ECE 3084

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
FEBRUARY 25, 2016

Name: ____

1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit. Exceptions are Prob. 1(c) and Prob. 4.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

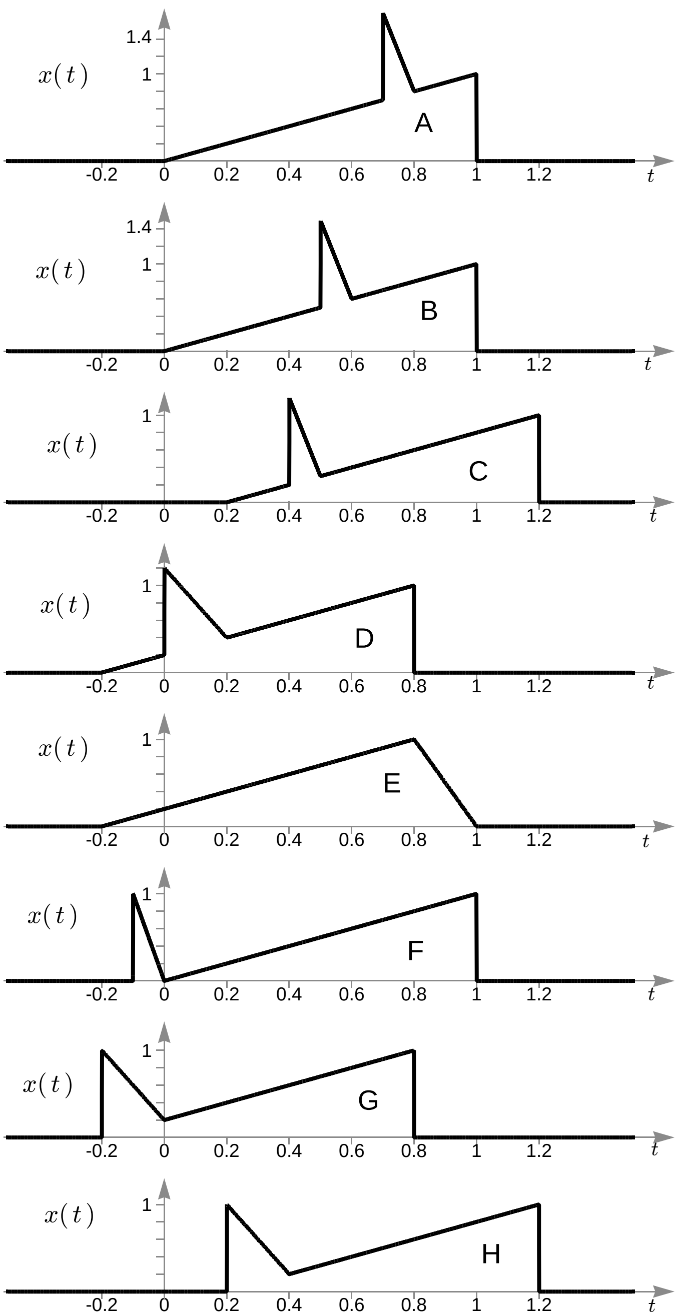
Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

PROBLEM 1. (20 points) Let $g(t) = t(u(t) - u(t-1))$.

(a) Its *energy* is $E = \boxed{}$.

(b) Evaluate the integral $\int_{-\infty}^{1} g(t)\delta(t - 0.6)dt = \boxed{}$.

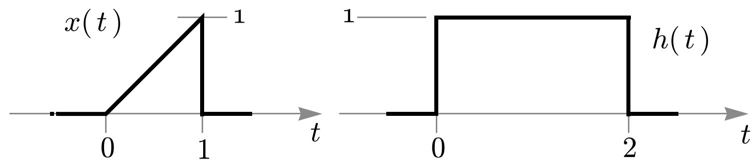
(c) Shown below are eight different plots of: $x(t) = g(t + T) + g(\tau - \beta t)$, labeled A through H. Match each plot to the corresponding set of constants $\{T, \beta, \tau\}$ by writing a letter (from A through H) in each answer box:



