Name: ____

1. Check that your exam includes all 6 pages (this cover sheet plus 5 pages). You should also have one additional sheet with Fourier transform pairs and properties.

2. Print your name on the front page only. Do not unstaple the test.

3. Work the problem in the space provided, and put a circle or box around your answer. If more space is needed for additional work, use the backs of previous pages, and clearly label this work with the problem number for it to be graded.

4. Unless stated otherwise, justify your reasoning to receive any partial credit.

5. You are permitted one sheet (8-1/2 x 11) of handwritten notes, double-sided. Use of any other notes, books, or other resources is prohibited. **NO CALCULATORS PERMITTED!**

6. This exam lasts for 80 minutes.

7. Put all backpacks, book bags, etc. on the floor. **TURN OFF CELL PHONES AND PUT THEM AWAY**.

Problem	**Value**	**Score**
1	20	
2	20	
3	20	
4	20	
5	20	
Total	**100**	

Honor Code: Possessing, using or exchanging improperly acquired written or verbal information is academic misconduct **- DON'T DO IT!** Also, the room is very crowded, so be sure to keep your eyes on your own work and not your neighbor's.

1. (5 points each) Short problem assortment – note that parts (a)-(d) are unrelated. None of the answers should have any summations, integrals, derivatives, etc., and should be simplified as much as possible.

(a) Simplify the following expression:

$$x(t) = \sum_{k=-1}^{1} t^2 \sin\left(\frac{\pi t}{2}\right)\delta\left(t-k\right)$$

(b) Evaluate the following integral:

$$x(t) = \int_{-\infty}^{t} \frac{1}{\tau}\left[\delta\left(\tau-2\right)+\delta\left(\tau-4\right)\right]d\tau$$

(c) Evaluate and simplify the following expression:

$$x(t) = \frac{d}{dt}\left\{\cos(\pi t)\left[u(t+1)-u(t-1)\right]\right\}$$

(d) Simplify the following expression (the * denotes convolution):

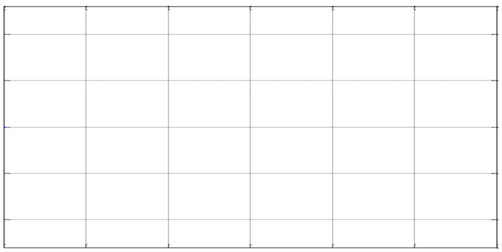
$$x(t) = \left[2e^{-2t}u(t)\right]*\left[\delta(t-1)+\delta(t-2)\right]$$

2. Consider the convolution of the following two signals, $x(t)$ and $h(t)$:

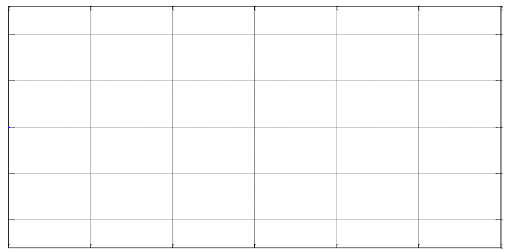
$$x(t) = u(t) - 2u(t-1) + u(t-2)$$
$$h(t) = u(t-1) - u(t-4)$$
$$y(t) = x(t) * h(t)$$

(a) (5 points) Plot $x(t)$ and $h(t)$ on the graphs below. Clearly label both axes for both plots.

