ECE 3084

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

GEORGIA INSTITUTE OF TECHNOLOGY

OCTOBER 4, 2018

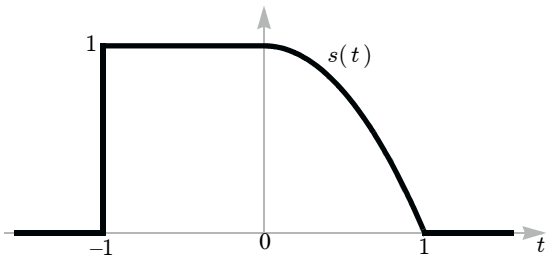
Name: _____

1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

PROBLEM 1. (20 points)

Consider the signal $s(t) = \begin{cases} 1, & -1 < t < 0 \\ 1 - t^2, & 0 < t < 1 \\ 0, & |t| > 1 \end{cases}$, as shown here:



(a) Evaluate the integral $\int_{-\infty}^{\infty} s(t)\delta(1-2t)dt =$ [ ].

(b) The signal $s(t)$ has energy $E =$ [ ], and power $P =$ [ ].

(c) The *odd* part of $s(t)$ is $s_o(t) =$

(specify missing functions of t)

$$\begin{cases} \boxed{}, & -1 < t < 0 \\ \boxed{}, & 0 < t < 1 \\ 0, & |t| > 1 \end{cases}$$

.

(d) Consider a system whose output $y(t)$ is related to its input $x(t)$ by $y(t) = x(t) + s(t)$.
In other words, the system *adds* the above $s(t)$ to its input. This system is (circle all that apply):

[causal][memoryless][stable][linear][time invariant][LTI][invertible].

PROBLEM 2. (20 points)

Consider the signal $x(t)$ shown at the top:

The remaining signals are labeled A through L.
The time scales are identical for all plots.
The scale of the y-axis is not specified.

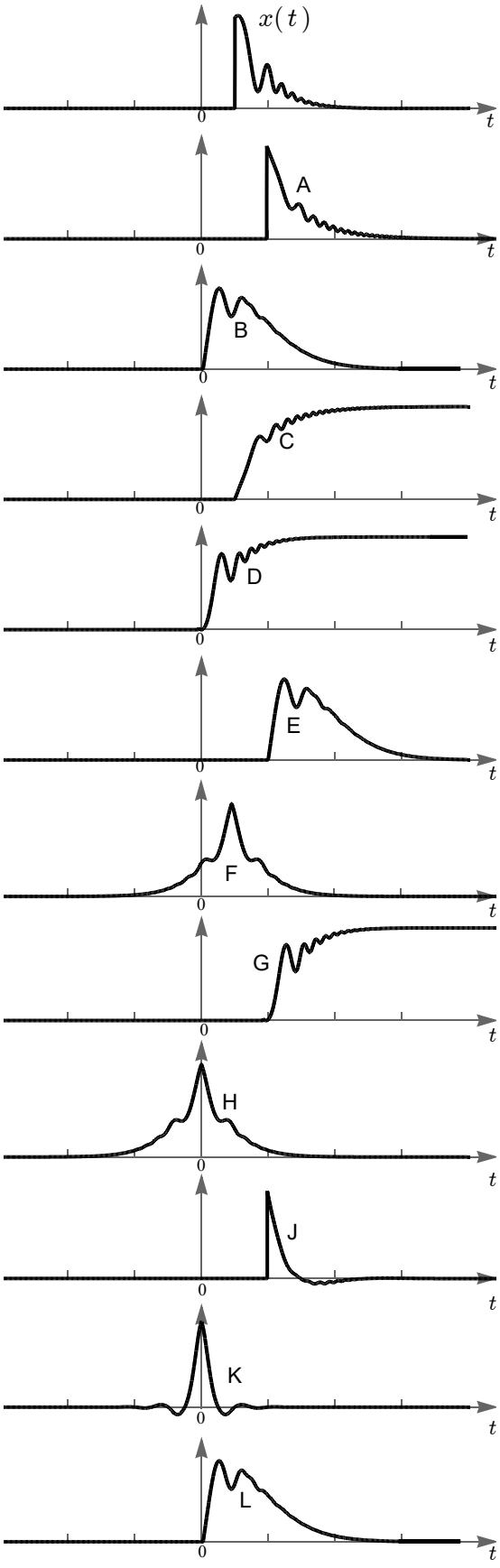
Match each equation below to its plot.
Identify your answers by writing a letter
from {A, B, ... L} into each answer box.