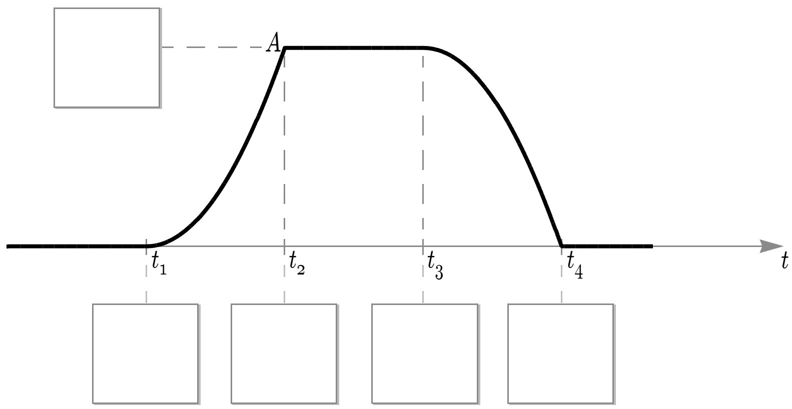
T	β	τ	ANSWER
0.2	5	0	
0	10	8	
0.2	5	1	
–0.2	5	2	
0	10	6	
0	10	0	
0.2	5	5	
–0.2	10	5	

PROBLEM 2. (20 points)

Consider the signals $x(t)$ and $h(t)$ shown on the right:



Here is a plot of the convolution $y(t) = x(t) * h(t)$, on a different scale:



(a) Specify the amplitude A and times t_1 through t_4 by writing numbers in the 5 answer boxes above.

(b) Specify *equations* as a function of t for the output $y(t)$ in the two missing regions below:

$$y(t) = \begin{cases} 0, & \text{for } t < t_1 \\ \boxed{\quad} & \text{for } t_1 < t < t_2 \\ A, & \text{for } t_2 < t < t_3 \\ \boxed{\quad} & \text{for } t_3 < t < t_4 \\ 0, & \text{for } t > t_4 \end{cases}$$

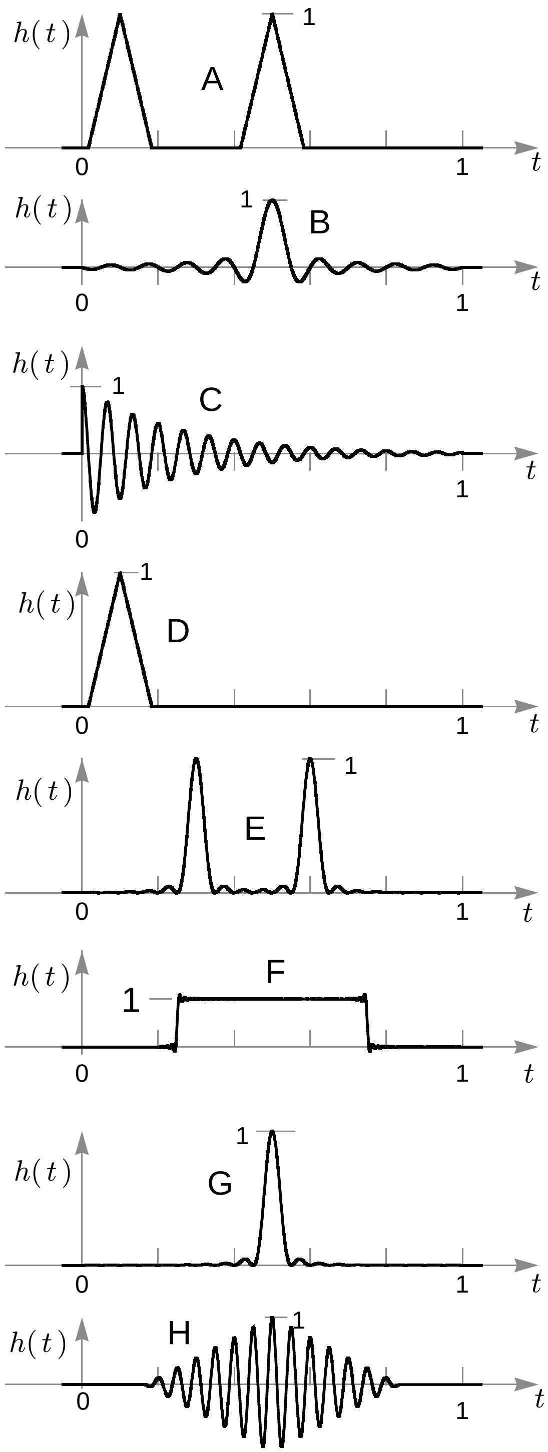
PROBLEM 3. (20 points)

