ECE 3084

QUIZ 1

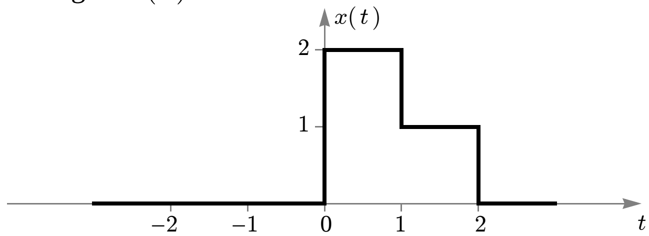
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
FEBRUARY 19, 2015

Name: ____

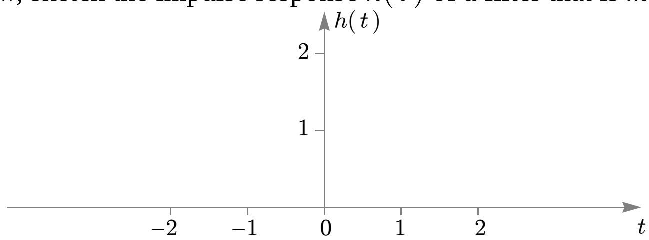
1. The quiz is closed book, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

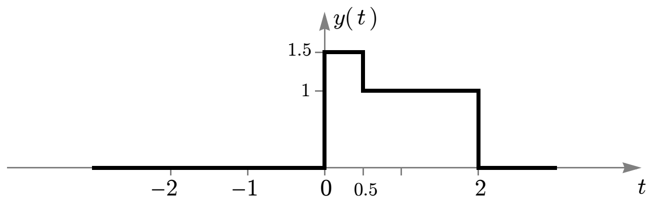
PROBLEM 1. Consider the signal $x(t)$ shown below:



(a) In the space below, sketch the impulse response $h(t)$ of a filter that is *matched* to $x(t)$:



(b) The signal $y(t)$ that is shown below:



can be written in terms of $x(t)$ at the top of the page according to: $y(t) = x(t) + Ax(Bt + C)$,

where: $A =$ ☐, $B =$ ☐, and $C =$ ☐:

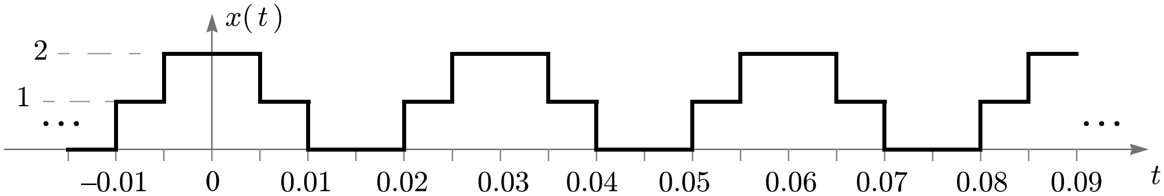
(c) The autocorrelation function $R_{xx}(t)$ for $x(t)$ achieves a maximum value of $R_{xx}(t_{\text{max}}) =$ ☐ at time $t_{\text{max}} =$ ☐.

(d) The convolution of $x(t)$ with itself is nonzero only for time between $t_{\text{start}} =$ ☐ and

$t_{\text{stop}} =$ ☐.

(e) Evaluate the integral $\int_{-\infty}^{\infty} x(t)\delta(t - 1/4)\sin(\pi t)dt =$ ☐.

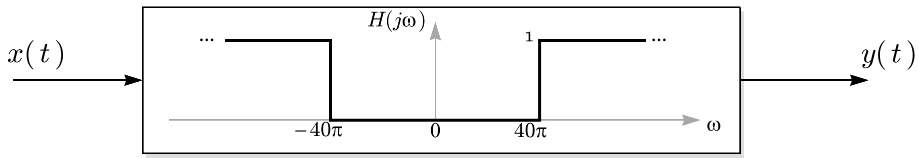
PROBLEM 2. Consider the periodic signal shown below:



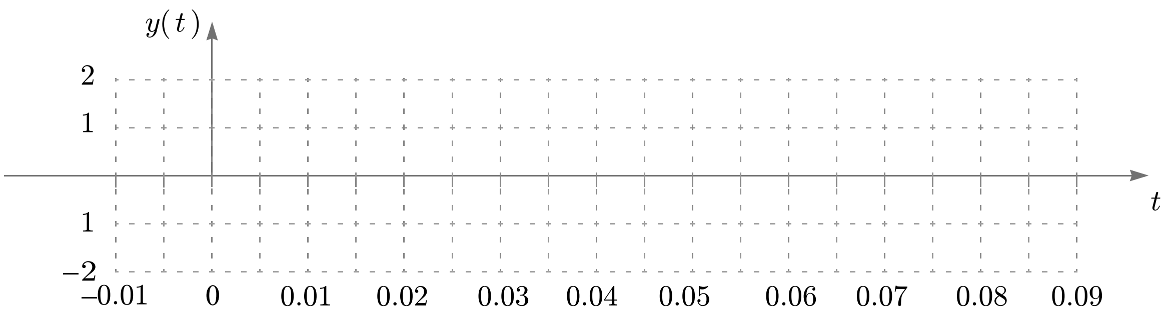
(a) The fundamental period of $x(t)$ is $T_0 =$ ☐ seconds.

(b) The zeroth coefficient in the Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T_0}$ is $a_0 =$ ☐.

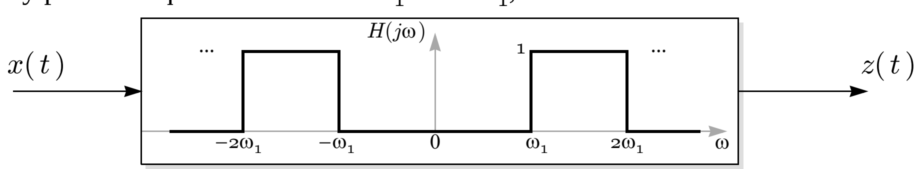
(c) Suppose that this periodic signal $x(t)$ is passed through an ideal *high-pass filter* with cutoff frequency of 40π rad/s, as shown below:



Carefully sketch the output signal $y(t)$ after the high-pass filter, for time $-0.01 < t < 0.09$:



(d) Instead, suppose that $x(t)$ is passed through an ideal *band-pass filter* that only passes frequencies between ω_1 and $2\omega_1$, as shown below:



If the output is a *single sinusoid*, of the form say $z(t) = A\cos(2\pi f_c t + \theta)$, then it must be that the filter parameter satisfies:

☐ $< \omega_1 <$ ☐,

and further it must be that $f_c =$ ☐, $\theta =$ ☐, and $A =$ ☐.

5PT BONUS
(no partial credit)

PROBLEM 3. For each impulse response given below, specify whether it is the impulse response of a low-pass filter (LPF), high-pass filter (HPF), bandpass filter (BPF), or none of the above (indicating your answer by circling one of the four options):

(a) $h(t) = \dfrac{\sin(40\pi t)}{\pi t}$ $\Rightarrow$ [LPF][HPF][BPF][none].

(b) $h(t) = \dfrac{\sin(880\pi t)}{\pi t} * \dfrac{\sin(440\pi t)}{\pi t}$ (convolution) $\Rightarrow$ [LPF][HPF][BPF][none].

(c) $h(t) = \dfrac{\sin(880\pi t)}{\pi t} - \dfrac{\sin(440\pi t)}{\pi t}$ $\Rightarrow$ [LPF][HPF][BPF][none].

(d) $h(t) = \dfrac{\sin(40\pi t)}{\pi t}\cos(880\pi t)$ $\Rightarrow$ [LPF][HPF][BPF][none].

