ECE 3084
QUIZ 1
SCHOOL OF ECE
GEORGIA INSTITUTE OF TECHNOLOGY

OCTOBER 1, 2020

NAME: _____

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

"My signature below attests that I have neither given nor received help during the taking of this exam, and that I am in complete compliance with the Georgia Tech honor code:"

_____
(your signature)

- The examination is open everything: open book/notes/calculators/MATLAB/computers/internet/etc, with the exception of no collaborating (that means no communications, electronic or otherwise) with anyone else.
- Only one exception to the no-communication rule: Use *piazza* to let me know if you spot any typos or other issues.
- Final answers must be entered into the answer box.
- Correct answers *must be* accompanied by concise justifications to receive full credit.
- This document has N pages.
- If you have a printer: Print it out, work directly on the printout.
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  - Take out N blank pages and show your work and answers on the corresponding page, in the corresponding space.
  - For example, anything related to Prob. 1 should be limited to page 2 of your submission, and only things related to Prob. 2 should be on page 3 of your submission.
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PROBLEM 1.

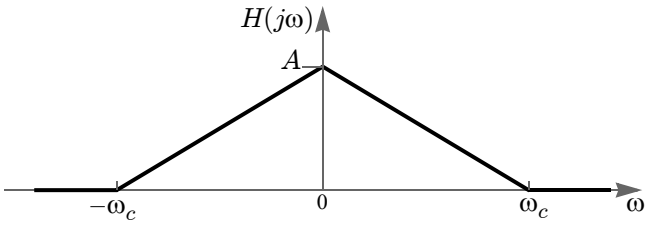
Consider the signal $x(t) = \sum_{k=-2}^{2} k^2 e^{jk^3\pi t}$. (Note k^3 here, not k)

(a) The signal $x(t)$ is [even][odd][neither] (circle one).

(b) The power of $x(t)$ is $P =$ [ ].

(c) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ [ ] Hz.

(d) Suppose $x(t)$ is fed as an input to an LTI system whose frequency response is real and triangular, as shown here:



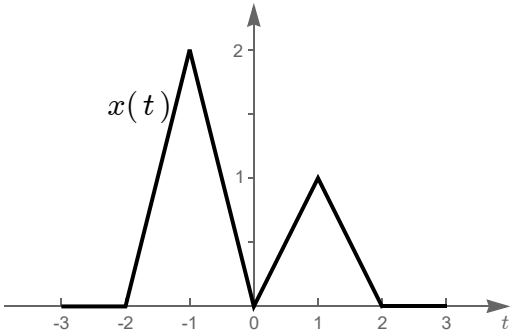
Find numeric values for the unspecified parameters A and ω_c so that the output is $y(t) = 4\cos(\pi t) + 2\cos(8\pi t)$.

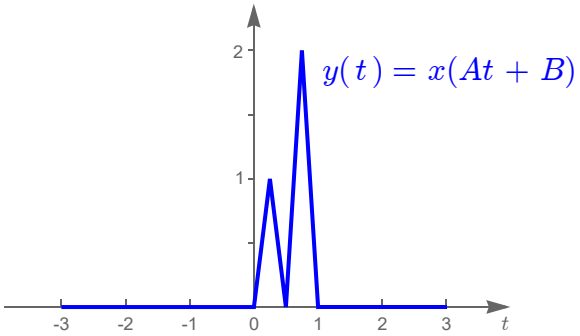
$A =$ [ ]

$\omega_c =$ [ ]

PROBLEM 2.

(a) Shown below are sketches of $x(t)$ and $y(t) = x(At + B)$. Find the constants A and B.

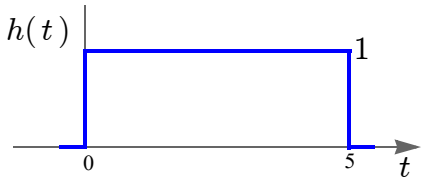




$A =$ ____

$B =$ ____

PROBLEM 3.



Consider an LTI system whose impulse response is $h(t) = u(t) - u(t-5)$, as sketched here:

Both parts below consider the same system, but with different inputs.

