ECE 3084

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
MAY 3, 2017

Name: ____

1. The exam is closed book & notes, except for three 2-sided sheets of handwritten notes.
2. Silence your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	10	
2	15	
3	15	
4	10	
5	15	
6	10	
7	10	
8	15	
TOTAL:	100	

PROBLEM 1. (10 points)

Let $X(s) = \dfrac{(1 - e^{-s})^3}{s^2}$.

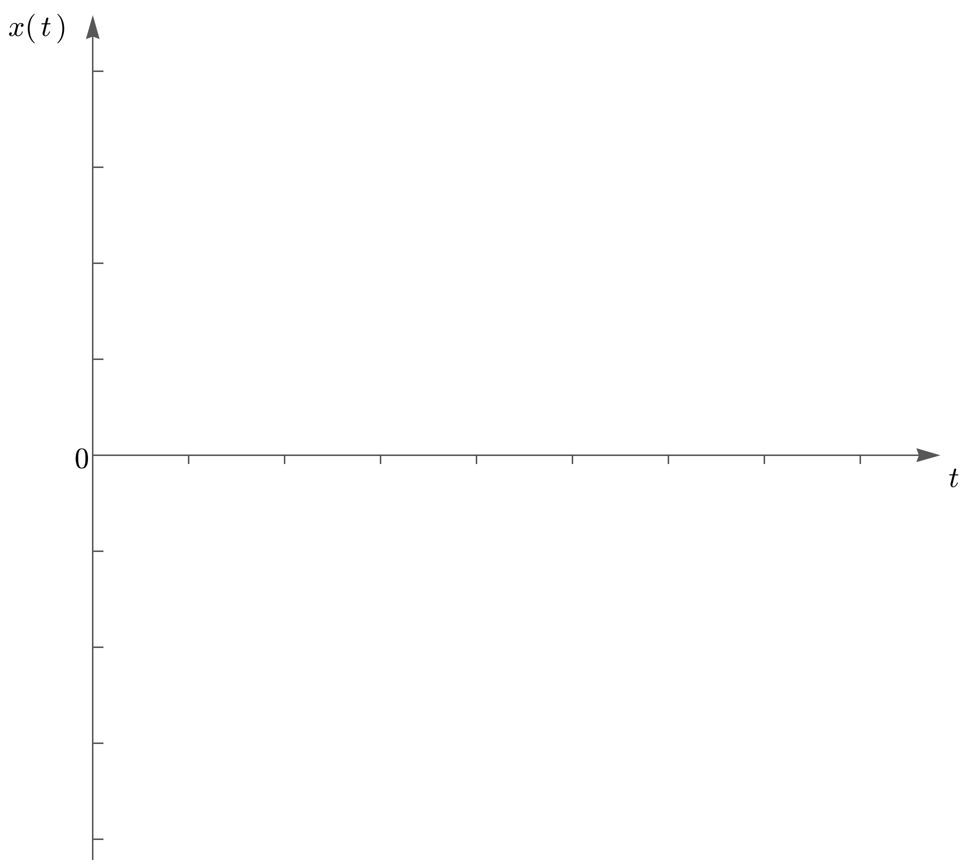
(a) In terms of $r(t) = tu(t)$, the inverse Laplace transform of $X(s)$ can be written as:

$$x(t) = Ar(t) + Br(t - C) + Dr(t - E) + Fr(t - G),$$

where (*hint:* all answers are integers!):

$A =$ ☐ , $B =$ ☐ , $C =$ ☐ , $D =$ ☐ , $E =$ ☐ , $F =$ ☐ , $G =$ ☐

(b) Provide a sketch of $x(t)$ in the space below, *taking care to carefully label both axes*:



PROBLEM 2. (15 points)

Below are three systems with input $x(t)$ and output $y(t)$. Specify which properties they satisfy by writing a “Y” (for yes) or “N” (for no) into each answer box:

(a) $y(t) = 2 + x(t)$

memoryless	causal	stable	linear	time-invariant	invertible

(b) $y(t) = (\pi - e^{-|t|})x(t)$

memoryless	causal	stable	linear	time-invariant	invertible

(c) $y(t) = \int_{-\infty}^{t} x(\tau) e^{-(t-\tau)} d\tau$

memoryless	causal	stable	linear	time-invariant	invertible

PROBLEM 3. (15 points)

Find an equation for the convolution of $x(t) = \left(\frac{\sin(8\pi t)}{\pi t}\right)^2$ with $h(t) = \cos(2\pi t)$, simplified as much as possible (no integrals!):

$$y(t) = x(t) * h(t) = \boxed{}\ .$$

PROBLEM 4. (10 points)

(a) In "I & Q", the "I" stands for ______________, and "Q" stands for ______________.

(b) In twelve words or less, what is "pulse compression"?

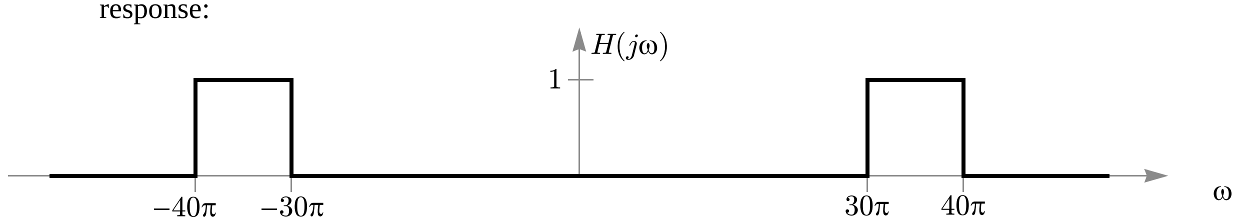
(c) TRUE FALSE

If $x_e(t)$ and $x_o(t)$ are the even and odd parts of a signal $x(t)$, then they must have the same energy: $\int_{-\infty}^{\infty} x_e^2(t)dt = \int_{-\infty}^{\infty} x_o^2(t)dt$.