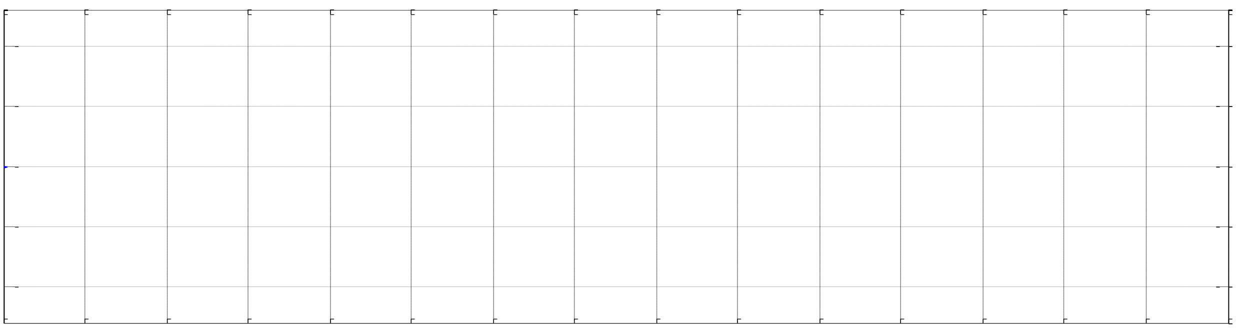
$x(t)$



$h(t)$



(b) (6 points) Plot $x(\tau)$ and $h(t-\tau)$ vs. τ for an arbitrary value of $t < 0$. Clearly identify each function. Carefully label both axes and all points of interest on the signal $h(t-\tau)$.



(c) (3 points each) Determine $y(t)$ for the following values of t. Put your answers in the table below. There is no partial credit.

t	$y(t)$
0	
2	
4	

3. (10 points each) Parts (a) and (b) are unrelated.

(a) You are given the following Fourier transform pair:

$$x(t) = \begin{cases} 1-|t| & \text{for } -1 < t < 1 \\ 0 & \text{otherwise} \end{cases} \Leftrightarrow X(j\omega) = \left[\frac{\sin(\omega/2)}{\omega/2}\right]^2$$

Use the scaling property (and any other properties needed) to find $Y(j\omega)$, the Fourier transform of

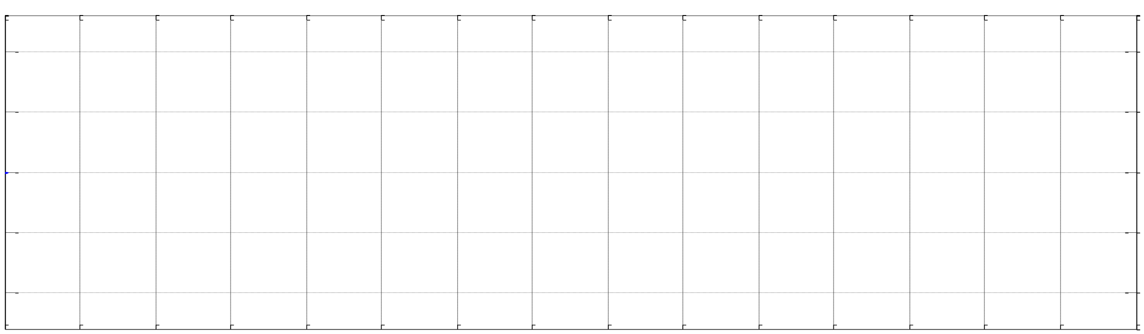
$$y(t) = \begin{cases} (5-|t|) & \text{for } -5 < t < 5 \\ 0 & \text{otherwise} \end{cases}$$

Be sure to show all work.

(b) Suppose $x(t)$ is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\{ \int_{0}^{8} \sin\left(\frac{\pi\tau}{2}\right) e^{-j\omega\tau} d\tau \right\} e^{j\omega t} d\omega$$

Sketch $x(t)$ for t between -10 and 10. Explain your reasoning. Rethink your approach before slogging through any complicated calculations.



4. Parts (a) and (b) are unrelated, but both consider the system below with input $x(t)$ and output $y(t)$.

(a) (10 points) Your friend insists that he has an LTI system, but you are skeptical and decide to test this premise. To do this test, you input the following three signals into the system:

$$x_1(t) = u(t) - u(t-1)$$
$$x_2(t) = 2\left[u(t-2) - u(t-3)\right]$$
$$x_3(t) = u(t) - 2u(t-1) + u(t-2)$$