(a) ☐ $y(t) = x(t) * x(t)$,
the convolution of $x(t)$ with itself.

(b) Explain your reasoning.

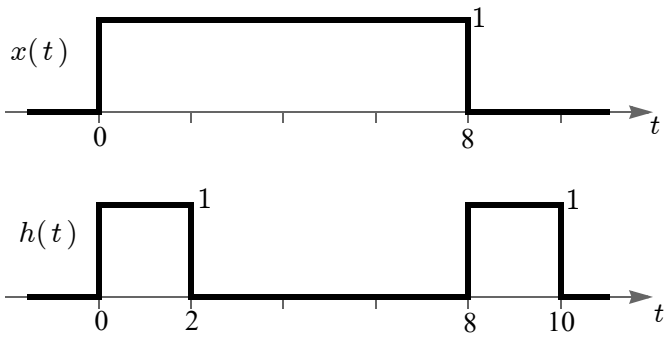
(c) ☐ $R_{xx}(t) = x(t) * x(-t)$.

(d) Explain your reasoning.



PROBLEM 3. (20 points)

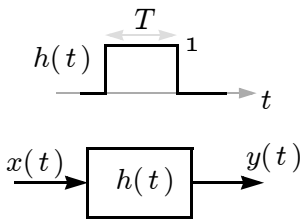
Consider the signals $x(t)$ and $h(t)$ shown below:



In the space below, carefully sketch the convolution $y(t) = x(t) * h(t)$, *carefully labeling important times and amplitudes*:

$y(t)$

PROBLEM 4. (20 points)



Consider an LTI system with impulse response
$h(t) = u(t) - u(t - T)$, a rectangle whose width T is unspecified:

The two parts below consider different input signals.

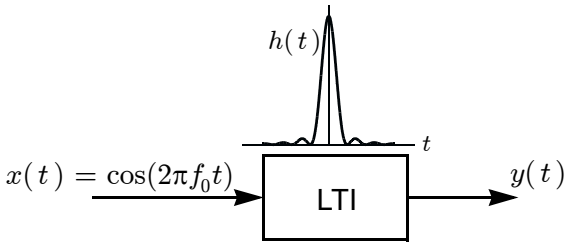
(a) Sketch the filter output in response to the input signal $x(t) = h(-t)$, *carefully labeling both axes* in terms of the unknown T:

(b) There are many values of T for which an input of the form $x(t) = 3084 + \sum_{k=7}^{3084} \cos(0.1k\pi t)$ results in a *constant* output. Name any three:

$T =$ [ ], or $T =$ [ ], or $T =$ [ ]

PROBLEM 5. (20 points)

Suppose a sinusoidal signal $x(t) = \cos(2\pi f_0 t)$ with some unspecified frequency $f_0 > 0$ is fed as an input into an LTI system whose impulse response is $h(t) = \left(\frac{\sin(16\pi t)}{\pi t}\right)^2$, as shown here:



(a) Evaluated at time 0, the impulse response is $h(0) =$ ____.

(b) Specify **all** values of f_0 (in Hz) for which the output is the zero signal, $y(t) = 0$ for all time:

$f_0 \in$ ____

(c) If the output has the specific form $y(t) = \cos(2\pi f_0 t + \theta)$, then it must be that:

$f_0 =$ ____ Hz, and $\theta =$ ____ radians.

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PROBLEM 1. (20 points)

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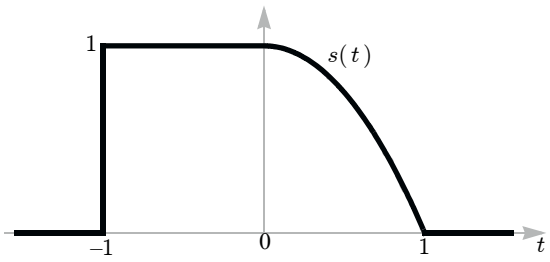


(a) Evaluate the integral $\int_{-\infty}^{\infty} s(t)\delta(1-2t)dt =$ $\frac{3}{8}$.

(b) The signal $s(t)$ has energy $E =$ $\frac{23}{15}$, and power $P =$ 0.