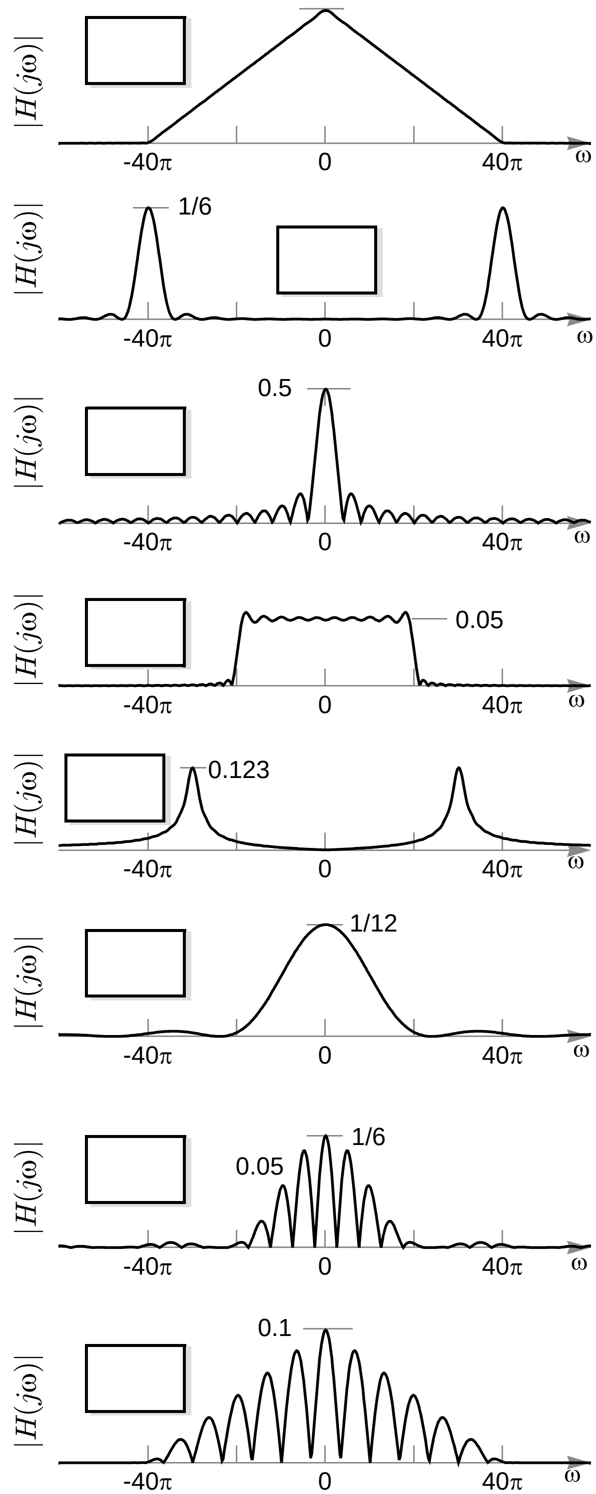
Shown below are the relationships between the input $x(t)$ and output $y(t)$ of two systems. Specify which properties listed on the left are satisfied by each: (*Brief* explanations are OK!)

	SYSTEM#1: $y(t) = x(\|t\|)$	SYSTEM#2: $y(t) = x(\cos(t))$
Linear?	YES ☐ NO ☐	YES ☐ NO ☐
Time Invariant?	YES ☐ NO ☐	YES ☐ NO ☐
Memoryless?	YES ☐ NO ☐	YES ☐ NO ☐
Invertible?	YES ☐ NO ☐	YES ☐ NO ☐
Causal?	YES ☐ NO ☐	YES ☐ NO ☐
Stable?	YES ☐ NO ☐	YES ☐ NO ☐

PROBLEM 4. (20 points)

Shown on the left are impulse responses of eight different LTI filters, labeled A through H. (The impulse responses are all zero before $t = 0$ and after $t = 1$.) Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magnitude response to its corresponding impulse response by writing a letter (A through H) in each answer box.





