GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 12-Feb-14 COURSE: ECE 3084A (Prof. Michaels)

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

- Write your name on the front page ONLY. Do not unstaple the test. You should have an additional sheet with Fourier transform tables.
- Calculators are permitted, but are to be used only for calculations, not to store notes, etc. No laptops, phones, or other electronic devices are allowed. Keep the desks clear of all backpacks, books, etc.
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Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem Q1.1:

(5 pts each) Your friend gives you four black boxes, each with a single input and a single output, and asks for your help to determine whether or not they are LTI systems. You tell your friend that the best you can do is to test the systems with a few inputs and try to identify which ones are definitely NOT LTI systems. Given the two inputs and corresponding two outputs for each of the four systems, classify each system as one of the following:

(A) Possibly LTI (linear and time-invariant)

(B) **NOT LINEAR** but possibly time-invariant

(C) **NOT TIME INVARIANT** but possibly linear

(D) **NEITHER LINEAR NOR TIME INVARIANT**

(a) System # 1

Input	*Output*
$x_1(t) = u(t)$	$y_1(t) = 2e^{-2(t-1)}u(t-1)$
$x_2(t) = 2u(t-10)$	$y_2(t) = 4e^{-2(t-12)}u(t-12)$

Circle One: A B C D

(b) System # 2

Input	*Output*
$x_1(t) = u(t)$	$y_1(t) = 10[u(t-5) - u(t-6)]$
$x_2(t) = 2u(t-10)$	$y_2(t) = 20[u(t-15) - u(t-16)]$

Circle One: A B C D

(c) System # 3

Input	*Output*
$x_1(t) = \cos(5t)$	$y_1(t) = 5\sin(5t)$
$x_2(t) = \cos(10t)$	$y_2(t) = 2\cos(10t)$

Circle One: A B C D

(d) System # 4

Input	*Output*
$x_1(t) = \cos(5t)$	$y_1(t) = 20\cos(5t) + \sin(10t)$
$x_2(t) = \cos(10t)$	$y_2(t) = 10\sin(10t) + \cos(20t)$

Circle One: A B C D

Problem Q1.2:
(5 pts each) The four parts of this problem are unrelated to each other.

(a) Simplify the expression $\{t^2[u(t+1) \times \delta(t-2)]\} * u(t-3) =$

(b) Simplify the expression $\displaystyle\int_{-\infty}^{t} \cos(0.5\pi\tau) \sum_{k=0}^{3} \delta(\tau - k)\, d\tau =$

(c) Find $X(j\omega)$, the Fourier transform of $x(t) = 4e^{-(t-1)}u(t-1)\cos(10t)$.

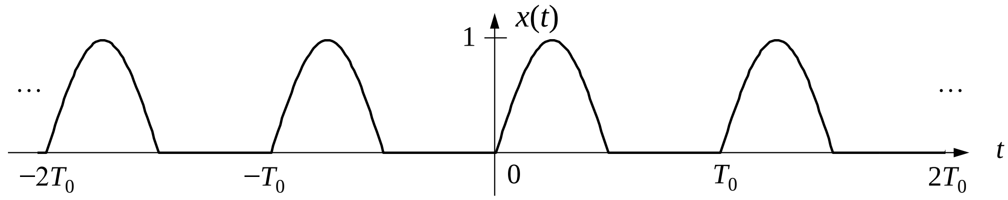
(d) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = 1 - [u(\omega+5) - u(\omega-5)]$. (Note that this is a perfect high pass filter with $\omega_c = 5$.)

Problem Q1.3:

(20 pts) Find and accurately sketch $y(t) = x(t) * h(t)$ where $x(t) = (1 - t))[u(t) - u(t - 2)]$ and $h(t) = u(t-1) - u(t-7)$. For full credit (and maximum partial credit), you need to find equations for $y(t)$ in each region as well as include a plot of $y(t)$ showing its correct shape with correct horizontal and vertical axis labels. Intermediate plots are also strongly recommended. **You must show your work to get full credit.** *Hint: If you find yourself doing a lot of algebra, STOP! It isn't necessary.*

Problem Q1.4:

Let $x_a(t)$ be the half-wave rectified sine wave shown below with period T_0 and Fourier series coefficients $a_1 = -j/4$, $a_{-1} = +j/4$, and $a_k = \dfrac{1+e^{-j\pi k}}{2\pi(1-k^2)}$ for all other k.