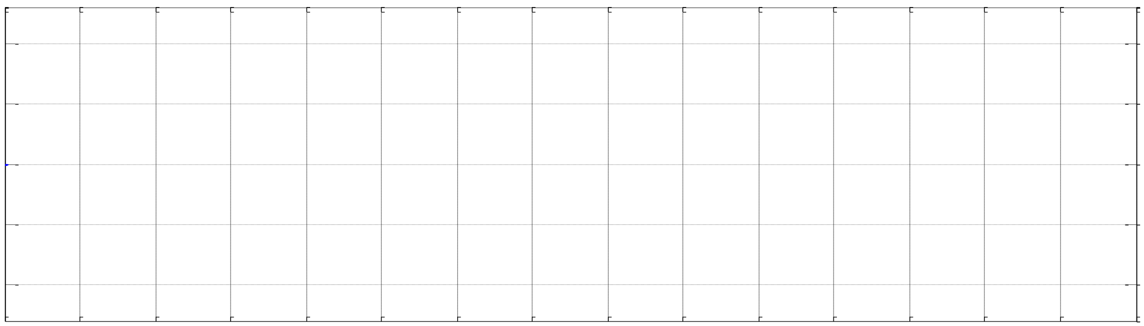
You measure the output $y_1(t) = 5e^{-2(t-3)}u(t-3)$ for the first input. If the system is linear and time-invariant, what would you expect to measure for $y_2(t)$ and $y_3(t)$?

(b) (10 points) Suppose a different system is LTI and its frequency response is $H(j\omega) = 2e^{-|\omega|}$. If the input is the periodic signal given by $x(t) = \sum_{k=-1}^{1} \frac{1}{|k|+1} e^{j2\pi kt}$, find the output $y(t)$ and express it as a real signal (no complex exponentials).

5. Consider a periodic signal with period $T_0 = 4$ that is defined as follows between $-2 < t < 2$:

$$x(t) = \begin{cases} 1 \text{ for } 0 \le t < 1 \\ -1 \text{ for } -1 < t < 0 \\ 0 \text{ otherwise} \end{cases}$$

(a) (5 points) Sketch this signal on the interval $-6 < t < 6$. Carefully label all axes and important values.



(b) (5 points) Set up the integral(s) to evaluate the Fourier series coefficients a_k for this specific $x(t)$. Do NOT actually evaluate any integrals.

(c) (10 points) Based upon any symmetry that this signal may have, circle the appropriate attributes for each coefficient named in the table below. Note that either "zero" or "non-zero" should be circled for every coefficient in the table; "real" or "imaginary" should be circled only if non-zero values are either purely real or purely imaginary.

a_0	zero	non-zero	real	imaginary
a_1	zero	non-zero	real	imaginary
a_2	zero	non-zero	real	imaginary

Name: Answer key – Section A

1. Check that your exam includes all 6 pages (this cover sheet plus 5 pages). You should also have one additional sheet with Fourier transform pairs and properties.

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1. (5 points each) Short problem assortment – note that parts (a)-(d) are unrelated. None of the answers should have any summations, integrals, derivatives, etc., and should be simplified as much as possible.

(a) Simplify the following expression:

$$x(t) = \sum_{k=-1}^{1} t^2 \sin\left(\frac{\pi t}{2}\right)\delta(t-k)$$

$$= (-1)^2 \sin(-\pi/2)\delta(t+1) + 0\,\delta(t) + (1)^2 \sin(\pi/2)\delta(t-1)$$

$$\boxed{x(t) = -\delta(t+1) + \delta(t-1)}$$

(b) Evaluate the following integral:

$$x(t) = \int_{-\infty}^{t} \frac{1}{\tau}\left[\delta(\tau-2)+\delta(\tau-4)\right]d\tau$$

$$= \int_{-\infty}^{t} \frac{1}{2}\delta(\tau-2)\,d\tau + \int_{-\infty}^{t} \frac{1}{4}\delta(\tau-4)\,d\tau$$

$$\boxed{x(t) = \frac{1}{2}u(t-2) + \frac{1}{4}u(t-4)}$$

(c) Evaluate and simplify the following expression:

$$x(t) = \frac{d}{dt}\left\{\cos(\pi t)\left[u(t+1)-u(t-1)\right]\right\}$$

$$= \cos(\pi t)\left[\delta(t+1)-\delta(t-1)\right] - \pi \sin(\pi t)\left[u(t+1)-u(t-1)\right]$$

$$\boxed{x(t) = -\delta(t+1) + \delta(t-1) - \pi\sin(\pi t)\left[u(t+1)-u(t-1)\right]}$$