PROBLEM 5. (20 points)

The signal $x(t) = \sum_{k=-2}^{2} 2^{|k|} e^{jk\pi t}$ is fed into the following cascade of three LTI systems:

$$x(t) = \sum_{k=-2}^{2} 2^{|k|} e^{jk\pi t} \longrightarrow \boxed{h_1(t) = \frac{\sin(Wt)}{\pi t}} \longrightarrow \boxed{H_2(j\omega) = e^{-j\omega t_0}} \longrightarrow \boxed{H_3(j\omega) = j\omega} \longrightarrow y(t)$$

The first system has impulse response $h_1(t) = \sin(Wt)/(\pi t)$; the second system is a delay-by-t_0 system with frequency response $H_2(j\omega) = e^{-j\omega t_0}$; the third system is a differentiator, with frequency response $H_3(j\omega) = j\omega$. The parameters W and t_0 are unspecified.

(a) Evaluate the integral $\int_{-10}^{60} x(t)dt = \boxed{}$.

(b) Choose the positive parameters W and t_0 so that the overall output will be zero ($y(t) = 0$):

$$W = \boxed{}, \qquad t_0 = \boxed{}.$$

(c) Choose the nonzero parameters W and t_0 so that the overall output will be a sinusoid of the form $y(t) = A\cos(\pi t)$, and also specify the sinusoid amplitude A:

$$W = \boxed{}, \quad t_0 = \boxed{}, \quad A = \boxed{}.$$

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Table of Fourier Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Frequency-Domain:* $X(j\omega)$
Right-sided exponential	$e^{-at}u(t) \quad (a > 0)$	$\dfrac{1}{a + j\omega}$
Left-sided exponential	$e^{bt}u(-t) \quad (b > 0)$	$\dfrac{1}{b - j\omega}$
Square pulse	$[u(t + T/2) - u(t - T/2)]$	$\dfrac{\sin(\omega T/2)}{\omega/2}$
"sinc" function	$\dfrac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega + \omega_0) - u(\omega - \omega_0)]$
Impulse	$\delta(t)$	1
Shifted impulse	$\delta(t - t_0)$	$e^{-j\omega t_0}$
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
General cosine	$A\cos(\omega_0 t + \phi)$	$\pi A e^{j\phi}\delta(\omega - \omega_0) + \pi A e^{-j\phi}\delta(\omega + \omega_0)$
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$
General periodic signal	$\displaystyle\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\displaystyle\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Impulse train	$\displaystyle\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\displaystyle\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/T)$

Table of Fourier Transform Properties		
Property Name	*Time-Domain* $x(t)$	*Frequency-Domain* $X(j\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert}X(j(\omega/a))$
Delay	$x(t - t_d)$	$e^{-j\omega t_d}X(j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$
Modulation	$x(t)\cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega - \omega_0)) + \frac{1}{2}X(j(\omega + \omega_0))$
Differentiation	$\dfrac{d^k x(t)}{dt^k}$	$(j\omega)^k X(j\omega)$
Convolution	$x(t) * h(t)$	$X(j\omega)H(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega) * P(j\omega)$

Date 14-Sept-2013

ECE 3084

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
FEBRUARY 25, 2016

Name: SOLUTIONS

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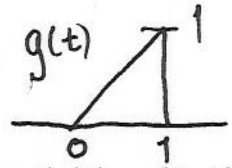
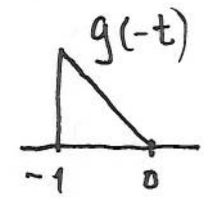
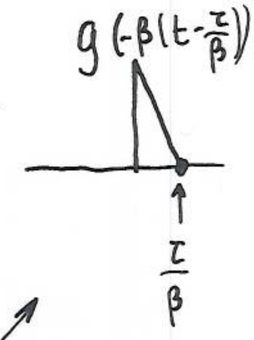
TOTAL:	100	



