ECE 3084

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

GEORGIA INSTITUTE OF TECHNOLOGY

FEBRUARY 17, 2017

Name: _____

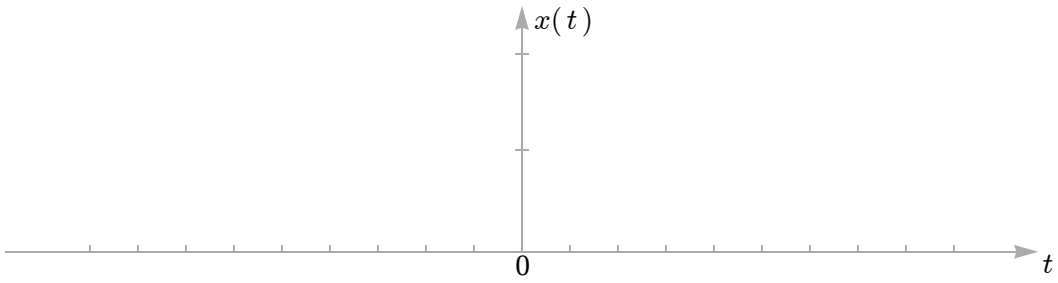
1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	30	
2	40	
3	30	
TOTAL:	100	

PROBLEM 1. (30 points)

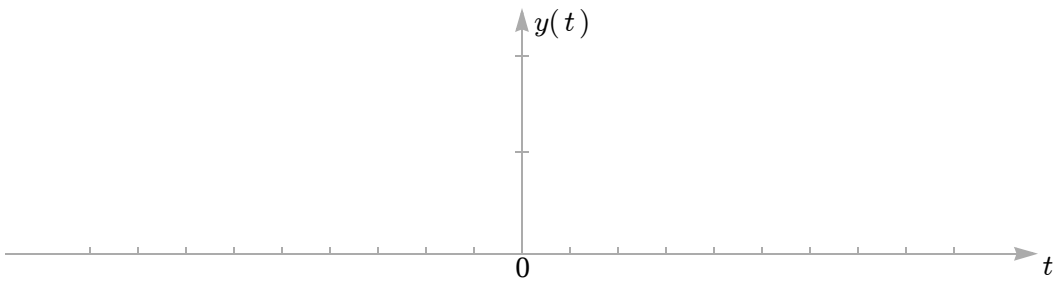
(a) If $X(j\omega) = e^{-j\omega/5}\left(\frac{\sin(5\omega)}{5\omega}\right)^2$,

sketch its inverse transform $x(t)$ in the space below, labeling carefully both axes.



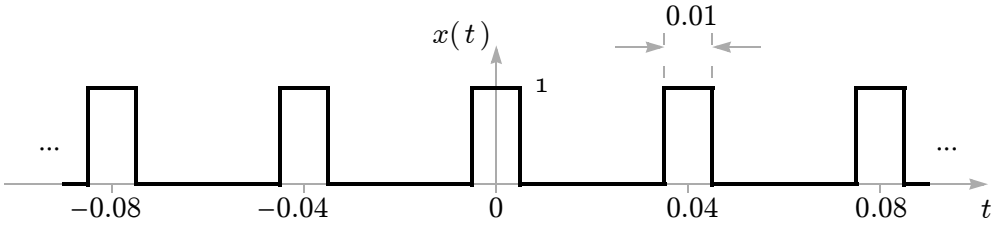
(b) Sketch the signal $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\int_{-1}^{1} e^{-\tau}e^{-j\omega\tau}d\tau\right\}e^{j\omega\tau}d\omega$ in the space below,

labeling carefully both axes. (*Hint*: integration not required!)

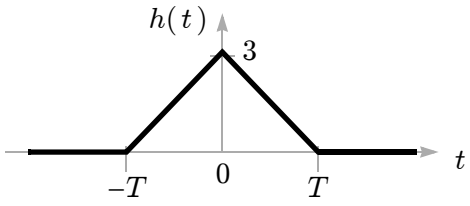


PROBLEM 2. (40 points)

Define $x(t)$ as the following 25%-duty-cycle periodic square wave:



and define $h(t)$ as the following triangular signal of width $2T$ and height 3:



(a) The power of $x(t)$ is $P =$ [ ].

