ECE 3084

QUIZ 1

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

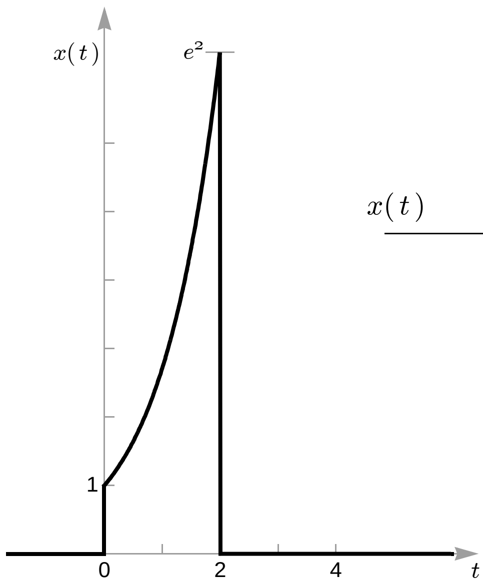
OCTOBER 5, 2017

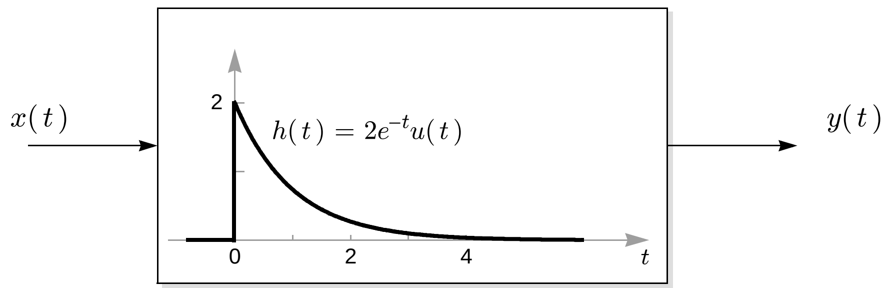
Name: ____

1. The quiz is closed book, closed notes, except for one 2-sided sheet of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL:	100	

PROBLEM 1. Suppose the signal $x(t) = e^t(u(t) - u(t-2))$ is fed as an input to an LTI system whose impulse response is $h(t) = 2e^{-t}u(t)$, as shown below:





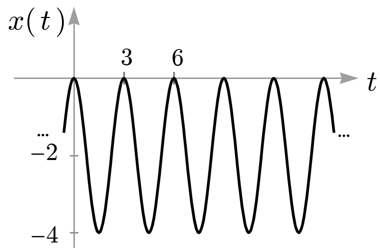
The energy of the input signal is: $E =$.

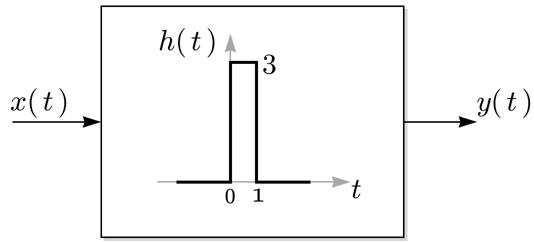
The power of the input signal is: $P =$.

The output $y(t)$ at time $t = 1$ is:[1] $y(1) =$.

1. The problem does not ask for the entire output signal, only the value it takes at time 1. The answer will thus be a real number, not a function of time.

PROBLEM 2. Suppose that a signal $x(t) = -2 + 2\cos(2\pi t/3)$ is passed through an LTI system whose impulse response is rectangular with duration 1 second and height 3, as shown below:





(a) The input is [even][odd][neither] (circle one).

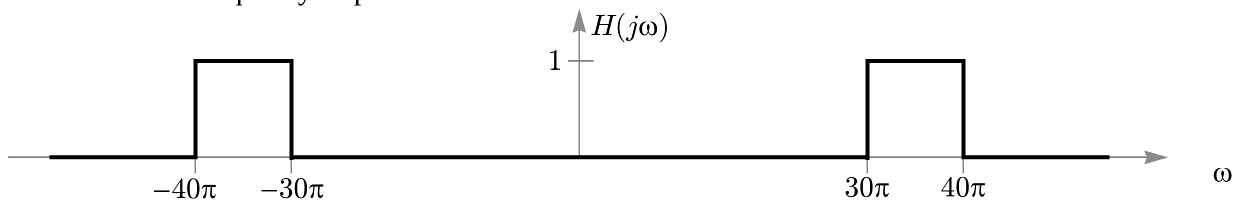
(b) The output can be written as:

$$y(t) = A + B\cos(Ct + D),$$

where

$A =$ ☐ $B =$ ☐ $C =$ ☐ rads/s $D =$ ☐ rads.