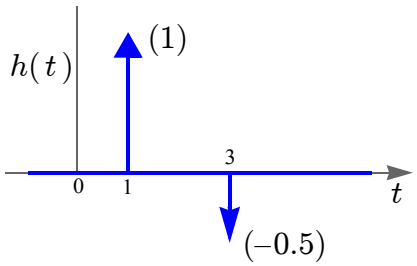
(a) There are many values of f_0 such that the output of this system in response to the sinusoidal input $x(t) = \cos(2\pi f_0 t + 0.3\pi)$ is $y(t) = 0$ for all t. Name any three, in Hz:

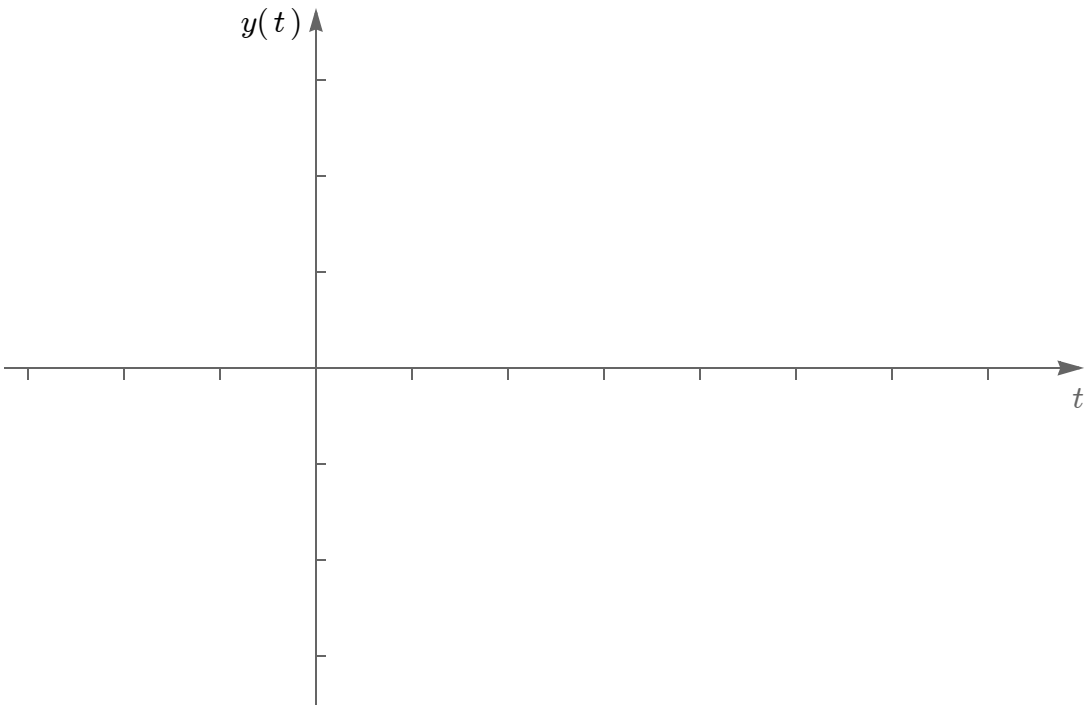
$f_0 =$ ☐ Hz

or $f_0 =$ ☐ Hz

or $f_0 =$ ☐ Hz .

(b) In the space below, carefully sketch the output $y(t)$ of this system when the input is $x(t) = \delta(t-1) - 0.5\delta(t-3)$, taking care to carefully label important times and important signal heights:



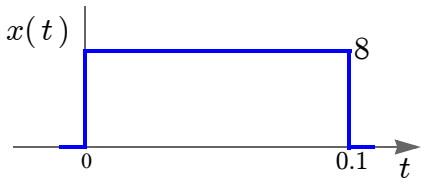


PROBLEM 4.

(a) Evaluate the integral $\int_{-\infty}^{5} \frac{1}{1+t^2}\,\delta(t-3)\,dt =$ $\boxed{}$.