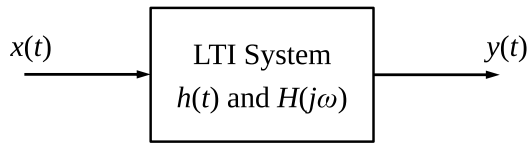
(a) (5 pts) Determine which, if any, Fourier coefficients of $x_a(t)$ are zero. Hint: $e^{j\pi k} = -1^k$.

(b) (10 pts) Use the appropriate property of Fourier series to find the the Fourier series coefficients b_k for the shifted signal $x_b(t) = x_a(t - T_0/2)$. Don't forget about the $k = \pm 1$ terms. For what values of k are the b_k equal to the a_k?

(c) (5 pts) Now let $x_c(t) = x_a(t) - x_b(t)$. Find the Fourier coefficients c_k of $x_c(t)$. Be sure to simplify your answer as much as possible. Now which coefficients are non-zero? Why?

Problem Q1.5:

Consider the general LTI system shown below with impulse response $h(t)$, frequency response $H(j\omega)$, input $x(t)$ and output $y(t)$. The impulse response is $h(t) = \dfrac{\sin(2t)}{\pi t} \times \cos(10t)$.



(a) (10 pts) Find $H(j\omega)$, the frequency response of the system, and sketch it. Make sure that the horizontal and vertical axes are correctly labeled, and that it is clear where $H(j\omega)$ is zero (if anywhere).

(b) (10 pts) Find the output $y(t)$ for $x(t) = 5\cos(t) + 10\cos(9t + \pi/4) - 2\delta(t-3)$.

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(a) System # 1

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$x_2(t) = 2u(t-10)$	$y_2(t) = 4e^{-2(t-12)}u(t-12)$

Circle One: A B C D

If LTI, $y_2(t) = 4e^{-2(t-11)}u(t-11)$

Not time-invariant

(b) System # 2

Input	*Output*
$x_1(t) = u(t)$	$y_1(t) = 10[u(t-5) - u(t-6)]$
$x_2(t) = 2u(t-10)$	$y_2(t) = 20[u(t-15) - u(t-16)]$

Circle One: A B C D

If LTI,

$y_2(t) = 20[u(t-15) - u(t-16)]$

Could be LTI

(c) System # 3

Input	*Output*
$x_1(t) = \cos(5t)$	$y_1(t) = 5\sin(5t)$
$x_2(t) = \cos(10t)$	$y_2(t) = 2\cos(10t)$

Circle One: A B C D

Sinusoid in $\rightarrow$ Sinusoid out

Could be LTI

(d) System # 4

Input	*Output*
$x_1(t) = \cos(5t)$	$y_1(t) = 20\cos(5t) + \sin(10t)$
$x_2(t) = \cos(10t)$	$y_2(t) = 10\sin(10t) + \cos(20t)$

Circle One: A B C D

Sinusoid in $\rightarrow$ 2 sinusoids out

Not linear, could be Time-invariant

Problem Q1.2:

(5 pts each) The four parts of this problem are unrelated to each other.