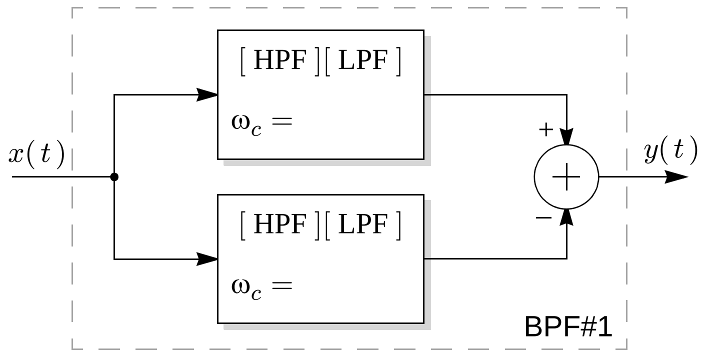
PROBLEM 3. You are an engineer charged with constructing *three* bandpass filters having the following frequency response:



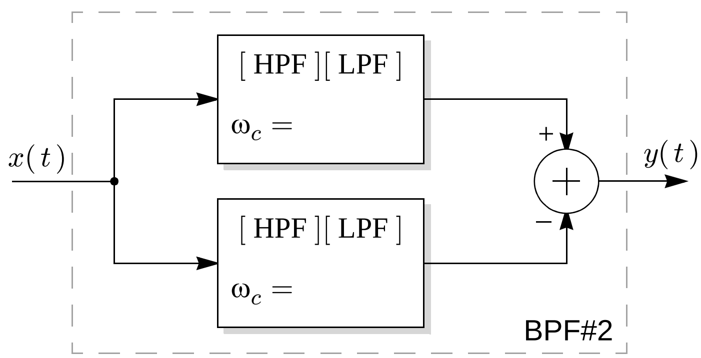
This is a rush job, you have no time to order new parts, so you must do the job with the parts you already have. The only parts available to you are:

- three tunable ideal low-pass filters (LPF's), unity gain
- three tunable ideal high-pass filters (HPF's), unity gain
- two subtractors

So you decide to construct the three BPF's as shown to the right.

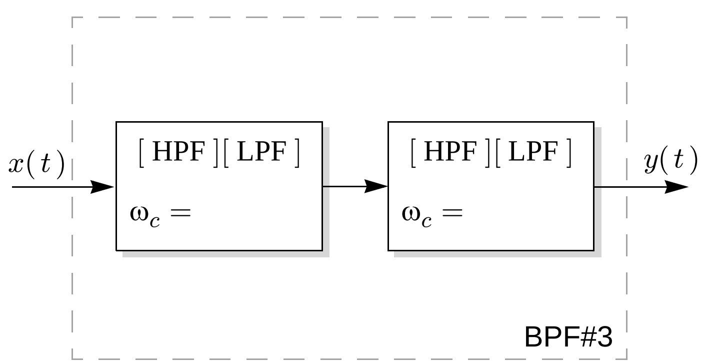


Specify which filters are LPF and which are HPF by circling the appropriate label. (Keep in mind you have only three of each type!)



Furthermore, specify the cutoff frequency ω_c (in rad/s) for each filter by writing it inside each box.[1]

Choose your answers so that all three of the dashed systems (labeled BPF#1, BPF#2, and BPF#3) have the desired frequency response shown at the top of the page.



1. The ideal LPF rejects all frequencies *above* its ω_c, while the ideal HPF rejects all frequencies *below* its ω_c.

PROBLEM 4. The different parts of this problem are unrelated.

(a) The frequency response of a system that delays by 0.1 seconds is:

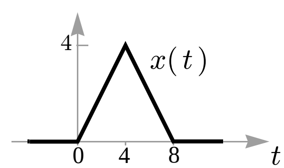
$$H(j\omega) = \boxed{}\,.$$

(b) Evaluate the following integral:

$$\int_{-\infty}^{\infty} \delta(t + 0.1)\sin(2.5\pi t)dt = \boxed{}\,.$$

.