(b) The energy of the rectangular pulse $x(t)$ shown here is

$$E = \boxed{}.$$



PROBLEM 5. (20 points)

Shown below are the relationships between the input $x(t)$ and output $y(t)$ of two systems. Specify which properties listed on the left are satisfied by each: (*Brief* explanations are OK!)

	SYSTEM#1: $y(t) = tx(t)$	SYSTEM#2: $y(t) = x(\cos(t))$
Linear?	YES ☐ NO ☐	YES ☐ NO ☐
Time Invariant?	YES ☐ NO ☐	YES ☐ NO ☐
Memoryless?	YES ☐ NO ☐	YES ☐ NO ☐
Invertible?	YES ☐ NO ☐	YES ☐ NO ☐
Causal?	YES ☐ NO ☐	YES ☐ NO ☐
Stable?	YES ☐ NO ☐	YES ☐ NO ☐

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PROBLEM 1.

Consider the signal $x(t) = \sum_{k=-2}^{2} k^2 e^{jk^3\pi t}$. (Note k^3 here, not k)

$= e^{j\pi t} + e^{-j\pi t} + 4e^{j8\pi t} + 4e^{-j8\pi t}$
$= 2\cos(\pi t) + 8\cos(8\pi t)$

(a) The signal $x(t)$ is [even][odd][neither] (circle one).

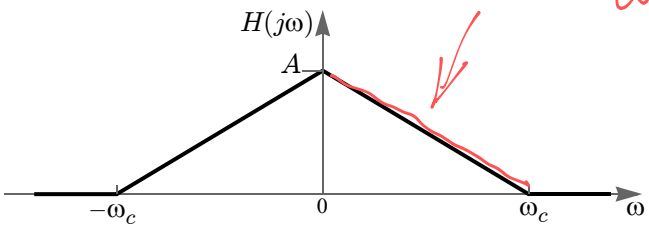
(b) The power of $x(t)$ is $P =$ 34 .

Parseval: Given FS coeffs $a_k = \begin{cases} k^2 \\ 0 \end{cases}$, $P = \sum |a_k|^2 = 4^2 + 1^2 + 0^2 + 1^2 + 4^2 = 34$

(c) The input $x(t)$ is periodic with fundamental frequency $f_0 =$ 0.5 Hz.

$f_0 = \gcd(0.5, 4)$

(d) Suppose $x(t)$ is fed as an input to an LTI system whose frequency response is real and triangular, as shown here:



equation for line: $A\left(1 - \frac{\omega}{\omega_c}\right)$

Find numeric values for the unspecified parameters A and ω_c so that the output is $y(t) = 4\cos(\pi t) + 2\cos(8\pi t)$.

$H(j\pi) = 2$ ↑ $H(j8\pi) = \frac{1}{4}$ ↑

$A =$ 2.25

$\omega_c =$ 9π

$A\left(1 - \frac{\pi}{\omega_c}\right) = 2$
$A\left(1 - \frac{8\pi}{\omega_c}\right) = \frac{1}{4}$
} divide ⇒ to eliminate A

$\frac{1 - \frac{\pi}{\omega_c}}{1 - \frac{8\pi}{\omega_c}} = 8$

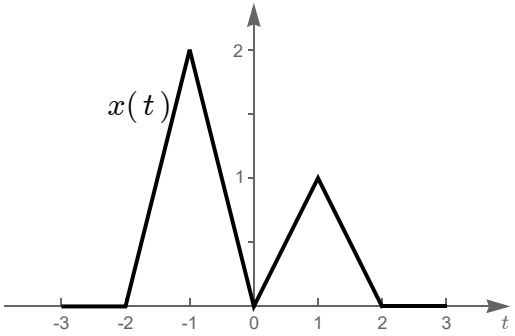
$\Rightarrow \frac{\omega_c - \pi}{\omega_c - 8\pi} = 8$

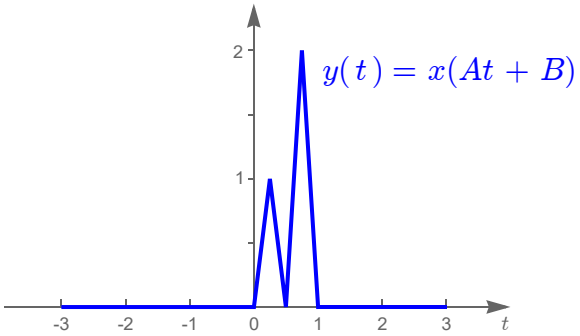
$\Rightarrow \omega_c = 9\pi$

$A\left(1 - \frac{\pi}{9\pi}\right) = 2 \Rightarrow A = \frac{9}{4}$

PROBLEM 2.