$\Rightarrow$



$\Rightarrow$



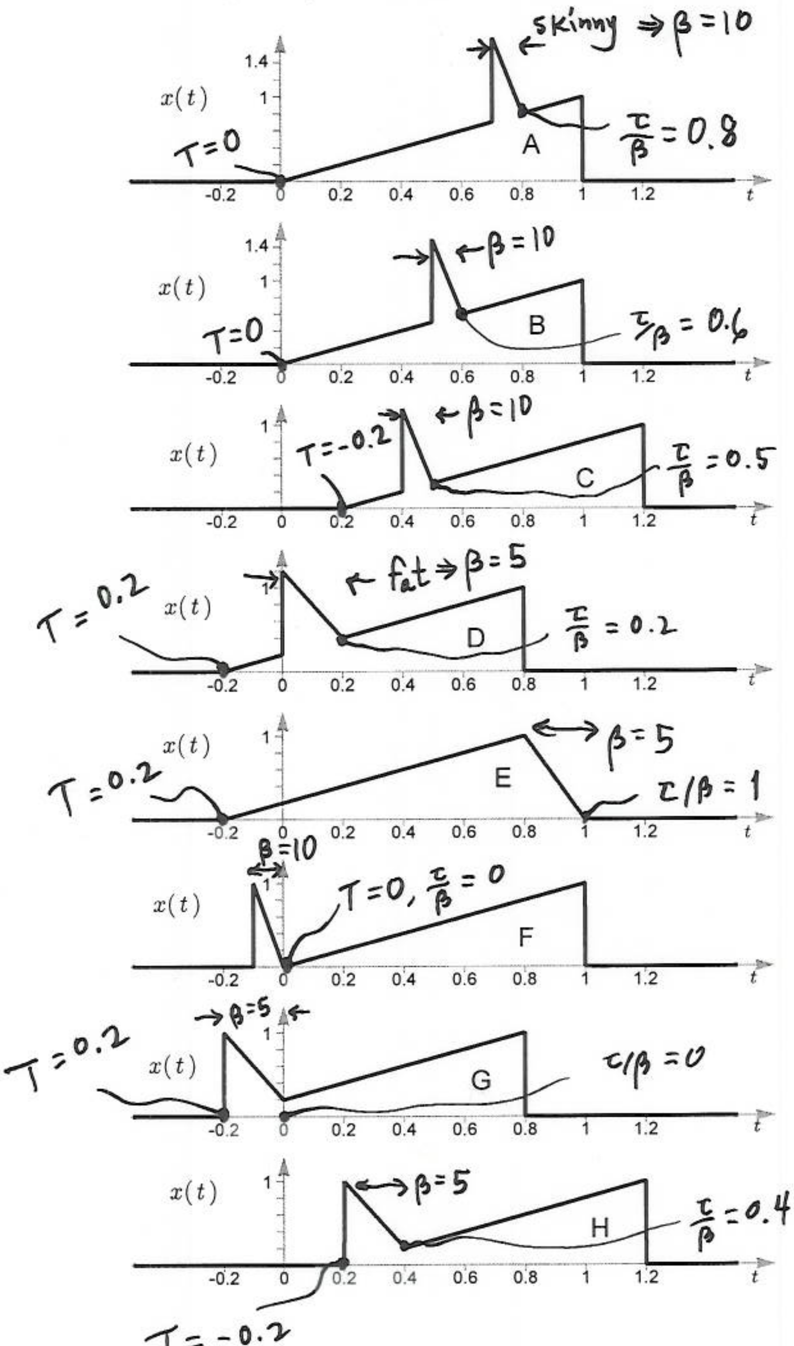
PROBLEM 1. (20 points) Let $g(t) = t(u(t) - u(t-1))$.

(a) Its *energy* is $E = \boxed{\frac{1}{3}}$. $= \int_0^1 t^2 dt$

(b) Evaluate the integral $\int_{-\infty}^{1} g(t)\delta(t-0.6)dt = \boxed{0.6}$.

$g(-\beta(t-\frac{\tau}{\beta}))$

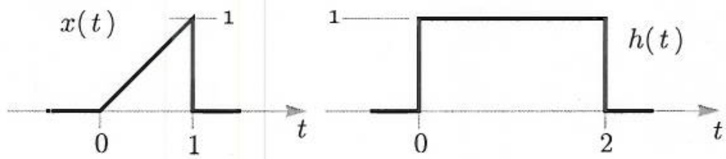
(c) Shown below are eight different plots of: $x(t) = g(t+T) + \underbrace{g(\tau - \beta t)}$,
labeled A through H. Match each plot to the corresponding set of constants $\{T, \beta, \tau\}$ by writing a letter (from A through H) in each answer box:



T	β	τ	ANSWER
0.2	5	0	G
0	10	8	A
0.2	5	1	D
–0.2	5	2	H
0	10	6	B
0	10	0	F
0.2	5	5	E
–0.2	10	5	C

PROBLEM 2. (20 points)