(c) If $x(t)$ is the triangle shown in the figure,
and if $y(t) = x(t) * x(t)$ is the convolution of this triangle with itself,
then the Fourier transform of $y(t)$ can be written as:



$$Y(j\omega) = Ae^{-jB\omega}\left(\frac{\sin(C\omega)}{\omega}\right)^D,$$

where

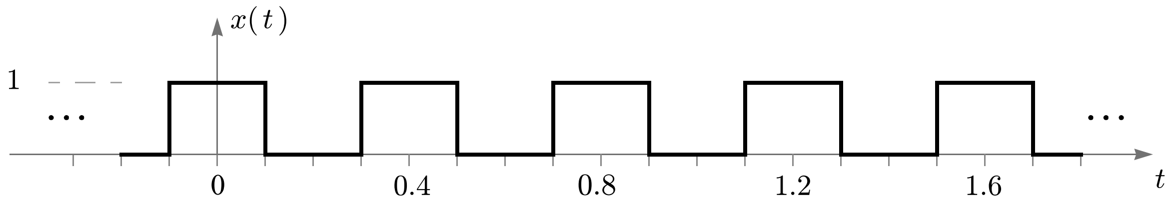
$A = \boxed{}$ $B = \boxed{}$ $C = \boxed{}$ $D = \boxed{}$.

(d) Consider the system $y(t) = (x(t) + x(-t))/2$, whose output $y(t)$ is the [even][odd] part of the input $x(t)$ (circle one!).

Specify whether or not this system satisfies each property below by writing “Y” (for yes) or “N” (for no) into each answer box:

memoryless	causal	stable	linear	time-invariant	invertible

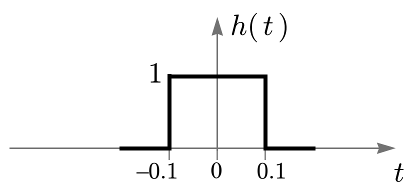
PROBLEM 5. Consider the periodic signal $x(t)$ shown below:



We saw in class that the k-th Fourier series coefficient for this signal is $a_k = \dfrac{\sin(0.5k\pi)}{k\pi}$.

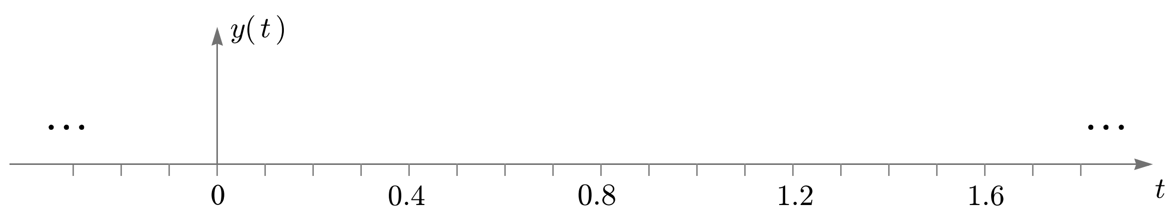
(a) The fundamental frequency of $x(t)$ is $f_0 =$ $\boxed{}$ Hz.

Suppose this $x(t)$ is fed as an input to an LTI filter whose impulse response is shown below:



(b) YES ☐ NO ☐ The filter output $y(t)$ will be periodic with the same fundamental frequency f_0.

(c) In the space below, carefully sketch the filter output $y(t)$:



(d) In the Fourier series representation $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk2\pi f_o t}$ of the filter output, the *first* coefficient (*i.e.*, b_k when $k = 1$) is:

$b_1 =$ $\boxed{}$

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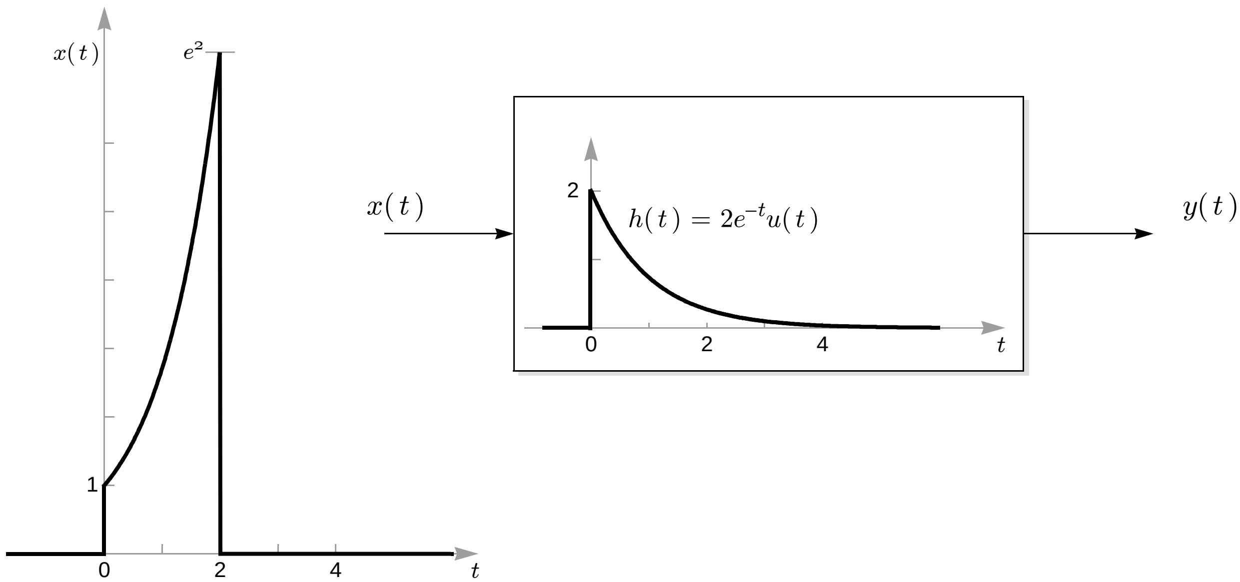

The energy of the input signal is:

$$E = \int_0^2 e^{2t} dt = \tfrac{1}{2}(e^4 - 1)$$

$E = \boxed{\dfrac{e^4 - 1}{2}}$.

The power of the input signal is:

$P = \boxed{0}$.

The output $y(t)$ at time $t = 1$ is:[1]

$$\begin{aligned} y(1) &= \int_0^1 h(\tau)x(1-\tau)d\tau \\ &= 2\int_0^1 e^{-\tau}e^{1-\tau}d\tau \\ &= 2e\int_0^1 e^{-2\tau}d\tau \\ &= e - e^{-1} \end{aligned}$$

$y(1) = \boxed{e - e^{-1}}$.

1. The problem does not ask for the entire output signal, only the value it takes at time 1. The answer will thus be a real number, not a function of time.

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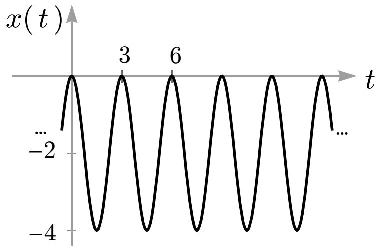

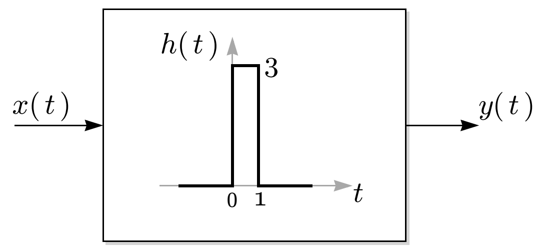

(a) The input is [even][odd][neither] (circle one).

(b) The output can be written as:

$$y(t) = A + B\cos(Ct + D),$$

where

$A = \boxed{-6}$ $B = \boxed{\dfrac{9\sqrt{3}}{\pi}}$ $C = \boxed{\dfrac{2\pi}{3}}$ rads/s $D = \boxed{\dfrac{-\pi}{3}}$ rads.