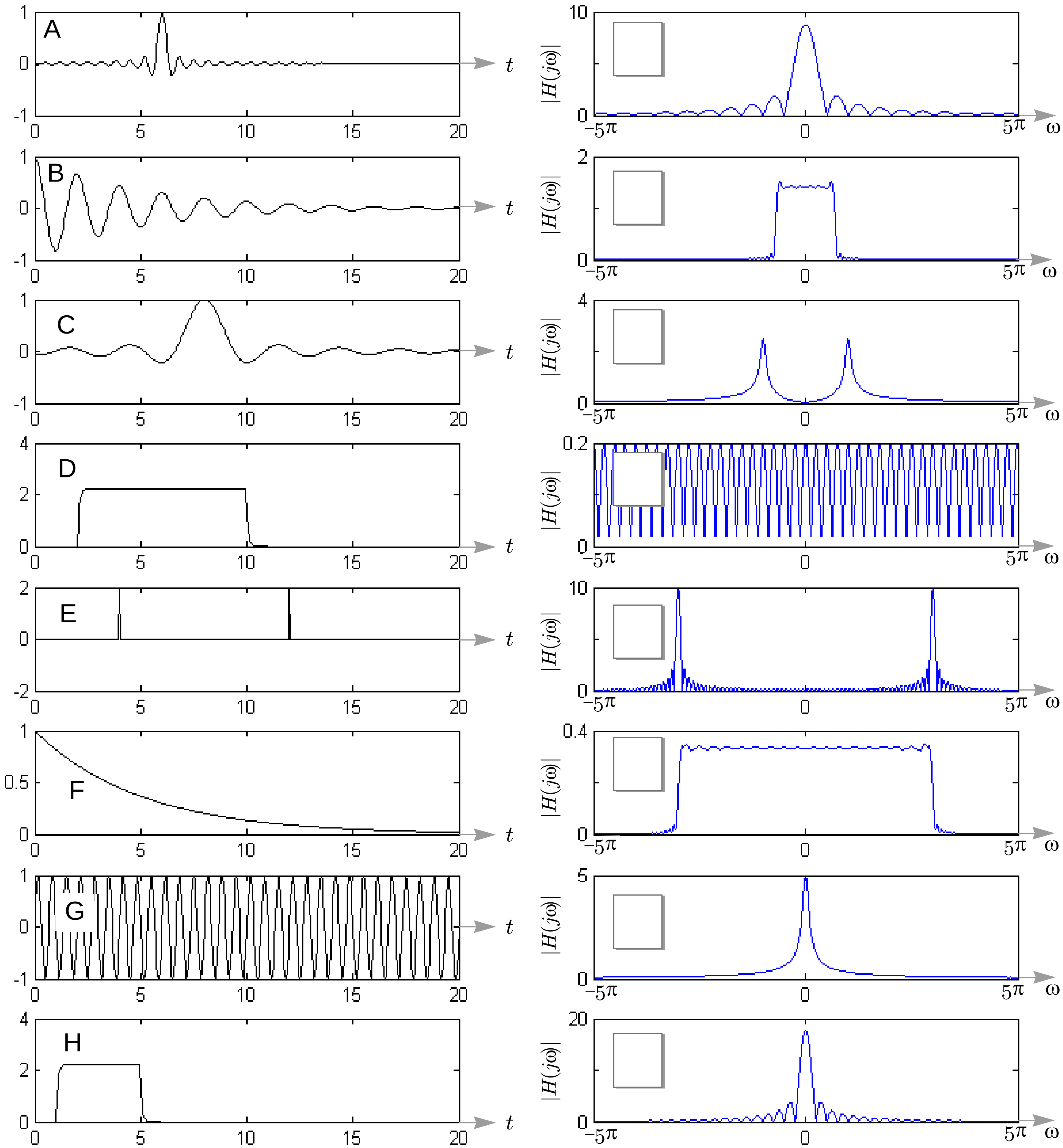
(e) $h(t) = \dfrac{\sin(40\pi t)}{\pi t} - \dfrac{\sin(440\pi t)}{\pi t} + \delta(t)$ $\Rightarrow$ [LPF][HPF][BPF][none].

(f) $h(t) = \dfrac{\sin(4\pi t)}{\pi t} + 2\cos(6\pi t)\dfrac{\sin(2\pi t)}{\pi t}$ $\Rightarrow$ [LPF][HPF][BPF][none].

(g) $h(t) = \delta(t) - \dfrac{\sin(300\pi t)}{\pi t}$ $\Rightarrow$ [LPF][HPF][BPF][none].

(h) $h(t) = \displaystyle\int_{-\infty}^{\infty} \left(\delta(\tau) - \frac{\sin(16\pi\tau)}{\pi\tau} \right)\left(\frac{\sin(300\pi(t-\tau))}{\pi(t-\tau)} \right) d\tau$ $\Rightarrow$ [LPF][HPF][BPF][none].