(a) Simplify the expression $\{t^2[u(t+1) \times \delta(t-2)]\} * u(t-3) =$ $\{t^2 \delta(t-2)\} * u(t-3)$

$= 4\,\delta(t-2) * u(t-3)$

$= 4\,u(t-5)$

(b) Simplify the expression $\int_{-\infty}^{t} \cos(0.5\pi\tau) \sum_{k=0}^{3} \delta(\tau - k) d\tau =$ $\int_{-\infty}^{t} \cos(\frac{\pi}{2}\tau)[\delta(\tau)+\delta(\tau-1)+\delta(\tau-2)+\delta(\tau-3)]d\tau$

$= \int_{-\infty}^{t} [1\,\delta(\tau) + 0\,\delta(\tau-1) - 1\,\delta(\tau-2) + 0\,\delta(\tau-3)]\, d\tau$

$= u(t) - u(t-2)$

(c) Find $X(j\omega)$, the Fourier transform of $x(t) = 4e^{-(t-1)}u(t-1)\cos(10t)$. $= \tilde{x}(t)\cos(10t)$

$\tilde{X}(j\omega) = \frac{4e^{-j\omega}}{1+j\omega}$ use modulation property

$X(j\omega) = \frac{2e^{-j(\omega+10)}}{1+j(\omega+10)} + \frac{2e^{-j(\omega-10)}}{1+j(\omega-10)}$

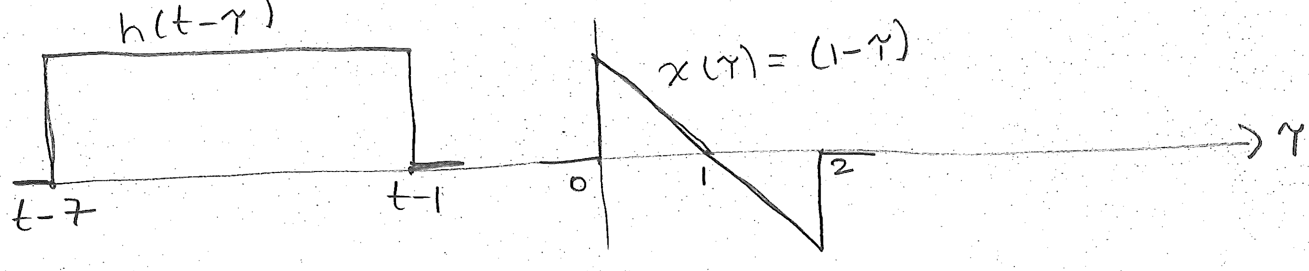
(d) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = 1 - [u(\omega+5) - u(\omega-5)]$. (Note that this is a perfect high pass filter with $\omega_c = 5$.)

$x(t) = \delta(t) - \frac{\sin(5t)}{\pi t}$

Problem Q1.3:

(20 pts) Find and accurately sketch $y(t) = x(t) * h(t)$ where $x(t) = (1-t))[u(t) - u(t-2)]$ and $h(t) = u(t-1) - u(t-7)$. For full credit (and maximum partial credit), you need to find equations for $y(t)$ in each region as well as include a plot of $y(t)$ showing its correct shape with correct horizontal and vertical axis labels. Intermediate plots are also strongly recommended. **You must show your work to get full credit.** *Hint: If you find yourself doing a lot of algebra, STOP! It isn't necessary.*

Flip $h(t)$ $\quad y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$



Region I. $t < 1$, $y(t) = 0$

Region II. $1 \le t < 3$ (partial overlap)

$$y(t) = \int_0^{t-1} (1-\tau)\, d\tau = \left(\tau - \frac{\tau^2}{2}\right)\Big|_0^{t-1} = (t-1) - \frac{(t-1)^2}{2} = -\frac{t^2}{2} + 2t - \frac{3}{2} = -\frac{1}{2}(t-1)(t-3)$$

Region III. $3 \le t < 7$ (full overlap)