Sinusoid-in, sinusoid-out at same freq $\Rightarrow C = \dfrac{2\pi}{3}$.

$$H(j\omega) = 3e^{-j\omega/2}\frac{\sin(\omega/2)}{\omega/2} \qquad \Rightarrow H(j0) = 3$$

$$\Rightarrow H(j\frac{2\pi}{3}) = 3e^{-j\pi/3}\frac{\sin(\pi/3)}{\pi/3} = \frac{9\sqrt{3}}{2\pi}e^{-j\pi/3}$$

$$\Rightarrow A = (-2)H(j0) = -6$$

$$\Rightarrow B = (2)|H(j\frac{2\pi}{3})| = \frac{9\sqrt{3}}{\pi},\ D = \text{angle}\{H(j\frac{2\pi}{3})\} = \frac{-\pi}{3}$$

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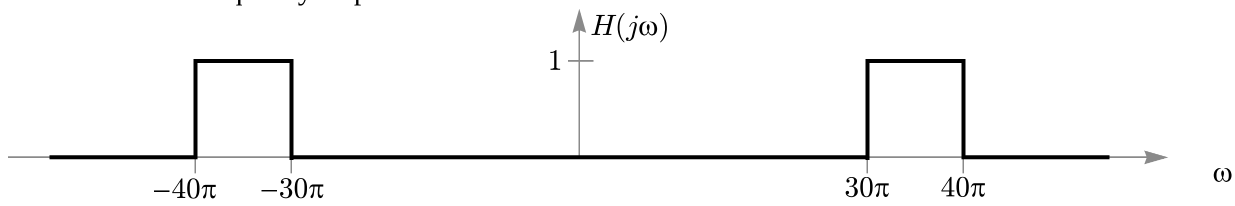

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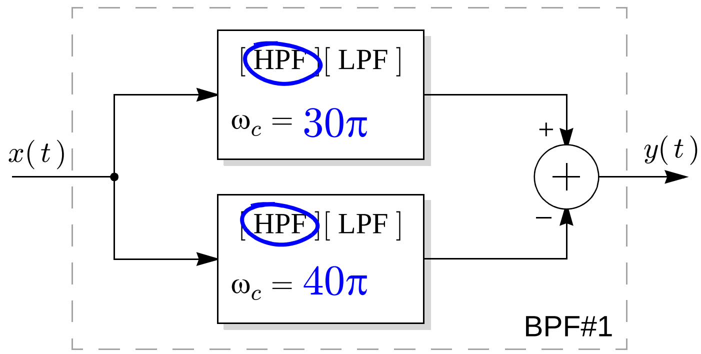

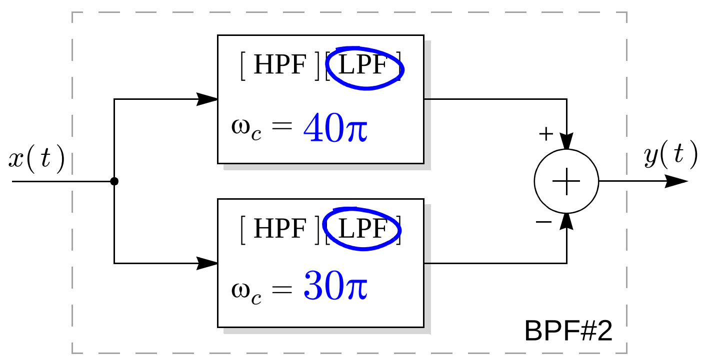

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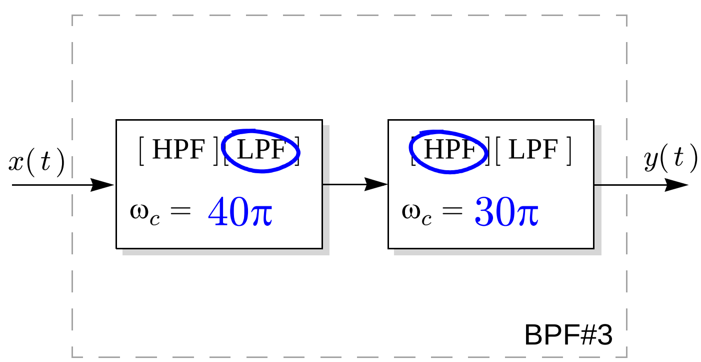

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(a) The frequency response of a system that delays by 0.1 seconds is:

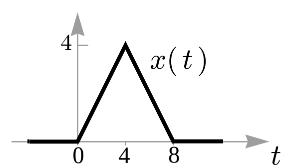
$$H(j\omega) = \boxed{e^{-j0.1\omega}}\ .$$

(b) Evaluate the following integral:

$$\int_{-\infty}^{\infty} \delta(t + 0.1)\sin(2.5\pi t)dt = \boxed{\sin\left(\frac{-\pi}{4}\right) = \frac{-1}{\sqrt{2}}}\ .$$

.

