ECE 3084

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
NOVEMBER 16, 2017

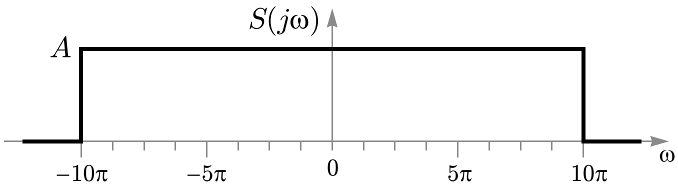
Name: ____

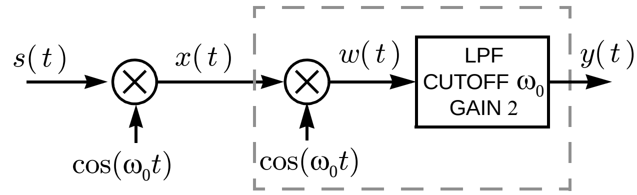
1. The quiz is *closed book*, closed notes, except for two 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers must be accompanied by concise justifications to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	15	
2	15	
3	25	
4	20	
5	25	
TOTAL:	100	

PROBLEM 1. (15 points) *Label amplitudes to receive full credit!*

Consider the system shown below, where the input $s(t) = \frac{\sin(10\pi t)}{10\pi t}$ with a rectangular Fourier transform is AM modulated with carrier frequency ω_0, producing $x(t)$. This AM signal is then passed through an AM demodulator (dashed box), producing an output $y(t)$:

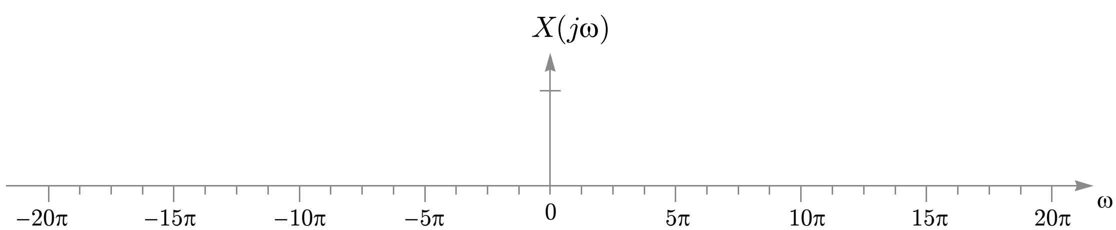


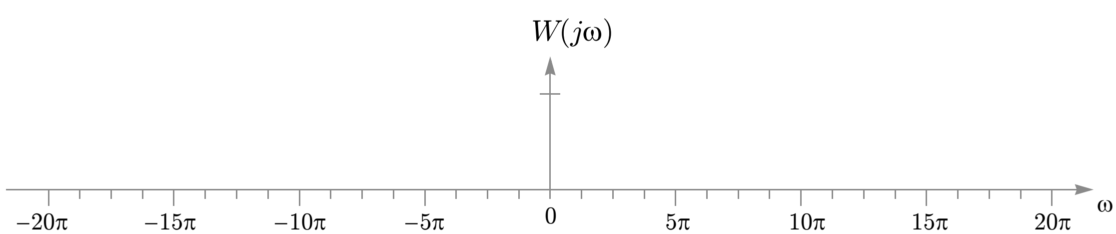


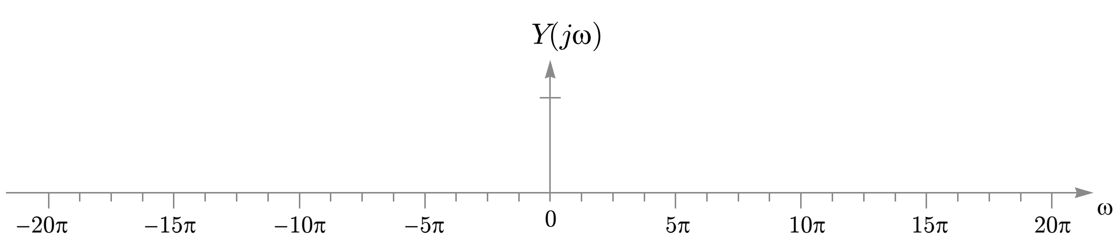
(a) The height of the rectangle is $A =$ $\boxed{}$.