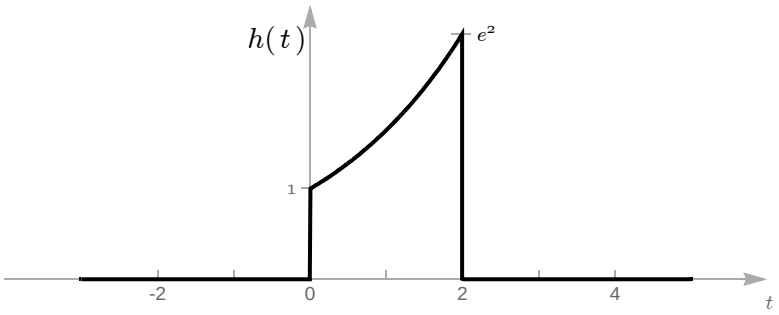
(b) The energy of $h(t)$ is $E =$ [ ] (as a function of T).

(c) For what values of the parameter T will the convolution $y(t) = x(t) * h(t)$ reduce to a *constant*, independent of t? (Hint: There is more than one such value of *T*. *Specify them all*.)

ANSWER

PROBLEM 3. (30 points)

Consider an LTI system whose impulse response is $h(t) = e^t(u(t) - u(t-2))$, as sketched below:



(a) Evaluate the integral $\int_{-3}^{\infty} h(t)u(t+1)u(t+2)\delta(t-1)\frac{e^{-1}}{3+t^5}\,dt = \boxed{}$.

(b) Let $s(t)$ denote the step response of the system, *i.e.*, the output in response to the unit step $u(t)$. Specify *equations* as a function of t for the step response $s(t)$ in the two large boxes below, and specify the times these equations change in the smaller boxes below:

$$s(t) = \begin{cases} 0, & \text{for } t < \boxed{} \\ \boxed{} & \text{for } \boxed{} < t < \boxed{} \\ \boxed{} & \text{for } t > \boxed{}\,. \end{cases}$$

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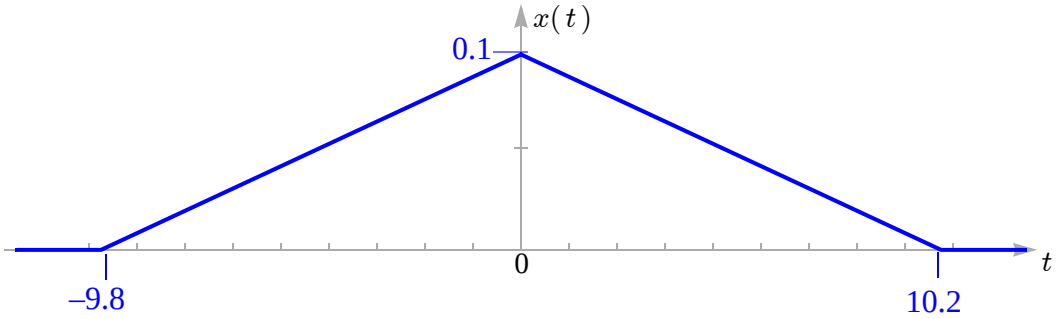

(b) Sketch the signal $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\int_{-1}^{1} e^{-\tau}e^{-j\omega\tau}d\tau\right\}e^{j\omega t}d\omega$ in the space below,

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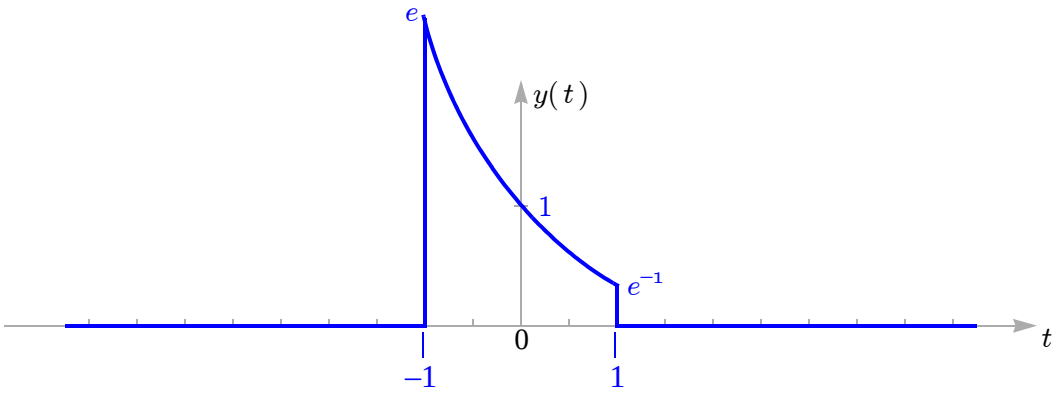