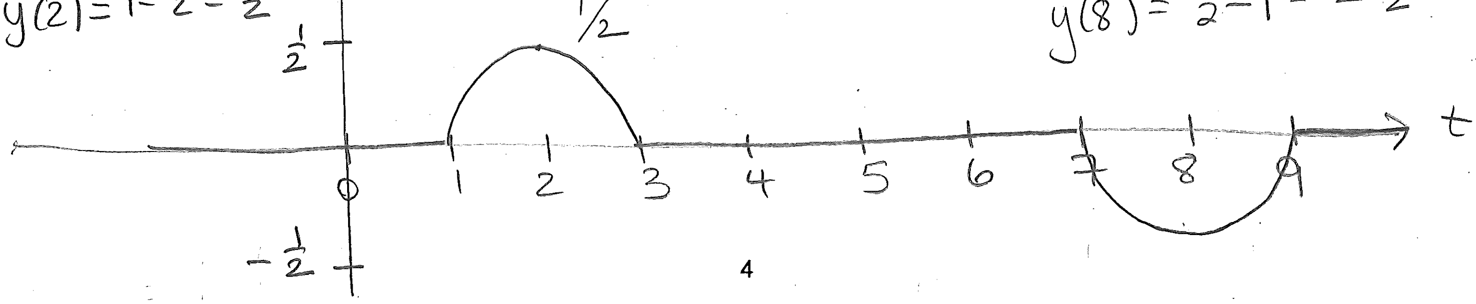
$y(t) = 0$ (positive & negative pieces of $x(\tau)$ cancel out)

Region IV. $7 \le t < 9$ (partial overlap)

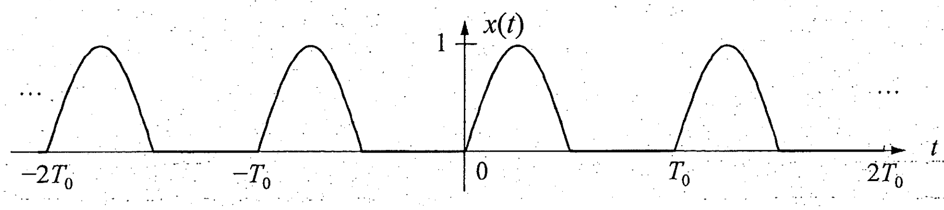
$$y(t) = \int_{t-7}^{2} (1-\tau)\, d\tau = \left(\tau - \frac{\tau^2}{2}\right)\Big|_{t-7}^{2} = 2 - \frac{2^2}{4} - (t-7) + \frac{(t-7)^2}{2} = \frac{(t-7)^2}{2} - (t-7) = \frac{t^2}{2} - 8t - \frac{63}{2} = \frac{1}{2}(t-7)(t-9)$$

Region V. $t \ge 9$, $y(t) = 0$

$y(2) = 1 - \frac{1}{2} = \frac{1}{2}$ $\qquad y(8) = \frac{1}{2} - 1 = -\frac{1}{2}$



Problem Q1.4:
Let $x_a(t)$ be the half-wave rectified sine wave shown below with period T_0 and Fourier series coefficients $a_1 = -j/4$, $a_{-1} = +j/4$, and $a_k = \dfrac{1+e^{-j\pi k}}{2\pi(1-k^2)}$ for all other k.



(a) (5 pts) Determine which, if any, Fourier coefficients of $x_a(t)$ are zero. Hint: $e^{j\pi k} = (-1)^k = e^{-j\pi k}$

$a_0 \neq 0$

$a_{\pm 1} \neq 0$

$a_{\text{even } k} = \dfrac{2}{2\pi(1-k^2)} \neq 0$

$\boxed{a_{\text{odd},\, k\neq \pm 1} = 0}$

(b) (10 pts) Use the appropriate property of Fourier series to find the the Fourier series coefficients b_k for the shifted signal $x_b(t) = x_a(t - T_0/2)$. Don't forget about the $k = \pm 1$ terms. For what values of k are the b_k equal to the a_k?

$b_k = a_k e^{-j\frac{2\pi k}{T_0}\cdot\frac{T_0}{2}} = a_k e^{-j\pi k}$

$b_0 = a_0$

$b_1 = -a_1 = \dfrac{j}{4}$

$b_{-1} = -a_{-1} = \dfrac{-j}{4}$

$b_{\text{even } k} = a_{\text{even } k}$

$b_{\text{odd},\, k\neq\pm 1} = 0$

$\boxed{\text{all } b_k = a_k \text{ except } b_{\pm 1} = -a_{\pm 1}}$

(c) (5 pts) Now let $x_c(t) = x_a(t) - x_b(t)$. Find the Fourier coefficients c_k of $x_c(t)$. Be sure to simplify your answer as much as possible. Now which coefficients are non-zero? Why?