Consider the signals $x(t)$ and $h(t)$ shown on the right:



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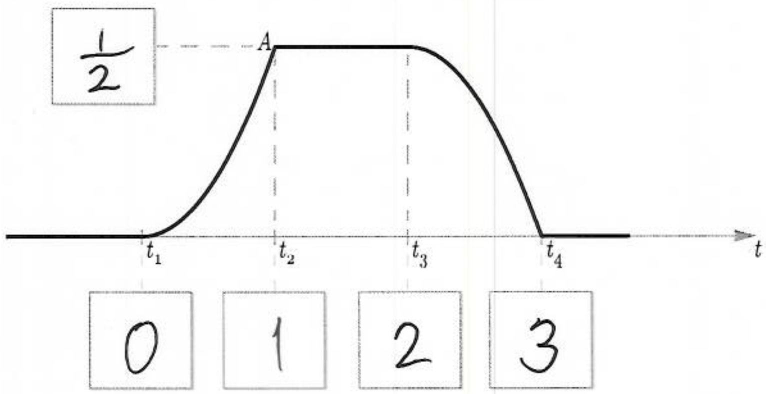
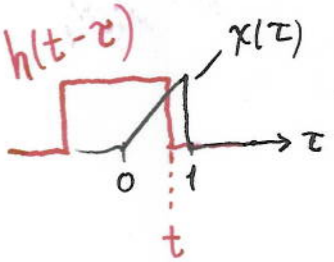
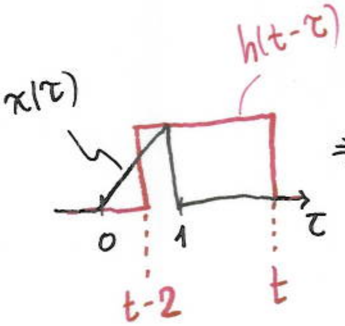


(a) Specify the amplitude A and times t_1 through t_4 by writing numbers in the 5 answer boxes above.

(b) Specify *equations* as a function of t for the output $y(t)$ in the two missing regions below:



$$\Rightarrow y(t) = \int_0^t \tau \, d\tau = \frac{t^2}{2}$$



$$\Rightarrow y(t) = \int_{t-2}^{1} \tau \, d\tau = \frac{1-(t-2)^2}{2}$$

$$y(t) = \begin{cases} 0, & \text{for } t < t_1 \\ \boxed{\dfrac{t^2}{2}} & \text{for } t_1 < t < t_2 \\ A, & \text{for } t_2 < t < t_3 \\ \boxed{\dfrac{1-(t-2)^2}{2}} & \text{for } t_3 < t < t_4 \\ 0, & \text{for } t > t_4 \end{cases}$$

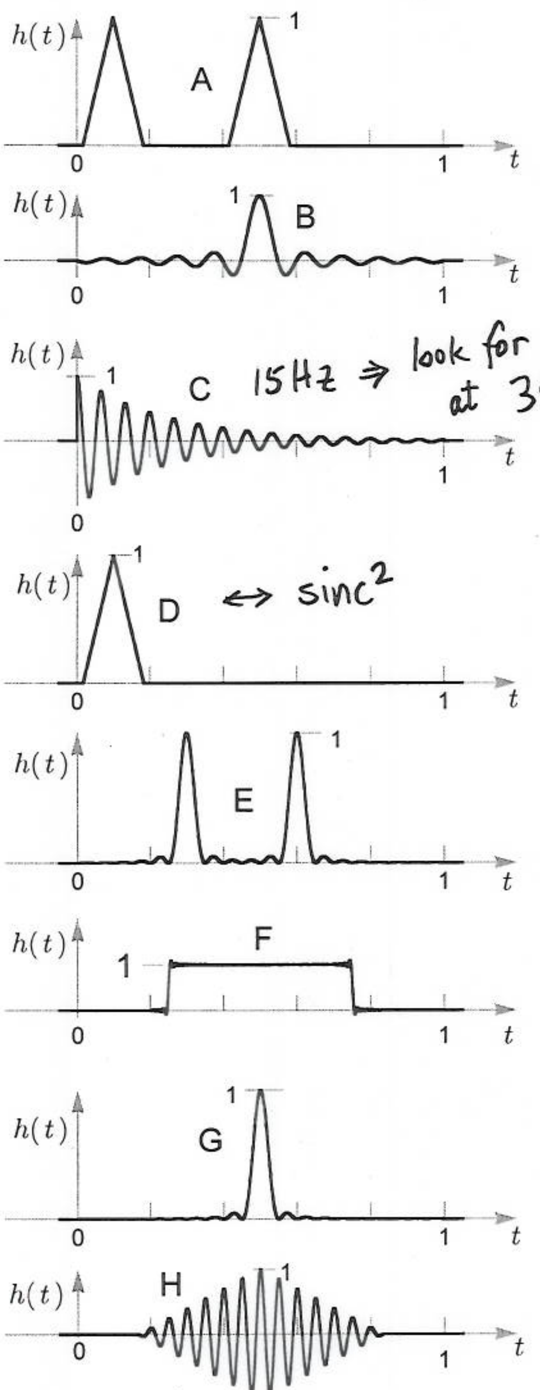
PROBLEM 3. (20 points)