(b) The AM demodulator will recover the input only when the carrier frequency is large enough:
In order for $y(t) = s(t)$,
the carrier frequency must satisfy $\omega_0 >$ $\boxed{}$ rad/s.

(c) Assuming that the carrier frequency is only $\omega_0 = 5\pi$, sketch the Fourier transforms $X(j\omega)$, $W(j\omega)$, and $Y(j\omega)$ in the space below:

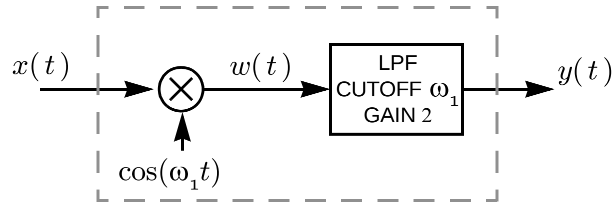




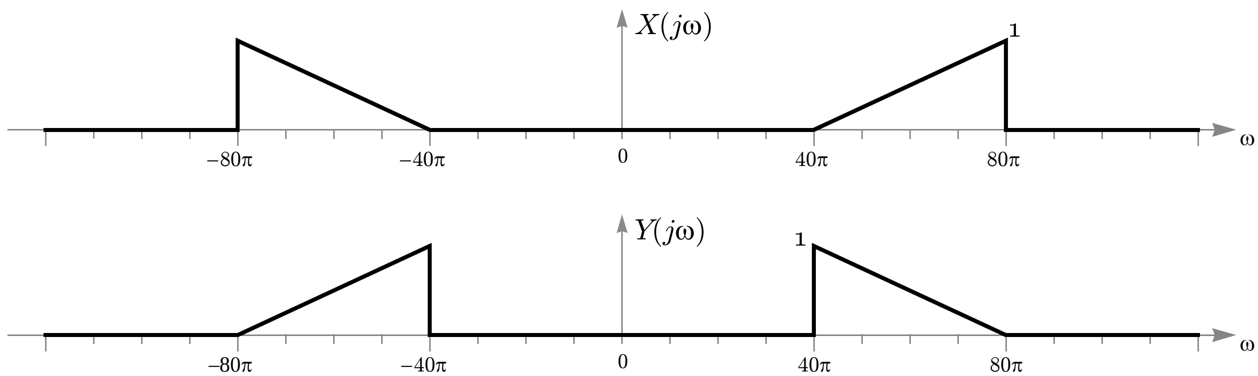


PROBLEM 2. (15 points)

Consider the following AM demodulator system (dashed box) with input $x(t)$ and output $y(t)$:



If the Fourier transforms $X(j\omega)$ and $Y(j\omega)$ of the input $x(t)$ and output $y(t)$ are as shown below:



then it must be that $\omega_1 =$ ☐ rad/s.

PROBLEM 3. (25 points)

An input $x(t)$ to an LTI system with transfer function $H(s)$ results in an output $y(t)$. For each combination of $x(t)$ and $H(s)$ given below, specify the *form* of the output signal $y(t)$ by crossing out terms that will not appear, and circling terms that will appear. No need to specify the constants!
For example, if the output is $y(t) = (30e^t + 84\sin(t))u(t)$, the answer would be simply:

$$\Rightarrow y(t) = \left(\cancel{A} + \cancel{Bt} + \cancel{Ct^2} + \cancel{De^{-t}} + \cancel{Ete^{-t}} + Fe^t + \cancel{Gte^t} + H\cos(t+\theta) + \cancel{Jt\cos(t+\varphi)}\right)u(t).$$

(a) $H(s) = \dfrac{1}{s+1}$ and $x(t) = e^t u(t)$.

$$\Rightarrow y(t) = \left(A + Bt + Ct^2 + De^{-t} + Ete^{-t} + Fe^t + Gte^t + H\cos(t+\theta) + Jt\cos(t+\varphi)\right)u(t).$$

(b) $H(s) = \dfrac{1}{s+1}$ and $x(t) = e^{-t} u(t)$.