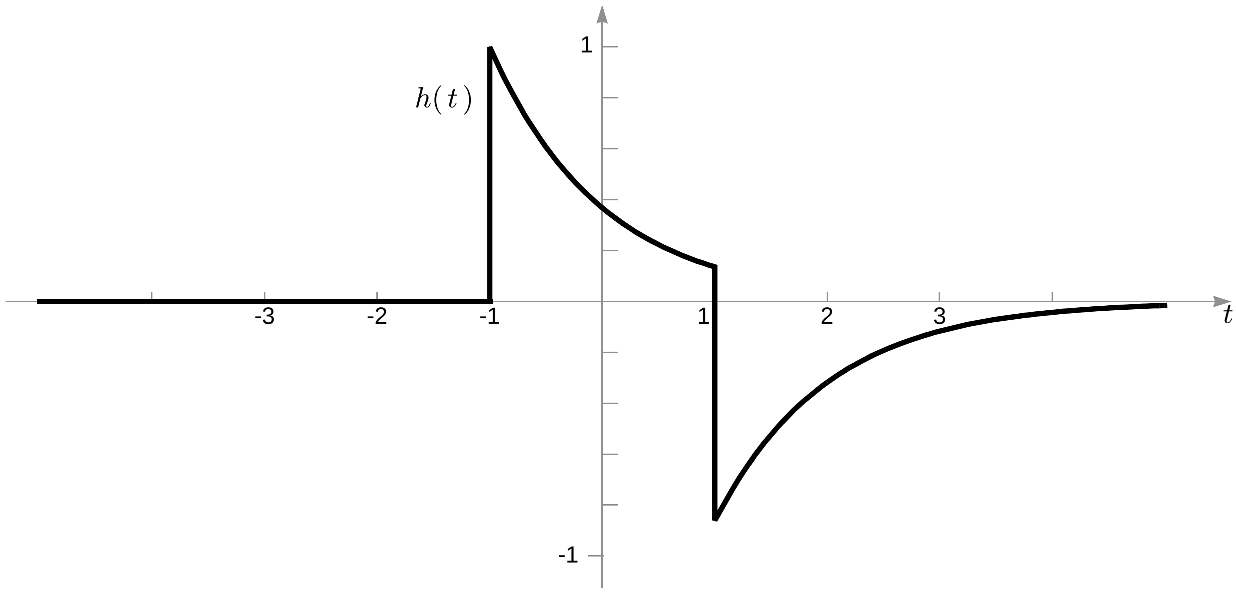
PROBLEM 4. Shown on the left are impulse responses of eight different LTI filters, labeled A through H. (The impulse responses are all zero before $t = 0$ and after $t = 20$.)
Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magntiude response to its corresponding impulse response by writing a letter (A through H) in each answer box.



PROBLEM 5. Consider an LTI system whose impulse response is:

$$h(t) = e^{-(t+1)}u(t+1) - e^{-(t-1)}u(t-1),$$

as sketched below:



(a) Suppose that the input to this system is a sinusoid of the form $x(t) = \cos(2\pi f_0 t)$. Specify *three* different values for the frequency f_0 that will result in an output that is zero, i.e. $y(t) = 0$ for all t:

$f_0 \in \{$ [ ] Hz, [ ] Hz, [ ] Hz $\}$.

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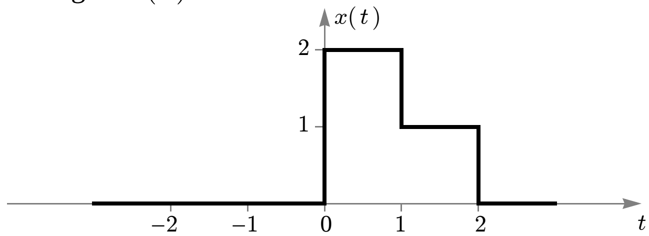

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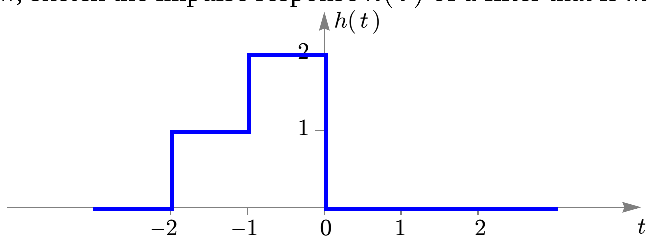

