GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 25-Sep-13 COURSE: ECE 3084A (Prof. Michaels)

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

- Write your name on the front page ONLY. Do not unstaple the test. You should have an additional sheet with Fourier transform tables.
- No calculators, laptops, phones, or other electronic devices allowed. Keep the desks clear of all backpacks, books, etc.
- This is a *closed book* exam. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes is permitted; it is OK to write on both sides.
- Unless stated otherwise, justify your reasoning clearly to receive any partial credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.
- The room is small for the number of students in this section. **BE CAREFUL TO NOT LET YOUR EYES WANDER.** Any sort of communication with your fellow students during this exam is strictly forbidden. Any attempt to read off of your neighbor's exam will result in unpleasant disciplinary action.
- Good luck!

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem Q1.1:

(5 pts each) Consider the four systems below for which the input/output relation is given for input $x(t)$ and output $y(t)$. Consider whether or not each system is **linear, time-invariant, causal, or memoryless**, and circle all that apply (if any). Circle "None" if the system does not have any of these properties.

(a)

$$y(t) = \frac{1}{1 + x(t+3)}$$

Linear Time-Invariant Causal Memoryless None

(b)

$$y(t) = (t+3)x(t-3)$$

Linear Time-Invariant Causal Memoryless None

(c)

$$y(t) = x(|t|) + |x(t)|$$

Linear Time-Invariant Causal Memoryless None

(d)

$$y(t) = x(\cos(t^2))$$

Linear Time-Invariant Causal Memoryless None

Problem Q1.2:

(5 pts each) The four parts of this problem are unrelated to each other.

(a) Simplify the expression $\int_{t}^{\infty} \tau[\delta(\tau+1) - \delta(\tau-3)]d\tau =$

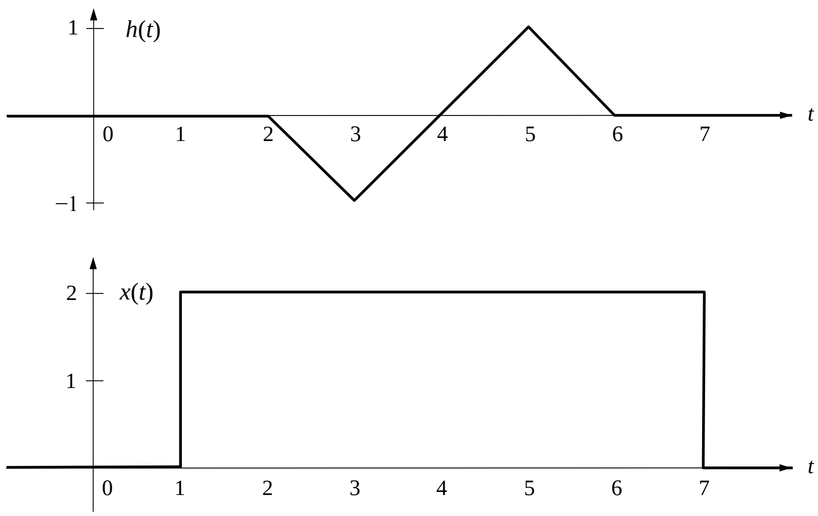
(b) Simplify the expression $\frac{d}{dt}\Big[\sin(3\pi t/4)\,[u(t) - u(t-2)]\Big] =$

(c) Find $X(j\omega)$, the Fourier transform of $x(t) = 10\sin(5t + \pi/4)$.

(d) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = \frac{j\omega}{5 + j\omega}$.

Problem Q1.3:

(5 pts each) In the following graphs, $h(t)$ is the impulse response of an LTI system and $x(t)$ is the input; the output of the system is $y(t)$.



(a) At what times does the output $y(t)$ *start* and *stop*?

$t_{\text{start}} =$

$t_{\text{stop}} =$

(b) What is the *complete* set of values of t for which $y(t)$ is exactly zero?

(c) Find $y(4)$.

(d) What is $y_{\max}$, the largest positive value of $y(t)$, and at what time $t_{\max}$ does it occur?

