the FT $X(j\omega)$ shown below is passed as an input into the illustrated AM demodulator (dashed box), producing an output $y(t)$:



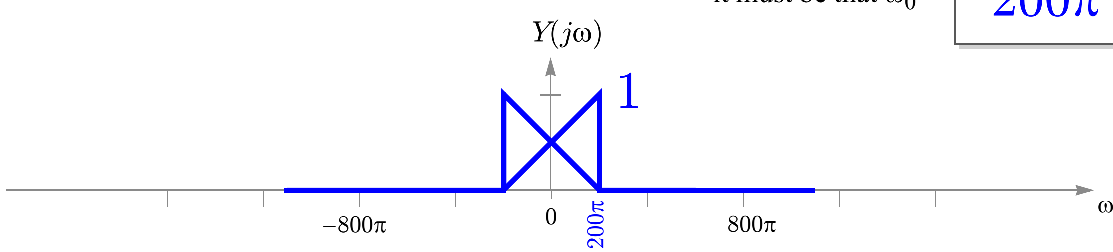


(a) Sketch the Fourier transform $Y(j\omega)$ of the demodulator output for the special case when the demodulator carrier frequency is $\omega_0 = 800\pi$ (labeling amplitudes carefully!):



(b) In order for the demodulator output $y(t)$ to be a sinc function,

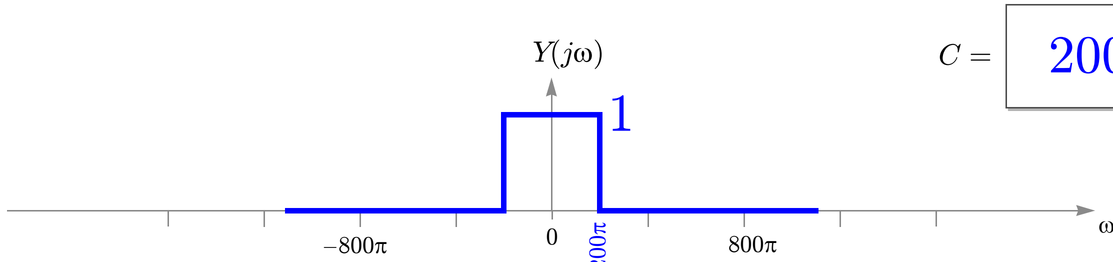
it must be that $\omega_0 =$ 200π rad/s.



(c) Using the value of ω_0 from part (b), the demodulator output has the form $y(t) = \dfrac{B\sin(Ct)}{t}$, where:

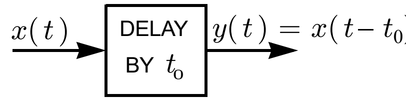
$B = \dfrac{1}{\pi}$

$C = 200\pi$



PROBLEM 2.

Suppose the signal $x(t) = 30\cos(15\pi t) - 84\sin(15\pi t)$ is fed as an input into a delay-by-t_0 system, producing the output $y(t) = x(t - t_0)$, as shown in the figure.
Let $\omega_0 = 15\pi$.



(a) The complex envelope of the input $x(t)$, with respect to ω_0, is $\tilde{x}(t) =$ $\boxed{30 + 84j}$.

(b) Find *the smallest** positive value for the delay $t_0 > 0$ so that the I & Q components of the delay output (with respect to ω_0) are $y_I(t) = 84$ and $y_Q(t) = -30$, respectively.

*there are multiple positive values of t_0 that do the job, but only one of them is the smallest

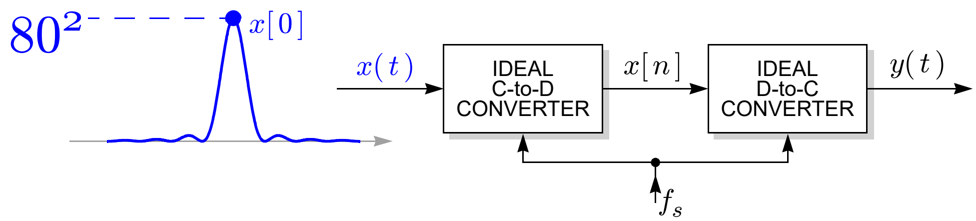
$t_0 = \boxed{\dfrac{1}{30}}$

$$y(t) = x(t - t_0)$$
$$= 30\cos(15\pi(t - t_0)) - 84\sin(15\pi(t - t_0))$$
$$= 30\cos(15\pi t - \theta) - 84\sin(15\pi - \theta)$$
$$= 84\cos(15\pi t) + 30\sin(15\pi)$$

$$\text{when } \theta = 15\pi t_0 = 0.5\pi \quad \Rightarrow \quad t_0 = \frac{0.5\pi}{15\pi} = \frac{1}{30}$$

PROBLEM 3.