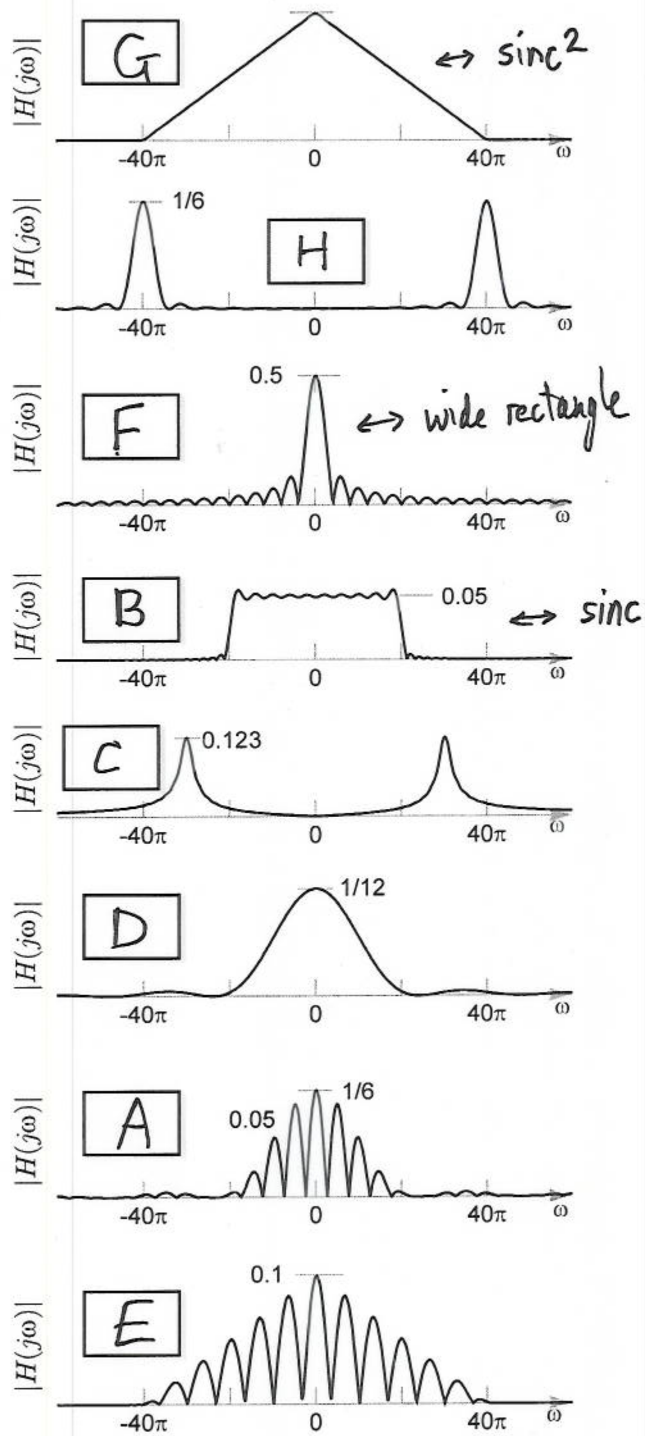
Shown below are the relationships between the input $x(t)$ and output $y(t)$ of two systems. Specify which properties listed on the left are satisfied by each: (*Brief* explanations are OK!)