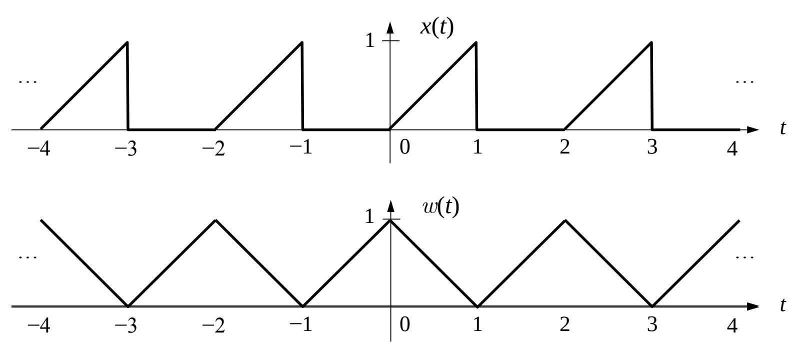
$y_{\max} =$

$t_{\max} =$

Problem Q1.4:

(a) (5 pts) Consider a general periodic signal $x(t)$ with period T_0 and Fourier series coefficients a_k. Show that the Fourier series coefficients b_k of the signal $y(t) = x(-t)$ are $b_k = a_k^*$.

(b) (5 pts) Now let $x(t)$ be the specific signal shown below with known Fourier series coefficients $a_0 = 0.25$ and $a_k = \dfrac{j(-1)^k}{2\pi k} - \dfrac{1-(-1)^k}{2\pi^2 k^2}$ for $k \neq 0$; $w(t)$ is the triangular signal below $x(t)$.

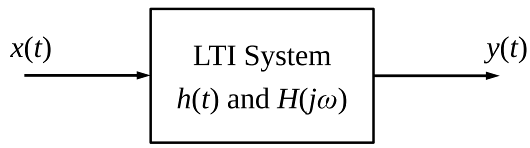


Express $w(t)$ as a sum of delayed, flipped and scaled versions of $x(t)$.

(c) (10 pts) Use the results of parts (a) and (b) along with other properties of Fourier series to obtain c_k, the Fourier series coefficients of $w(t)$.

Problem Q1.5:

Consider the general LTI system shown below with impulse response $h(t)$, frequency response $H(j\omega)$, input $x(t)$ and output $y(t)$. Let $h(t) = e^{-(t+1)}u(t+1) - e^{-(t-1)}u(t-1)$.



(a) (5 pts) Show that $H(j\omega) = \dfrac{2j\sin(\omega)}{1+j\omega}$.

(b) (10 pts) Find the output $y(t)$ for $x(t) = 5\cos(t) + 10\sin(2\pi t) + 3\delta(t-2)$.

(c) (5 pts) Now consider an input $x(t)$ that is a *completely general* periodic signal with period T_0. Specify at least *two* specific values of T_0 for which $y(t) \equiv 0$.

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None of these systems are memryless because they look into either the past or future.

Problem Q1.1:

(5 pts each) Consider the four systems below for which the input/output relation is given for input $x(t)$ and output $y(t)$. Consider whether or not each system is **linear, time-invariant, causal, or memoryless**, and circle all that apply (if any). Circle "None" if the system does not have any of these properties.

(a)

$$y(t) = \frac{1}{1 + x(t+3)}$$

Linear   (Time-Invariant)   Causal   Memoryless   None

non-linear because of reciprocal

$x(t+3)$ looks into the future

(b)

$$y(t) = (t+3)x(t-3)$$

(Linear)   Time-Invariant   (Causal)   Memoryless   None

$(t+3)$ makes it time-invariant

looks into the past

(c)

$$y(t) = x(|t|) + |x(t)|$$

Linear   Time-Invariant   Causal   Memoryless   (None)

$|x(t)|$ is nonlinear

$x(|t|)$ is time-varying

counter example: $y(-1) = x(1) + |x(-1)|$ ↑ looks into the future

(d)

$$y(t) = x(\cos(t^2))$$

(Linear)   Time-Invariant   Causal   Memoryless   None

$\cos(t^2)$ depends upon time

Counterexample $y(t = -\sqrt{2\pi}) = x(\cos(2\pi)) = x(1)$

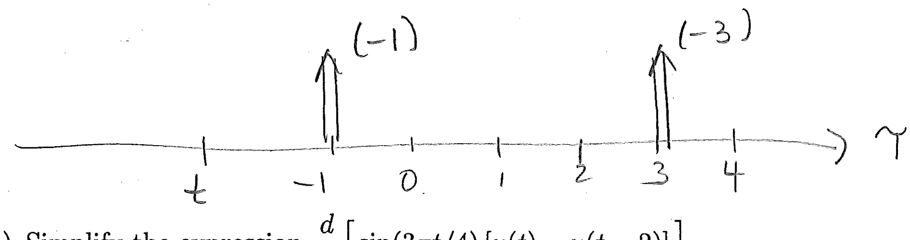
looks into the future

Problem Q1.2:

(5 pts each) The four parts of this problem are unrelated to each other.