Suppose that the continuous-time sinc-squared signal $x(t) = \left(\frac{\sin(80\pi t)}{\pi t}\right)^2$ is sampled at an unspecified sampling rate f_s, and that the samples are immediately fed to an ideal D-to-C converter (or ideal DAC, with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:

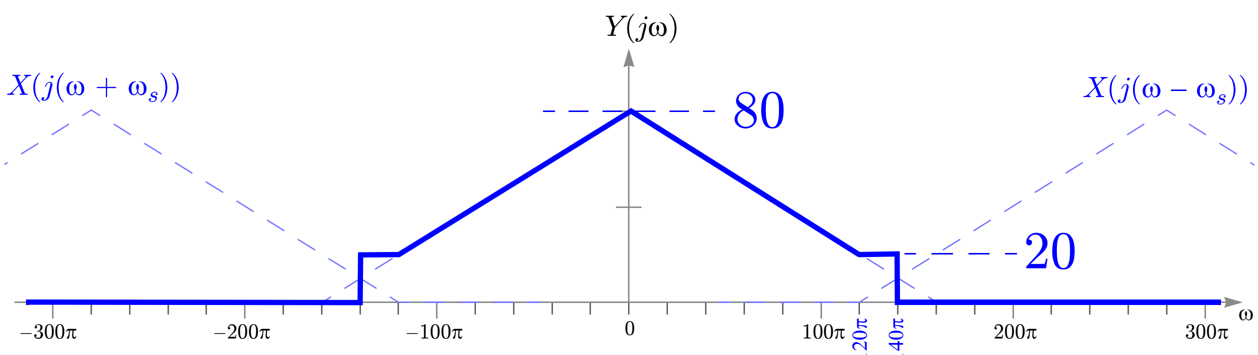


(a) The zero-th sample is $x[0] =$ 80^2.

(b) In order for the D-to-C converter to reconstruct the original signal (i.e., to achieve $y(t) = x(t)$), the sampling frequency must satisfy: $f_s >$ 160 Hz.

$$2f_{\max} = 2(80) = 160$$

(c) In the space below, carefully sketch the output Fourier transform $Y(j\omega)$ when $f_s = 140$ Hz, *carefully labeling important amplitudes*:



PROBLEM 4.

Consider a signal $x(t)$ that obeys the following differential equation:

$$\frac{d^2}{dt^2}x(t) - 9x(t) = 0,$$

where one initial condition is specified, namely $x(0) = 2$,
while the other initial condition $\dot{x}(0)$ is not specified and may be nonzero.

If the solution to this differential equation has the form $x(t) = Be^{-Ct}u(t)$, where $C > 0$, then it must be that:

$\dot{x}(0) =$	-6
$B =$	2
$C =$	3

On the one hand, taking the LT of the given form yields:

$$X(s) = \frac{B}{s+C}$$

On the other hand, taking the LT of both sides of the diff eqn yields:

$$s^2X(s) - sx(0) - \dot{x}(0) - 9X(s) = 0$$

$$\Rightarrow X(s) = \frac{sx(0) + \dot{x}(0)}{s^2 - 9} = \frac{2(s + 0.5\dot{x}(0))}{(s+3)(s-3)}$$

Equating both forms, it must be that the numerator *cancels* the 2nd factor in the denominator:

$$\Rightarrow \quad s + 0.5\dot{x}(0) = s - 3 \quad \Rightarrow \dot{x}(0) = -6$$

$$\Rightarrow X(s) \text{ reduces to } \frac{2}{s+3} \quad \Rightarrow B = 2,\ C = 3.$$

PROBLEM 5.

An LTI system (initially at rest, zero initial conditions) with input $x(t)$ and output $y(t)$ obeys the following differential equation:

$$\frac{d^2}{dt^2}y(t) = 2x(t) - 9y(t) - 4\frac{d}{dt}y(t).$$

Taking LT, solving for $Y(s)/X(s) \Rightarrow H(s) = \dfrac{2}{s^2 + 4s + 9}$

Equate denom to $s^2 + 2\zeta\omega_n s + \omega_n^2$

$\Rightarrow \omega_n = 3,\ \zeta = 2/3$

(a) Circle one: The system is [undamped][overdamped][underdamped][critically damped]?

$0 < \zeta < 1$

(b) Circle one: It most resembles a [LPF][HPF][BPF][notch] filter.

numerator is constant

(c) Its d.c. gain is $H_0 = \boxed{\frac{2}{9}}$.

set s = 0

(d) Its natural frequency is $\omega_n = \boxed{3}$ (in rad/s).

(e) Its damping ratio is $\zeta = \boxed{\frac{2}{3}}$.

(f) The *impulse* response of this system is $h(t) = Ae^{-Bt}\sin(Ct)$, where:

$$A = \frac{H_0\,\omega_n}{\sqrt{1-\zeta^2}} = \frac{(2/9)(3)}{\sqrt{5}/3}$$

$$B = \zeta\omega_n = (2/3)(3)$$

$$C = \omega_d = \sqrt{1-\zeta^2}\,\omega_n = \frac{\sqrt{5}}{3}(3)$$

$A = \boxed{\frac{2}{\sqrt{5}}}$

$B = \boxed{2}$

$C = \boxed{\sqrt{5}}$