(c) The *odd* part of $s(t)$ is $s_o(t) =$ (specify missing functions of t)

$$\begin{cases} \frac{t^2}{2}, & -1 < t < 0 \\ \frac{-t^2}{2}, & 0 < t < 1 \\ 0, & |t| > 1 \end{cases}$$

(d) Consider a system whose output $y(t)$ is related to its input $x(t)$ by $y(t) = x(t) + s(t)$. In other words, the system *adds* the above $s(t)$ to its input. This system is (circle all that apply):

[causal][memoryless][stable][linear][time invariant][LTI][invertible].

PROBLEM 2. (20 points)

Consider the signal $x(t)$ shown at the top:

The remaining signals are labeled A through L.
The time scales are identical for all plots.
The scale of the y-axis is not specified.

Match each equation below to its plot.
Identify your answers by writing a letter
from {A, B, ... L} into each answer box.

(a) E $y(t) = x(t) * x(t)$,
the convolution of $x(t)$ with itself.

(b) Explain your reasoning.

Only A, E, G, and J start at time 2.

Of these, only E and G are zero there.

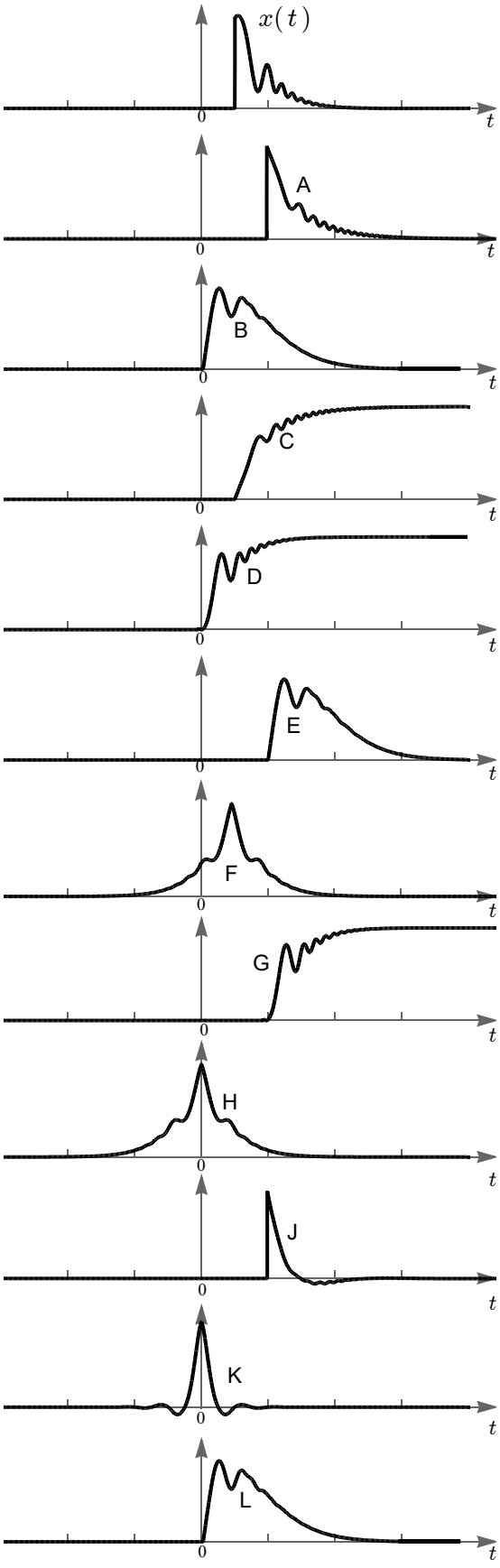
Of these, only E goes to 0 as $t \to \infty$

(c) H $R_{xx}(t) = x(t) * x(-t)$.

(d) Explain your reasoning.

Only H and K are even.

Of these, only H is everywhere positive.