(c) If $x(t)$ is the triangle shown in the figure, and if $y(t) = x(t) * x(t)$ is the convolution of this triangle with itself, then the Fourier transform of $y(t)$ can be written as:



$$Y(j\omega) = Ae^{-jB\omega}\left(\frac{\sin(C\omega)}{\omega}\right)^D,$$

where

$A = \boxed{16}$ $B = \boxed{8}$ $C = \boxed{2}$ $D = \boxed{4}$.

(d) Consider the system $y(t) = (x(t) + x(-t))/2$, whose output $y(t)$ is the [even][odd] part of the input $x(t)$ (circle one!).

Specify whether or not this system satisfies each property below by writing "Y" (for yes) or "N" (for no) into each answer box:

memoryless	causal	stable	linear	time-invariant	invertible
N	N	Y	Y	N	N

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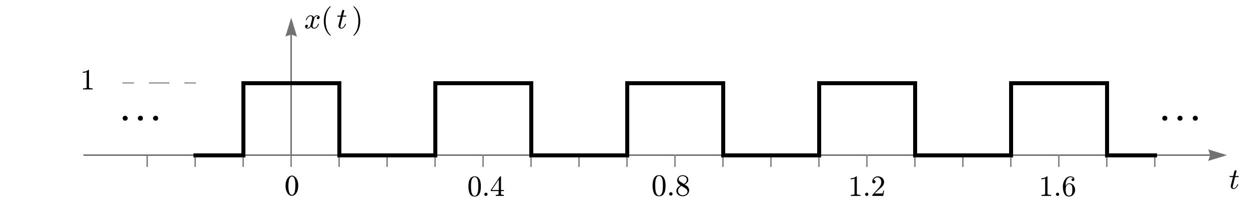

We saw in class that the k-th Fourier series coefficient for this signal is $a_k = \dfrac{\sin(0.5k\pi)}{k\pi}$.

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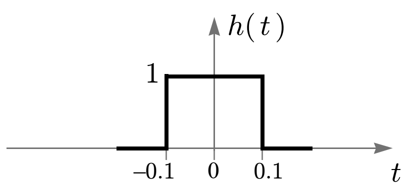
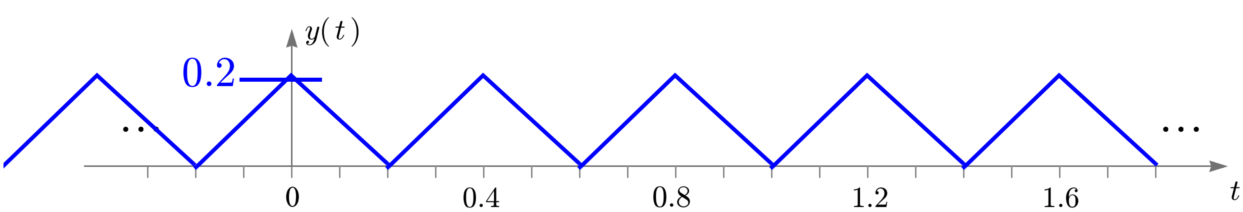


$$\longleftrightarrow \quad H(j\omega) = \frac{\sin(0.1\omega)}{\omega/2}$$

(b) YES ☒ NO ☐ The filter output $y(t)$ will be periodic with the same fundamental frequency f_0.

(c) In the space below, carefully sketch the filter output $y(t)$:



(d) In the Fourier series representation $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk2\pi f_0 t}$ of the filter output, the *first* coefficient (*i.e.*, b_k when $k = 1$) is:

$$b_1 = a_1 H(j5\pi)$$

$$= \frac{1}{\pi} \cdot \frac{\sin(0.5\pi)}{2.5\pi}$$

$$= \frac{1}{2.5\pi^2}$$

$b_1 = \boxed{\dfrac{1}{2.5\pi^2}}$