(d) Simplify the following expression (the * denotes convolution):

$$x(t) = \boxed{2e^{-2t}u(t)} * \left[\delta(t-1)+\delta(t-2)\right]$$

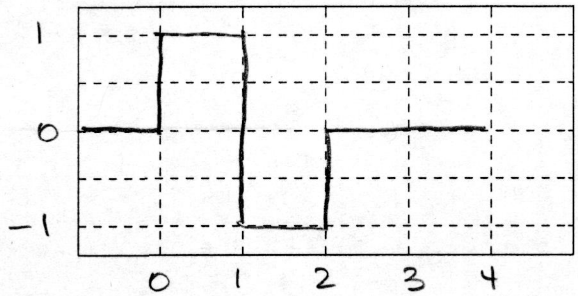
$$\boxed{x(t) = 2e^{-2(t-1)}u(t-1) + 2e^{-2(t-2)}u(t-2)}$$

2. Consider the convolution of the following two signals, $x(t)$ and $h(t)$:

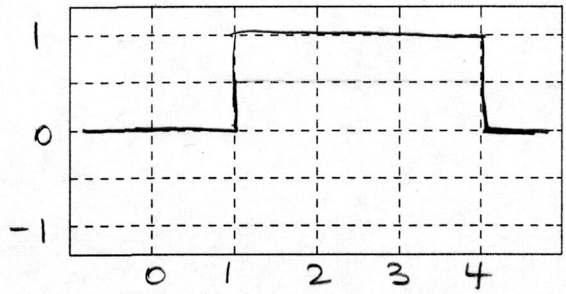
$$x(t) = u(t) - 2u(t-1) + u(t-2)$$
$$h(t) = u(t-1) - u(t-4)$$
$$y(t) = x(t) * h(t)$$

(a) (5 points) Plot $x(t)$ and $h(t)$ on the graphs below. Clearly label both axes for both plots.

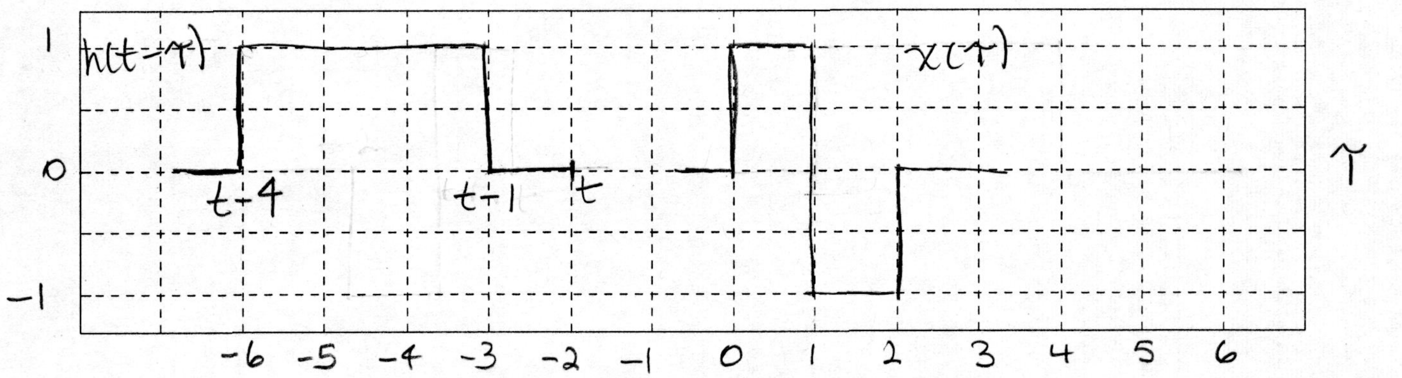
$x(t)$



$h(t)$



(b) (6 points) Plot $x(\tau)$ and $h(t-\tau)$ vs. τ for an arbitrary value of $t < 0$. Clearly identify each function. Carefully label both axes and all points of interest on the signal $h(t-\tau)$.



Plotted for $t = -2$

(c) (3 points each) Determine $y(t)$ for the following values of t. Put your answers in the table below. There is no partial credit.

t	$y(t)$
0	0
2	1
4	0

No overlap

Overlap from 0 to 1 ⇒ Area = 1

Overlap from 0 to 2 ⇒ Area = 0

3. (10 points each) Parts (a) and (b) are unrelated.