	SYSTEM#1: $y(t) = x(\lvert t \rvert)$	SYSTEM#2: $y(t) = x(\cos(t))$
Linear?	YES: X / NO: $\alpha x_1(t) + \beta x_2(t) \to \alpha y_1(t) + \beta y_2(t)$	YES: X / NO: $\alpha x_1 + \beta x_2 \to \alpha y_1 + \beta y_2$
Time Invariant?	YES: / NO: X $x_1(t)$ -2 -1 $\to y_1(t) = 0$ $x_1(t-2) \to$ -1 1 $\neq y_1(t-2)$	YES: / NO: X $x_1(t)$ -3 -2 $\to y_1(t) = 0$ $x_1(t-3) \to \cdots$ square wave $\neq y_1(t-3)$
Memoryless?	YES: / NO: X $y(-1) = x(+1)$	YES: / NO: X $y(0) = x(1)$
Invertible?	YES: / NO: X $x(t)$ for $t<0$ is unobserved	YES: / NO: X $x(t)$ for $t \notin [-1,1]$ is unobserved
Causal?	YES: / NO: X $y(-1) = x(+1)$ looks into future	YES: / NO: X $y(0) = x(1)$ looks into future
Stable?	YES: X / NO: BIBO	YES: X / NO: BIBO

PROBLEM 4. (20 points)

Shown on the left are impulse responses of eight different LTI filters, labeled A through H. (The impulse responses are all zero before $t = 0$ and after $t = 1$.) Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magnitude response to its corresponding impulse response by writing a letter (A through H) in each answer box.





triangle × cos(·) ↔ sinc²(ω ± 40π)

Euler $\Rightarrow$ $x(t) = 1 + 4\cos(\pi t) + 8\cos(2\pi t)$

PROBLEM 5. (20 points)

The signal $x(t) = \sum_{k=-2}^{2} 2^{|k|} e^{jk\pi t}$ is fed into the following cascade of three LTI systems:

$$x(t) = \sum_{k=-2}^{2} 2^{|k|} e^{jk\pi t} \longrightarrow \boxed{h_1(t) = \frac{\sin(Wt)}{\pi t}} \longrightarrow \boxed{H_2(j\omega) = e^{-j\omega t_0}} \longrightarrow \boxed{H_3(j\omega) = j\omega} \longrightarrow y(t)$$

The first system has impulse response $h_1(t) = \sin(Wt)/(\pi t)$; the second system is a delay-by-t_0 system with frequency response $H_2(j\omega) = e^{-j\omega t_0}$; the third system is a differentiator, with frequency response $H_3(j\omega) = j\omega$. The parameters W and t_0 are unspecified.

(a) Evaluate the integral $\int_{-10}^{60} x(t)dt = \boxed{70}$.

$$\int_{-10}^{60} 1\,dt + \underbrace{\int_{-10}^{60} 4\cos(\pi t)\,dt}_{0} + \underbrace{\int_{-10}^{60} 8\cos(2\pi t)\,dt}_{0} = \int_{-10}^{60} 1\,dt = 70.$$

(b) Choose the positive parameters W and t_0 so that the overall output will be zero ($y(t) = 0$):

$W = \boxed{\text{e.g. } \frac{\pi}{2}}$, $t_0 = \boxed{\text{does not matter}}$.

input is constant plus a sinusoid at π plus a sinusoid at 2π.

Differentiator kills the constant.

Choose W to reject the two sinusoids. Any $\boxed{W < \pi}$ does the job.

(c) Choose the nonzero parameters W and t_0 so that the overall output will be a sinusoid of the form $y(t) = A\cos(\pi t)$, and also specify the sinusoid amplitude A:

$W = \boxed{\text{e.g. } 1.5\pi}$, $t_0 = \boxed{0.5}$, $A = \boxed{4\pi}$.

Choose W in range $\boxed{\pi < W < 2\pi}$ to reject high-freq sinusoid, and keep low-freq sinusoid.

Output will then be a delayed version of the derivative of the low-freq sinusoid:

$$y(t) = \frac{d}{dt} 4\cos(\pi t)\Big|_{t-t_0} = -4\pi \sin(\pi(t-t_0)) = \overset{A}{4\pi}\cos(\pi t)$$

when $t_0 = 0.5$