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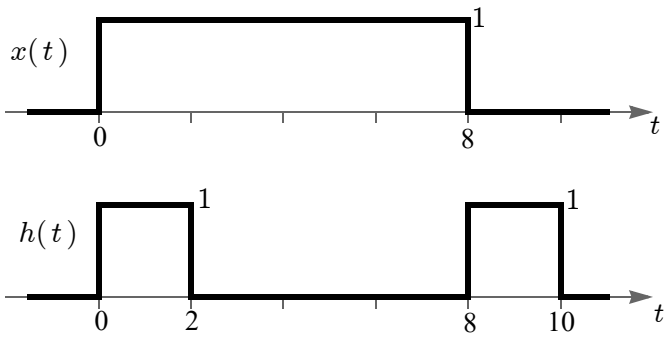

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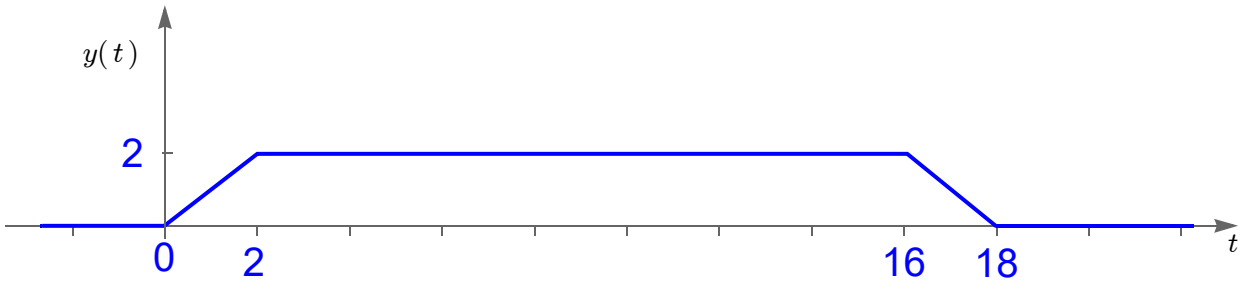

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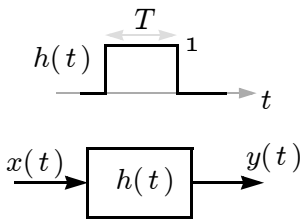

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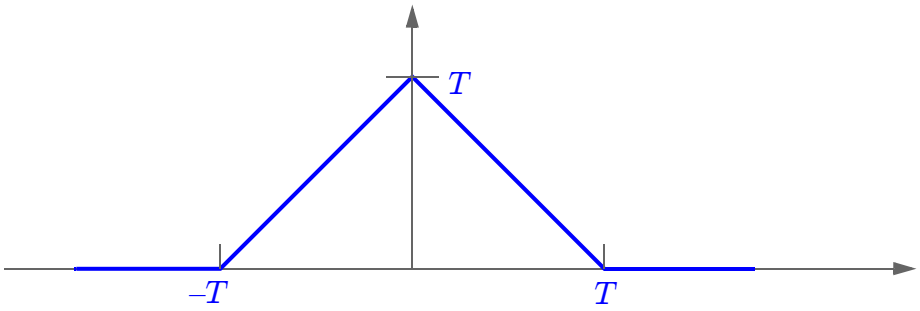
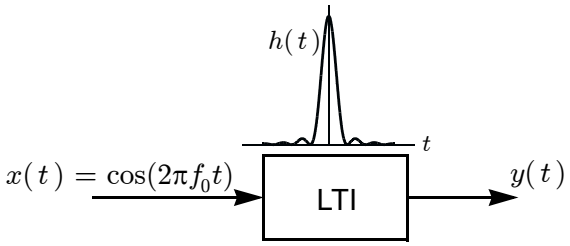


(b) There are many values of T for which an input of the form $x(t) = 3084 + \sum_{k=7}^{3084} \cos(0.1k\pi t)$ results in a *constant* output. Name any three:

$T =$ 20 , or $T =$ 40 , or $T =$ 60

PROBLEM 5. (20 points)

Suppose a sinusoidal signal $x(t) = \cos(2\pi f_0 t)$ with some unspecified frequency $f_0 > 0$ is fed as an input into an LTI system whose impulse response is $h(t) = \left(\frac{\sin(16\pi t)}{\pi t}\right)^2$, as shown here:



(a) Evaluated at time 0, the impulse response is $h(0) =$ $16^2 = 256$.

(b) Specify **all** values of f_0 (in Hz) for which the output is the zero signal, $y(t) = 0$ for all time:

$f_0 \in$ $|f_0| > 16$ Hz

(c) If the output has the specific form $y(t) = \cos(2\pi f_0 t + \theta)$, then it must be that:

$f_0 =$ 15 Hz, and $\theta =$ 0 radians.