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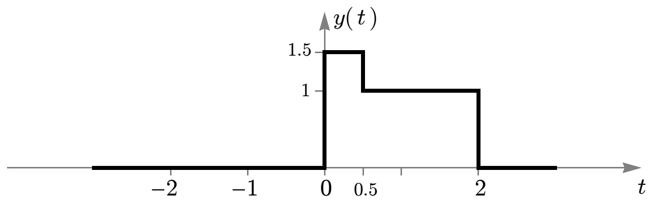


can be written in terms of $x(t)$ at the top of the page according to: $y(t) = x(t) + Ax(Bt + C)$,

where: $A =$ $-1/2$, $B =$ -2 , and $C =$ 2 :

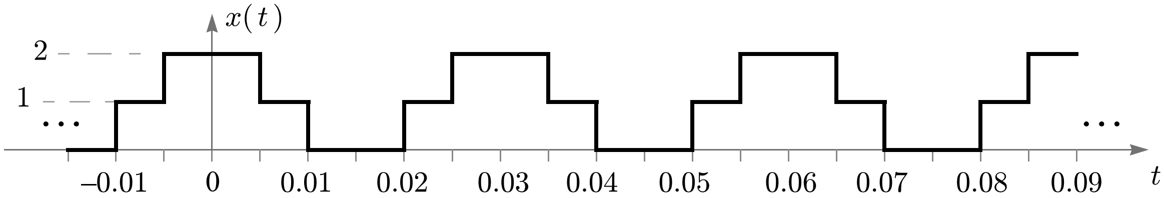
(c) The autocorrelation function $R_{xx}(t)$ for $x(t)$ achieves a maximum value of $R_{xx}(t_{\text{max}}) =$ 5 at time $t_{\text{max}} =$ 0 .

(d) The convolution of $x(t)$ with itself is nonzero only for time between $t_{\text{start}} =$ 0 and

$t_{\text{stop}} =$ 4 .

(e) Evaluate the integral $\int_{-\infty}^{\infty} x(t)\delta(t - 1/4)\sin(\pi t)dt =$ $\sqrt{2}$.

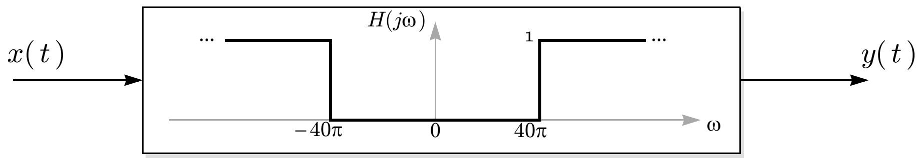
PROBLEM 2. Consider the periodic signal shown below:



(a) The fundamental period of $x(t)$ is $T_0 =$ 0.03 seconds.

(b) The zeroth coefficient in the Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T_0}$ is $a_0 =$ 1.

(c) Suppose that this periodic signal $x(t)$ is passed through an ideal *high-pass filter* with cutoff frequency of 40π rad/s, as shown below:



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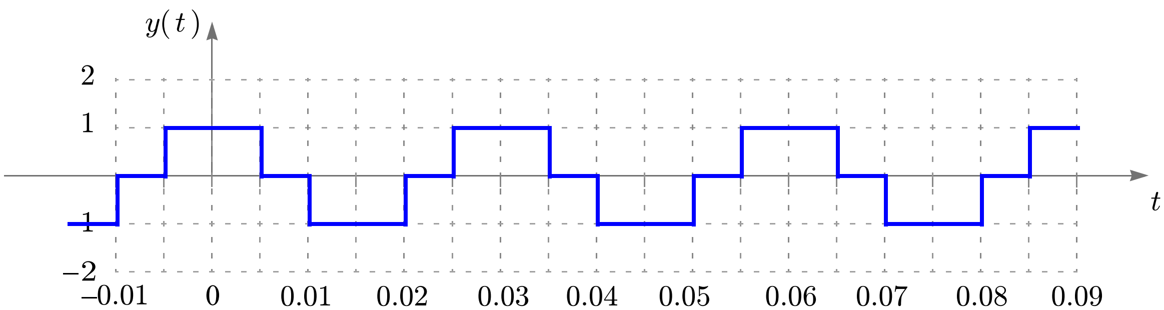