(a) Simplify the expression $\int_t^{\infty} \tau[\delta(\tau+1) - \delta(\tau-3)]d\tau =$ $\int_t^{\infty} [-\delta(\tau+1) - 3\delta(\tau-3)]\, d\tau$

$= -u(-(t+1)) - 3u(-(t-3)) = -4 + u(t+1) + 3u(t+3)$



(b) Simplify the expression $\frac{d}{dt}\Big[\sin(3\pi t/4)\,[u(t) - u(t-2)]\Big] =$

$\sin\left(\frac{3\pi t}{4}\right)[\delta(t) - \delta(t-2)] + \frac{3\pi}{4}\cos\left(\frac{3\pi t}{4}\right)[u(t) - u(t-2)]$

$= +\delta(t-2) + \frac{3\pi}{4}\cos\left(\frac{3\pi t}{4}\right)[u(t) - u(t-2)]$

(c) Find $X(j\omega)$, the Fourier transform of $x(t) = 10\sin(5t + \pi/4)$.

$x(t) = \frac{10}{2j}\left[e^{j\pi/4}e^{j5t} - e^{-j\pi/4}e^{-j5t}\right]$

$X(j\omega) = -5j \cdot 2\pi\left[e^{j\pi/4}\delta(\omega-5) - e^{-j\pi/4}\delta(\omega+5)\right]$

$= 10\pi e^{-j\pi/4}\delta(\omega-5) + 10\pi e^{j\pi/4}\delta(\omega+5)$

(d) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = \frac{j\omega}{5 + j\omega}$.

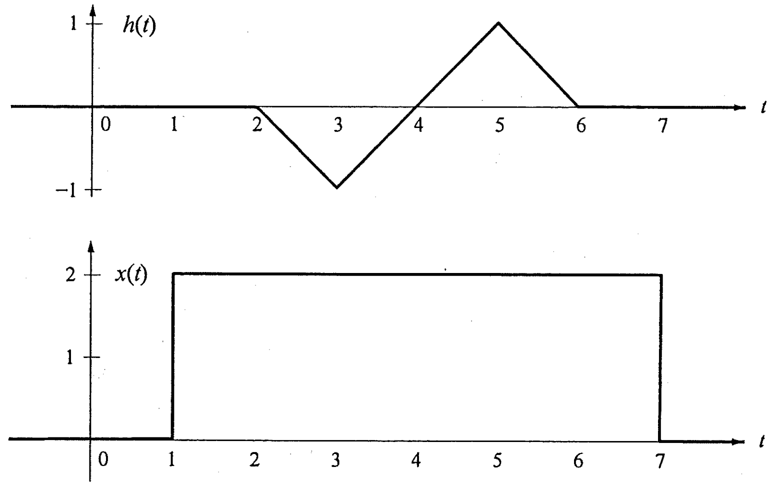
$x(t) = \frac{d}{dt}\left[e^{-5t}u(t)\right]$

$= e^{-5t}\delta(t) - 5e^{-5t}u(t)$

$= \delta(t) - 5e^{-5t}u(t)$

Problem Q1.3:

(5 pts each) In the following graphs, $h(t)$ is the impulse response of an LTI system and $x(t)$ is the input; the output of the system is $y(t)$.



(a) At what times does the output $y(t)$ *start* and *stop*?

$t_{\text{start}} =$ 3

$t_{\text{stop}} =$ 13

(b) What is the *complete* set of values of t for which $y(t)$ is exactly zero?

$t \leq 3$
$t \geq 13$
$7 \leq t \leq 9$

$t-1=6, \; t=7$
$t-7=2, \; t=9$

(c) Find $y(4)$. The flipped $x(t-\tau)$ overlaps $h(t)$ from 2 to 3.

$y(4) = -\frac{1}{2}(2) = -1$

(d) What is y_{max}, the largest positive value of $y(t)$, and at what time t_{max} does it occur?

$y_{\text{max}} =$ $1(2) = 2$

$t_{\text{max}} =$ 11

Max value occurs when $x(t-\tau)$ overlaps $h(t)$ from 4 to 6.

$t-7=4$
$t=11$

Problem Q1.4:

(a) (5 pts) Consider a general periodic signal $x(t)$ with period T_0 and Fourier series coefficients a_k. Show that the Fourier series coefficients b_k of the signal $y(t) = x(-t)$ are $b_k = a_k^*$.