$$\Rightarrow y(t) = \left(A + Bt + Ct^2 + De^{-t} + Ete^{-t} + Fe^t + Gte^t + H\cos(t+\theta) + Jt\cos(t+\varphi)\right)u(t).$$

(c) $H(s) = \dfrac{s+1}{s(s^2+1)}$ and $x(t) = u(t)$.

$$\Rightarrow y(t) = \Big(A + Bt + Ct^2 + De^{-t} + Ete^{-t} + Fe^{t} + Gte^{t} + H\cos(t+\theta) + Jt\cos(t+\varphi)\Big)u(t).$$

(d) $H(s) = \dfrac{s+1}{s^2+1}$ and $x(t) = t^2e^{-t}u(t)$.

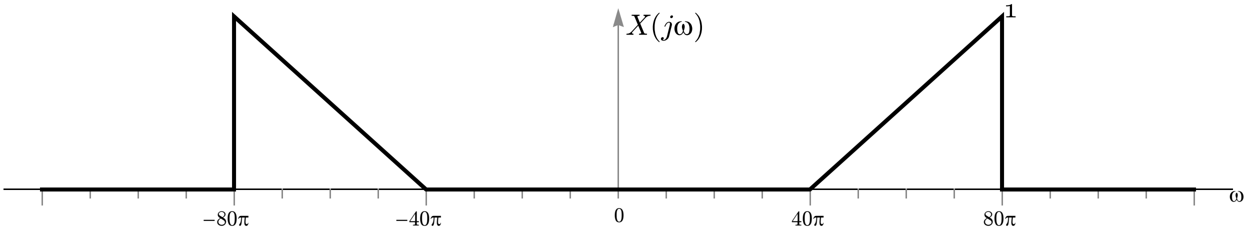
$$\Rightarrow y(t) = \Big(A + Bt + Ct^2 + De^{-t} + Ete^{-t} + Fe^{t} + Gte^{t} + H\cos(t+\theta) + Jt\cos(t+\varphi)\Big)u(t).$$

(e) $H(s) = \dfrac{s+3}{s+1}$ and $x(t) = \cos(t)u(t) + \sin(t)u(t)$.

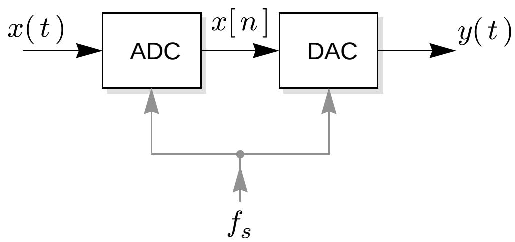
$$\Rightarrow y(t) = \Big(A + Bt + Ct^2 + De^{-t} + Ete^{-t} + Fe^{t} + Gte^{t} + H\cos(t+\theta) + Jt\cos(t+\varphi)\Big)u(t).$$

PROBLEM 4. (20 points)

Consider a continuous-time signal $x(t)$ whose Fourier transform is as sketched below:



Suppose this signal is passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter (with the same f_s parameter), producing the continuous-time output signal $y(t)$, as shown below:



(a) In order for the DAC to reconstruct the original signal (i.e., to achieve $y(t) = x(t)$), the sampling frequency must satisfy:

$f_s >$ $\boxed{}$ Hz.

(b) Find an equation for the output $y(t)$ when the sampling frequency is $f_s = 30$ Hz:

$y(t) =$ $\boxed{}$

PROBLEM 5. (25 points)

The differential equation relating the input $x(t)$ to the output $y(t)$ of an LTI system whose impulse response is $h(t) = (t + 1)e^{-\pi t}u(t)$ can be written as:

$$\frac{d^2}{dt^2}y(t) + a_1\frac{d}{dt}y(t) + a_0 y(t) = b_2\frac{d^2}{dt^2}x(t) + b_1\frac{d}{dt}x(t) + b_0 x(t),$$

where $b_2 = \boxed{}$, $b_1 = \boxed{}$, $b_0 = \boxed{}$.

and $a_1 = \boxed{}$, $a_0 = \boxed{}$.

(a) If the output of this system is $y(t) = te^{-\pi t}u(t)$,

the input must be $x(t) = \boxed{}$.