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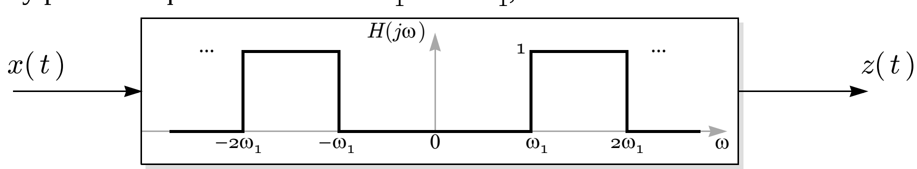

If the output is a *single sinusoid*, of the form say $z(t) = A\cos(2\pi f_c t + \theta)$, then it must be that the filter parameter satisfies:

There are 2 solutions: $\frac{100\pi}{3} < \omega_1 < \frac{200\pi}{3}$, or $\frac{200\pi}{3} < \omega_1 < 100\pi$,

(The 1st solution extracts the 1st harmonic, the 2nd solution extracts the 2nd harmonic.)

and further it must be that $f_c = \frac{1}{0.03}$, $\theta = 0$, and $A = \frac{\sqrt{3}}{\pi}$.

5PT BONUS
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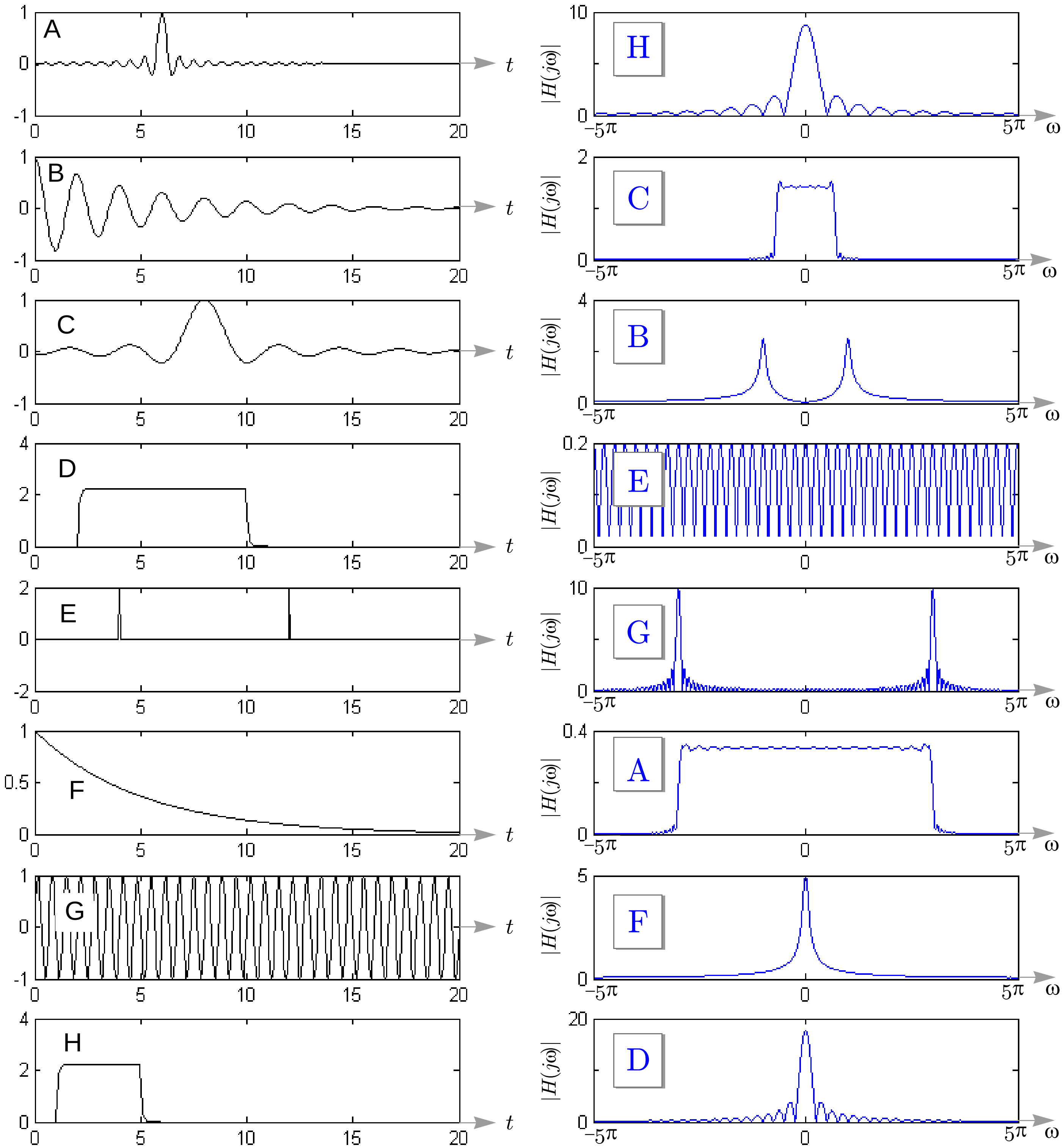