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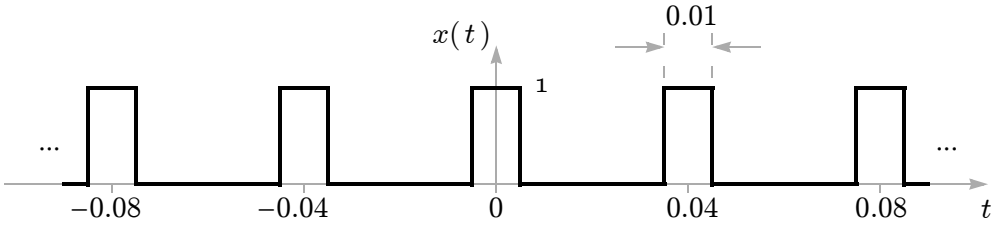

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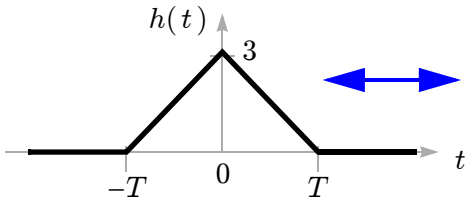


$$\frac{3}{T}\left(\frac{\sin(\omega T/2)}{\omega/2}\right)^2$$

(a) The power of $x(t)$ is $P =$ $\boxed{0.25}$.

(b) The energy of $h(t)$ is $E =$ $\boxed{6T}$ (as a function of T).

$$E = \int_{-\infty}^{\infty} h^2(t)\,dt$$

$$= 2\int_0^T (3t/T)^2\,dt$$

$$= \frac{18}{T^2}\int_0^T t^2\,dt$$

$$= 6T$$

(c) For what values of the parameter T will the convolution $y(t) = x(t) * h(t)$ reduce to a *constant*, independent of t? (Hint: There is more than one such value of T. *Specify them all*.)

$T = 0.04\ell = \{0.04,\ 0.08,\ 0.12,\ 0.16,\ \ldots\}$ ANSWER

Freq response $H(j\omega) = \frac{3}{T}\left(\frac{\sin(\omega T/2)}{\omega/2}\right)^2$ is zero when

$$\omega T/2 = \ell\pi$$

$$\Rightarrow \omega = \ell\frac{2\pi}{T} \text{ for nonzero } \ell$$

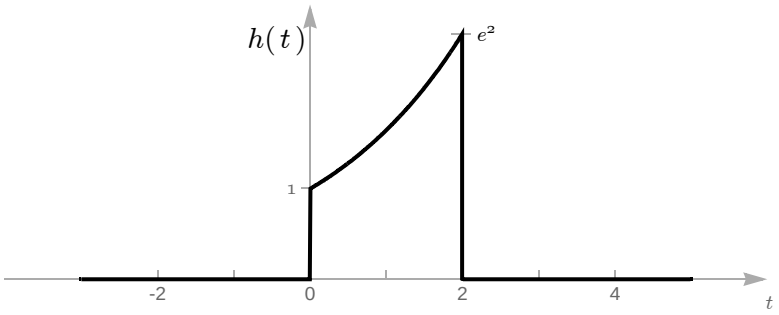
$$\Rightarrow \frac{2\pi}{T} = \frac{\omega_0}{\ell}$$

$$\Rightarrow \frac{1}{T} = \frac{f_0}{\ell} = \frac{1}{\ell T_0}$$

$$\Rightarrow T = \ell T_0 = 0.04\ell, \text{ for nonzero } \ell$$

PROBLEM 3. (30 points)

Consider an LTI system whose impulse response is $h(t) = e^t(u(t) - u(t-2))$, as sketched below:



(a) Evaluate the integral $\int_{-3}^{\infty} h(t)u(t+1)u(t+2)\delta(t-1)\frac{e^{-1}}{3+t^5}\,dt = \boxed{0.25}$.

(b) Let $s(t)$ denote the step response of the system, *i.e.*, the output in response to the unit step $u(t)$. Specify *equations* as a function of t for the step response $s(t)$ in the two large boxes below, and specify the times these equations change in the smaller boxes below:

$$s(t) = \begin{cases} 0, & \text{for } t < \boxed{0} \\ \boxed{e^t - 1} & \text{for } \boxed{0} < t < \boxed{2} \\ \boxed{e^2 - 1} & \text{for } t > \boxed{2}. \end{cases}$$

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