(a) Shown below are sketches of $x(t)$ and $y(t) = x(At + B)$. Find the constants A and B.





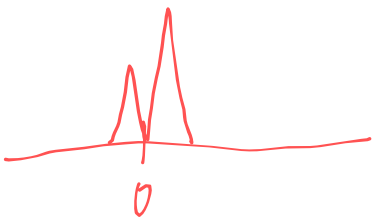
$A =$ -4

$B =$ 2

time reversed $\Rightarrow$ A negative

duration compressed by 4 $\Rightarrow A = -4$

Let $w(t) = x(-4t)$:



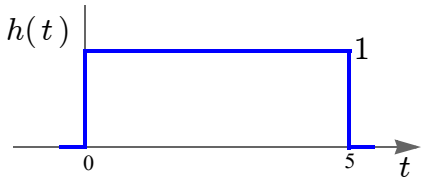
To get $y(t)$, shift to right by 0.5

$\Rightarrow y(t) = w(t-0.5) = x(-4(t-0.5))$

$= x(-4t+2) \Rightarrow B = 2$

PROBLEM 3.

Consider an LTI system whose impulse response is $h(t) = u(t) - u(t-5)$, as sketched here:



Both parts below consider the same system, but with different inputs.

(a) There are many values of f_0 such that the output of this system in response to the sinusoidal input $x(t) = \cos(2\pi f_0 t + 0.3\pi)$ is $y(t) = 0$ for all t. Name any three, in Hz:

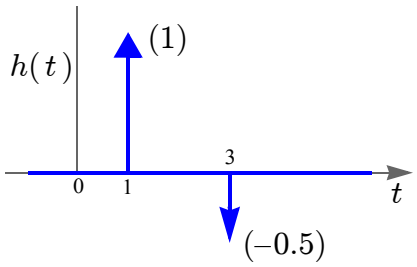
$f_0 =$ 0.2 Hz

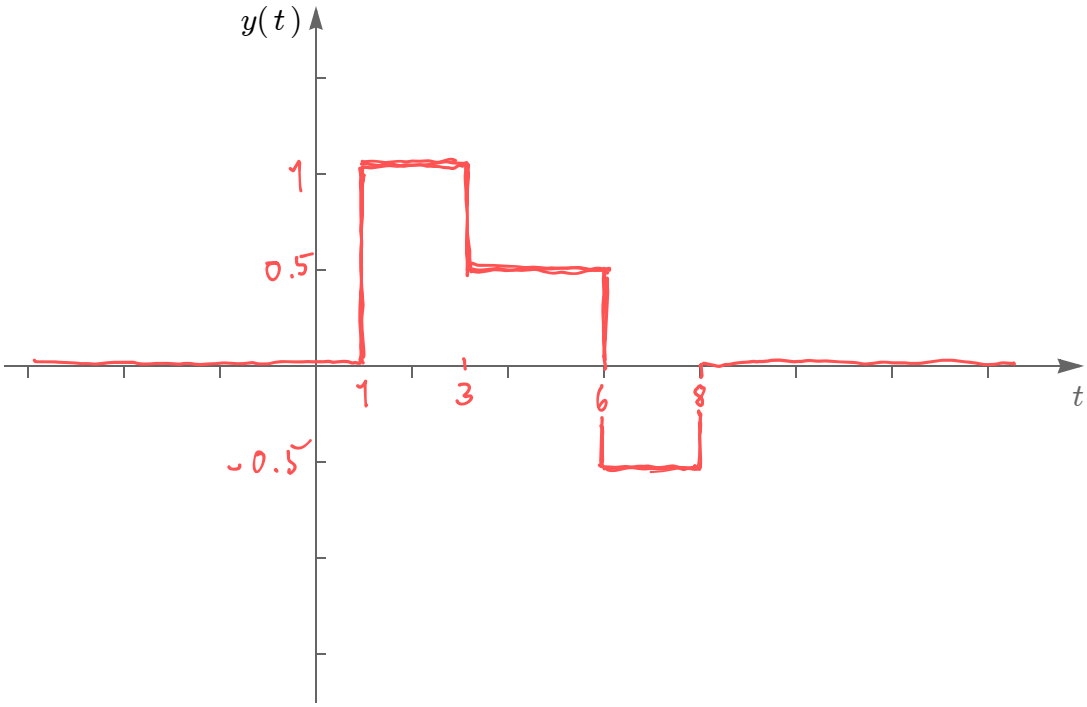
or $f_0 =$ 0.4 Hz

or $f_0 =$ 0.6 Hz.

$f_0 = \frac{k}{5}$ for any integer k.

(b) In the space below, carefully sketch the output $y(t)$ of this system when the input is $x(t) = \delta(t-1) - 0.5\delta(t-3)$, taking care to carefully label important times and important signal heights:





PROBLEM 4.

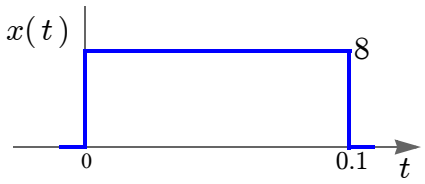
(a) Evaluate the integral $\int_{-\infty}^{5} \frac{1}{1+t^2}\delta(t-3)dt =$ 0.1 .

Integrand reduces to $\frac{1}{1+3^2}\delta(t-3) = 0.1\,\delta(t-3)$

$$\Rightarrow \int_{-\infty}^{5} 0.1\,\delta(t-3)\,dt = 0.1$$

(b) The energy of the rectangular pulse $x(t)$ shown here is

$E =$ 6.4 .



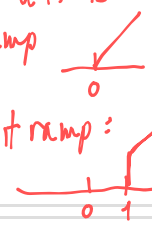
$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{0.1} 8^2\,dt$$

$$= 6.4$$

PROBLEM 5. (20 points)

Shown below are the relationships between the input $x(t)$ and output $y(t)$ of two systems. Specify which properties listed on the left are satisfied by each: (*Brief* explanations are OK!)

	SYSTEM#1: $y(t) = tx(t)$	SYSTEM#2: $y(t) = x(\cos(t))$
Linear?	YES [X] NO [ ] response to $\alpha x_1(t) + \beta x_2(t)$ is $\alpha y_1(t) + \beta y_2(t)$	YES [X] NO [ ] response to $\alpha x_1(t) + \beta x_2(t)$ is $\alpha y_1(t) + \beta y_2(t)$
Time Invariant?	YES [ ] NO [X] response to $u(t)$ is unit ramp; response to $u(t-1)$ is not a delayed unit ramp: 	YES [ ] NO [X] response to $u(t)$ is nonzero:  response to $u(t-2)$ is zero, not a delayed version of
Memoryless?	YES [X] NO [ ] output at a given moment depends only on current input	YES [ ] NO [X] time zero: $y(0) = x(1)$
Invertible?	YES [ ] NO [ ] All of $x(t)$ except for $x(0)$ can be recovered using $x(t) = \frac{y(t)}{t}$; therefore either answer was accepted	YES [ ] NO [X] since $\lvert\cos(t)\rvert \le 1$, only $x(t)$ for $\lvert t\rvert \le 1$ can be recovered from $y(t)$
Causal?	YES [X] NO [ ] memoryless implies causal	YES [ ] NO [X] future $y(0) = x(1)$
Stable?	YES [ ] NO [X] unit step input is bounded unit ramp output is not	YES [X] NO [ ] if $x(t)$ for all t is bounded then $x(t)$ for $\lvert t\rvert \le 1$ is too

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