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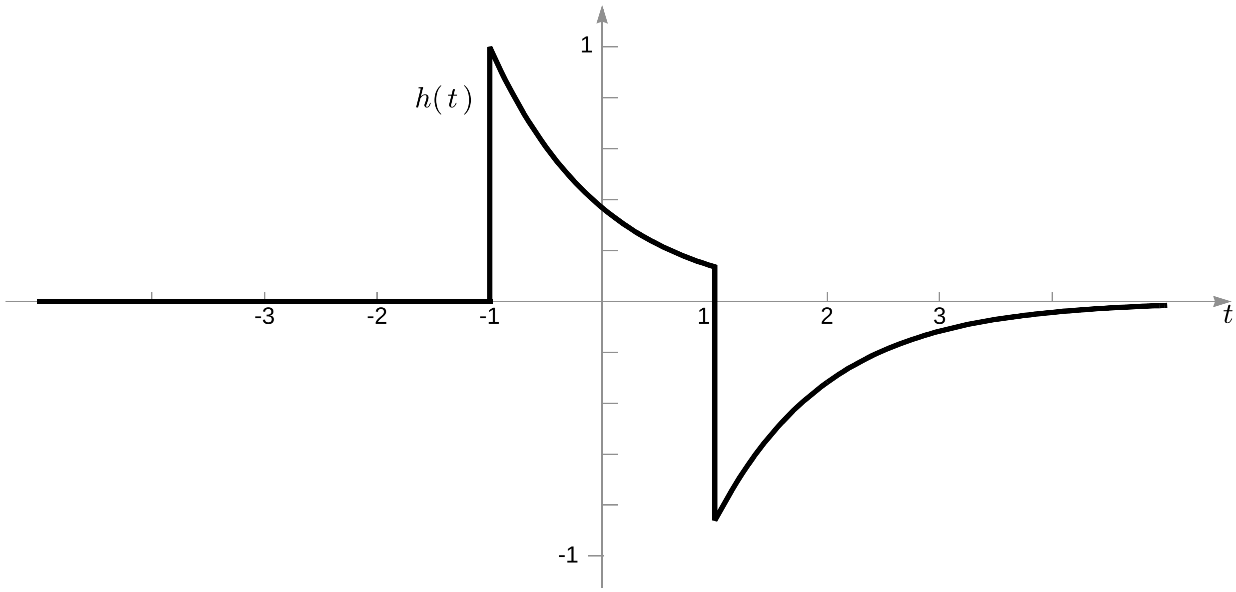

(a) Suppose that the input to this system is a sinusoid of the form $x(t) = \cos(2\pi f_0 t)$. Specify *three* different values for the frequency f_0 that will result in an output that is zero, i.e. $y(t) = 0$ for all t:

$f_0 \in \{$ 0.5 Hz, 1 Hz, 1.5 Hz $\}$.

$$H(j\omega) = \frac{1}{1+j\omega}(e^{j\omega} - e^{-j\omega}) = \frac{2j\sin(\omega)}{1+j\omega}$$

$H(j\omega)$ is zero when its numerator is zero

$\Rightarrow$ when $\omega = m\pi$ for any integer m

$\Rightarrow$ when $f = \frac{m}{2}$ for any integer m