(a) You are given the following Fourier transform pair:

$$x(t) = \begin{cases} 1-|t| & \text{for } -1<t<1 \\ 0 & \text{otherwise} \end{cases} \Leftrightarrow X(j\omega) = \left[\frac{\sin(\omega/2)}{\omega/2}\right]^2$$

Use the scaling property (and any other properties needed) to find $Y(j\omega)$, the Fourier transform of

$$y(t) = \begin{cases} (5-|t|) & \text{for } -5<t<5 \\ 0 & \text{otherwise} \end{cases}$$

Be sure to show all work.

$$y(t) = \begin{cases} 5(1-|0.2t|) & -5<t<5 \\ 0 & \text{otherwise} \end{cases} = 5x(0.2t)$$

$$Y(j\omega) = \frac{1}{0.2} 5X(j5\omega)$$

$$= \frac{1}{0.2} 5 \left[\frac{\sin(5\omega/2)}{5\omega/2}\right]^2$$

$$\boxed{Y(j\omega) = 25\left[\frac{\sin(5\omega/2)}{5\omega/2}\right]^2} = 4\left(\frac{\sin(5\omega/2)}{\omega^2}\right)^2$$

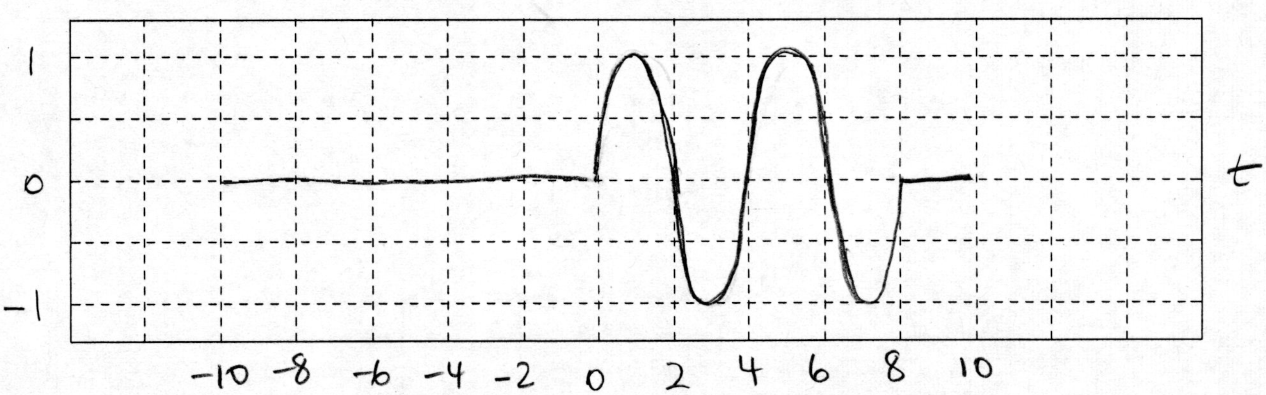
(b) Suppose $x(t)$ is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\int_0^8 \sin\left(\frac{\pi\tau}{2}\right) e^{-j\omega\tau} d\tau\right\} e^{j\omega t} d\omega$$

$\omega = \frac{\pi}{2}$

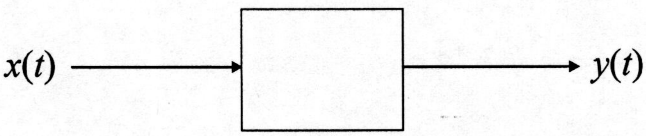
$T = \frac{2\pi}{\omega} = 4$

Sketch $x(t)$ for t between -10 and 10. Explain your reasoning. Rethink your approach before slogging through any complicated calculations.



The quantity in curly brackets $\{\cdot\}$ is $X(j\omega)$. Thus, $x(t) = \sin(\pi t/2)$ for $0<t<8$ and is zero otherwise.

4. Parts (a) and (b) are unrelated, but both consider the system below with input $x(t)$ and output $y(t)$.



(a) (10 points) Your friend insists that he has an LTI system, but you are skeptical and decide to test this premise. To do this test, you input the following three signals into the system:

$$x_1(t) = u(t) - u(t-1)$$
$$x_2(t) = 2[u(t-2) - u(t-3)]$$
$$x_3(t) = u(t) - 2u(t-1) + \cancel{u(t+1)}\, u(t-2)$$

You measure the output $y_1(t) = 5e^{-2(t-3)}u(t-3)$ for the first input. If the system is linear and time-invariant, what would you expect to measure for $y_2(t)$ and $y_3(t)$?