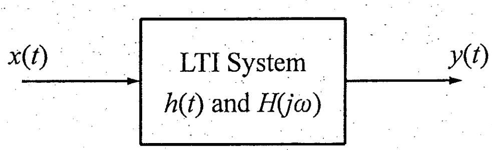
$c_k = a_k - b_k$ All $c_k = 0$ except $k = \pm 1$

$$\left.\begin{aligned} c_1 &= -\frac{j}{4} - \frac{j}{4} = -\frac{j}{2} \\ c_{-1} &= \frac{j}{4} + \frac{j}{4} = \frac{j}{2} \end{aligned}\right\} \text{these are the only non-zero coefficients because } x_c(t) = \sin(\omega_0 t)$$

$$x_c(t) = -\frac{j}{2}e^{j\omega_0 t} + \frac{j}{2}e^{-j\omega_0 t} = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \sin(\omega_0 t)$$

Problem Q1.5:

Consider the general LTI system shown below with impulse response $h(t)$, frequency response $H(j\omega)$, input $x(t)$ and output $y(t)$. The impulse response is $h(t) = \frac{\sin(2t)}{\pi t} \times \cos(10t)$.



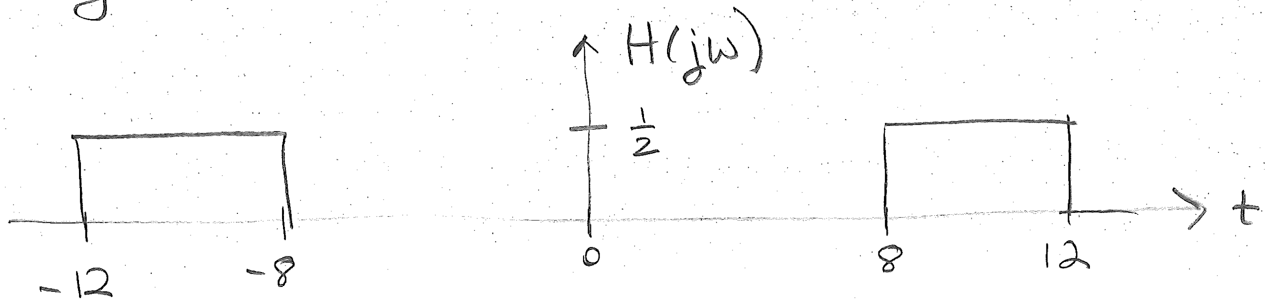
(a) (10 pts) Find $H(j\omega)$, the frequency response of the system, and sketch it. Make sure that the horizontal and vertical axes are correctly labeled, and that it is clear where $H(j\omega)$ is zero (if anywhere).

$$h(t) = \tilde{h}(t)\cos(10t)$$

$$H(j\omega) = \tfrac{1}{2}\tilde{H}(\omega+10) + \tfrac{1}{2}\tilde{H}(\omega-10)$$

$$\tilde{H}(j\omega) = u(\omega+2) - u(\omega-2)$$

$$H(j\omega) = \tfrac{1}{2}\left[u(\omega+12) - u(\omega+8)\right] + \tfrac{1}{2}\left[u(\omega-8) + u(\omega-12)\right]$$



(b) (10 pts) Find the output $y(t)$ for $x(t) = 5\cos(t) + 10\cos(9t + \pi/4) - 2\delta(t-3)$.

$$y(t) = 5\underbrace{|H(j1)|}_{0}\cos(t + \angle H(j1)) + 10\underbrace{|H(j9)|}_{1/2}\cos(9t + \pi/4 + \underbrace{\angle H(j9)}_{0}) - 2h(t-3)$$

$$\boxed{y(t) = 5\cos(9t + \pi/4) - \frac{2\sin(2(t-3))}{\pi(t-3)}\cos(10(t-3))}$$