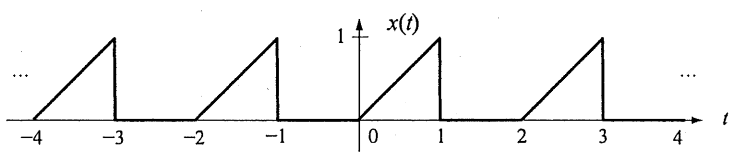
$$b_k = \frac{1}{T_0}\int_0^{T_0} x(-t) e^{-j\omega_0 k t}\,dt \qquad \text{let } \tau = -t$$

$$= \frac{1}{T_0}\int_0^{-T_0} x(\tau) e^{+j\omega_0 k \tau}(-d\tau)$$

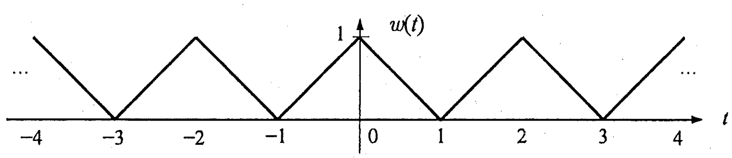
$$= \frac{1}{T_0}\int_{-T_0}^{0} x(\tau) e^{j\omega_0 k \tau}\,d\tau = a_k^*$$

$\leftarrow$ can integrate over any period

(b) (5 pts) Now let $x(t)$ be the specific signal shown below with known Fourier series coefficients $a_0 = 0.25$ and $a_k = \dfrac{j(-1)^k}{2\pi k} - \dfrac{1-(-1)^k}{2\pi^2 k^2}$ for $k \neq 0$; $w(t)$ is the triangular signal below $x(t)$.



$$\omega_0 = \frac{2\pi}{T_0} = \pi$$



Express $w(t)$ as a sum of delayed, flipped and scaled versions of $x(t)$.

$w(t) = x(t+1) + x(-(t+1))$ (other shifts are possible)

or $w(t) = x(t-1) + x(-(t-1))$

(c) (10 pts) Use the results of parts (a) and (b) along with other properties of Fourier series to obtain c_k, the Fourier series coefficients of $w(t)$.

$$c_k = (a_k + a_k^*)e^{-j\omega_0 k} \qquad \text{time shift property}$$

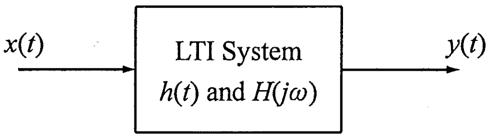
$$= 2\,\mathrm{Re}(a_k)e^{-j\pi k}$$

$$= \frac{(-1)^k - 1}{\pi^2 k^2}(-1)^k \qquad c_0 = a_0 + a_0 = 2a_0 = 0.5$$

$$= \frac{1-(-1)^k}{\pi^2 k^2}\;;\; k \neq 0$$

Problem Q1.5:

Consider the general LTI system shown below with impulse response $h(t)$, frequency response $H(j\omega)$, input $x(t)$ and output $y(t)$. Let $h(t) = e^{-(t+1)}u(t+1) - e^{-(t-1)}u(t-1)$.



(a) (5 pts) Show that $H(j\omega) = \dfrac{2j\sin(\omega)}{1+j\omega}$.

$$H(j\omega) = \frac{1}{1+j\omega} e^{j\omega} - \frac{1}{1+j\omega} e^{-j\omega}$$

$$= \frac{2j \sin(\omega)}{1+j\omega}$$

(b) (10 pts) Find the output $y(t)$ for $x(t) = 5\cos(t) + 10\sin(2\pi t) + 3\delta(t-2)$.

freq domain ↖ time domain

$$H(j1) = \frac{2j \sin(1)}{1+j}$$

$$= \frac{2e^{j\pi/2}\sin(1)}{\sqrt{2}e^{j\pi/4}}$$

$$= \sqrt{2}\sin(1) e^{j\pi/4}$$

$$H(j2\pi) = 0$$

$$y(t) = 5\sqrt{2}\sin(1)\cos(t + \pi/4)$$
$$3e^{-(t-1)}u(t-1) - 3e^{-(t-3)}u(t-3)$$

(c) (5 pts) Now consider an input $x(t)$ that is a *completely general* periodic signal with period T_0. Specify at least *two* specific values of T_0 for which $y(t) \equiv 0$.

$y(t) = 0$ if $H(j\omega) = 0$ for all harmonics of $x(t)$.

This happens if:

$$\omega_0 = n\pi$$

$$\frac{2\pi}{T_0} = n\pi$$

$$T_0 = \frac{2}{n}$$

Two values of T_0 are $T_0 = 2$ $(n=1)$ and $T_0 = 1$ $(n=2)$