$x_2(t) = 2x_1(t-2)$

$\boxed{y_2(t) = 2y_1(t-2) = 10e^{-2(t-5)}u(t-5)}$

$x_3(t) = x_1(t) - x_1(t-1)$

$\boxed{y_3(t) = y_1(t) - y_1(t-1) = 5e^{-2(t-3)}u(t-3) - 5e^{-2(t-4)}u(t-4)}$

(b) (10 points) Suppose a different system is LTI and its frequency response is $H(j\omega) = 2e^{-|\omega|}$. If the input is the periodic signal given by $x(t) = \sum_{k=-1}^{1} \frac{1}{|k|+1} e^{j2\pi kt}$, find the output $y(t)$ and express it as a real signal (no complex exponentials).

2 frequencies: $\omega = 0$ and $\omega = 2\pi$

$H(j0) = 2 \qquad H(j2\pi) = 2e^{-2\pi}$

$$y(t) = 2e^{-2\pi}\frac{1}{|-1|+1}e^{-j2\pi t} + 2 + 2e^{-2\pi}\frac{1}{1+1}e^{j2\pi t}$$
$$= e^{-2\pi}(e^{-j2\pi t} + e^{j2\pi t}) + 2$$

$$\boxed{y(t) = 2e^{-2\pi}\cos(2\pi t) + 2}$$

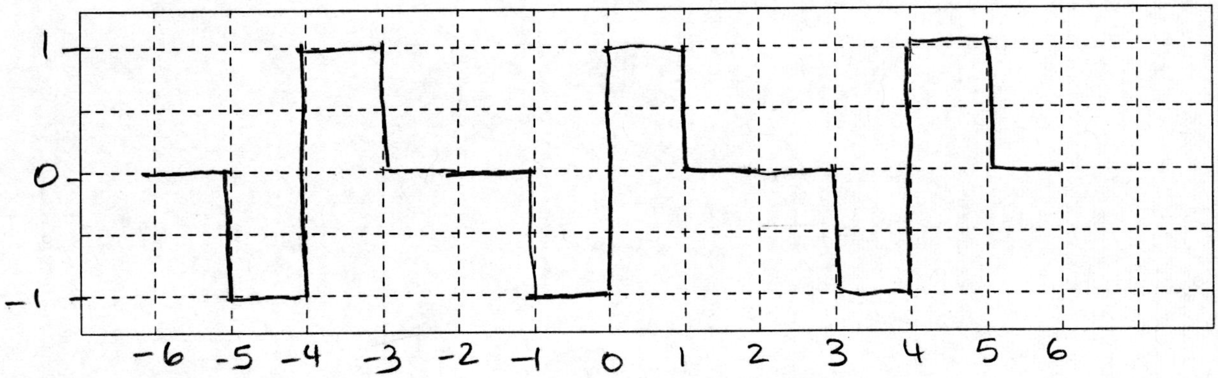
This problem can also be worked using Fourier transforms.

5. Consider ~~a~~ periodic signal with period $T_0 = 4$ that is defined as follows between $-2 < t < 2$:

$$x(t) = \begin{cases} 1 \text{ for } 0 \le t < 1 \\ -1 \text{ for } -1 < t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$$

(a) (5 points) Sketch this signal on the interval $-6 < t < 6$. Carefully label all axes and important values.



(b) (5 points) Set up the integral(s) to evaluate the Fourier series coefficients a_k for this specific $x(t)$. Do NOT actually evaluate any integrals.

$$a_k = -\frac{1}{4}\int_{-1}^{0} e^{-j\frac{\pi}{2}kt}\,dt + \frac{1}{4}\int_{0}^{1} e^{-j\frac{\pi}{2}kt}\,dt$$

(c) (10 points) Based upon any symmetry that this signal may have, circle the appropriate attributes for each coefficient named in the table below. Note that either "zero" or "non-zero" should be circled for every coefficient in the table; "real" or "imaginary" should be circled only if non-zero values are either purely real or purely imaginary.

a_0	zero	non-zero	real	imaginary
a_1	zero	non-zero	real	imaginary
a_2	zero	non-zero	real	imaginary

$a_0 = 0$ since there is clearly no dc term.
a_1 and a_2 are non-zero and imaginary since $x(t)$ is odd and there is no symmetry to cause a_1 or a_2 to be zero.