Explain!

PROBLEM 5. (10 points)

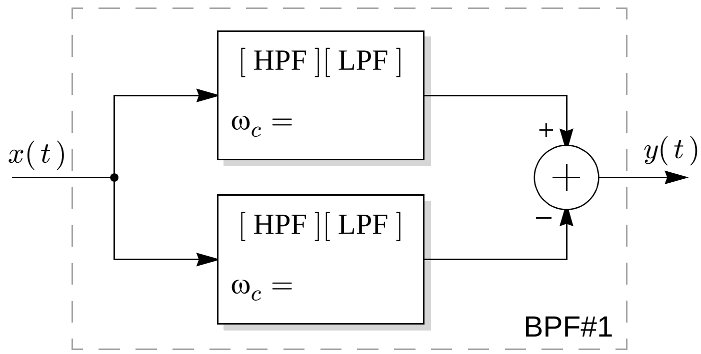
You are an engineer charged with constructing three bandpass filters having the following frequency response:

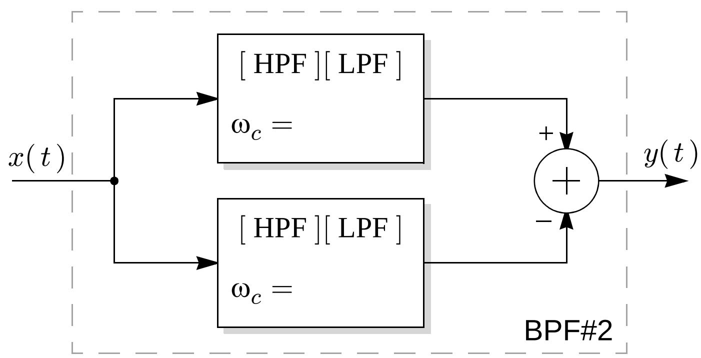


This is a rush job, you have no time to order new parts, so you must do the job with the parts you already have.
The only parts avalable to you are:

- three tunable ideal low-pass filters (LPF's), unity gain
- three tunable ideal high-pass filters (HPF's), unity gain
- two subtractors

So you decide to construct the three BPF's as shown to the right.

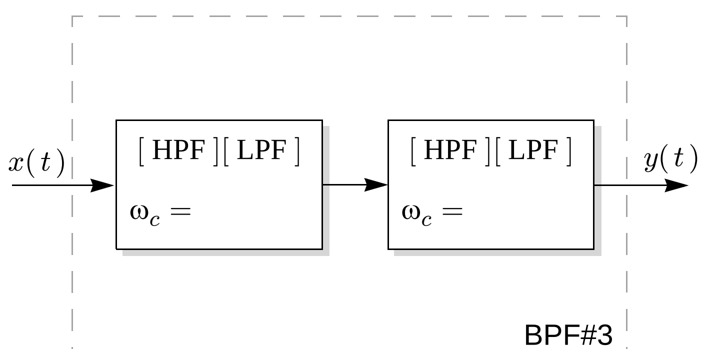




Specify which filters are LPF and which are HPF by circling the appropriate label. (Keep in mind you have only three of each type!)

Furthermore, specify the cutoff frequency ω_c (in rad/s) for each filter by writing it inside each box.[1]

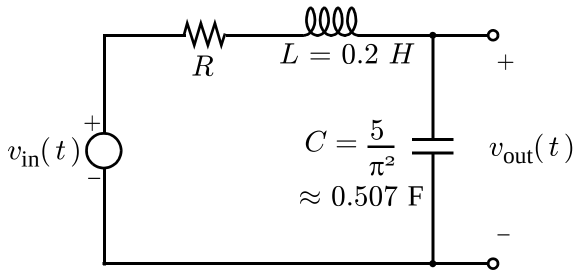
Choose your answers so that all three of the dashed systems (labeled BPF#1, BPF#2, and BPF#3) have the desired frequency response shown at the top of the page.

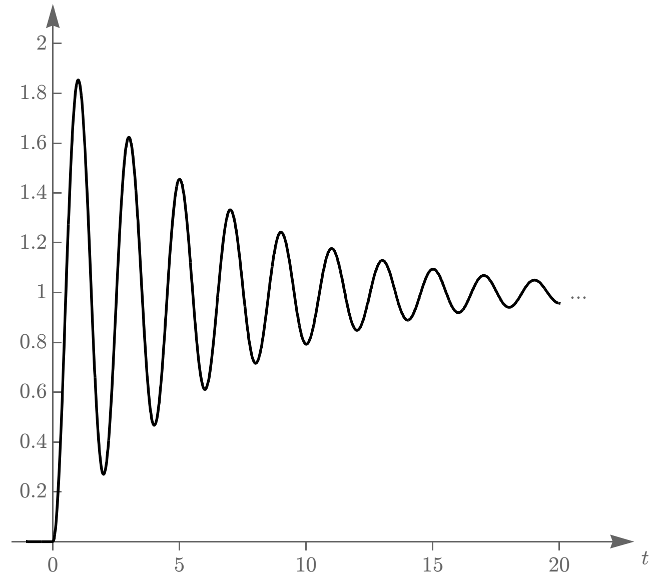


1. The ideal LPF rejects all frequencies *above* its ω_c, while the ideal HPF rejects all frequencies *below* its ω_c.

PROBLEM 6. (10 points)

Shown below is a circuit diagram of a [LPF][BPF][HPF] (circle one), with a $L = 0.2$ H inductor and a $C = 5/\pi^2 \approx 0.507$ F capacitor:



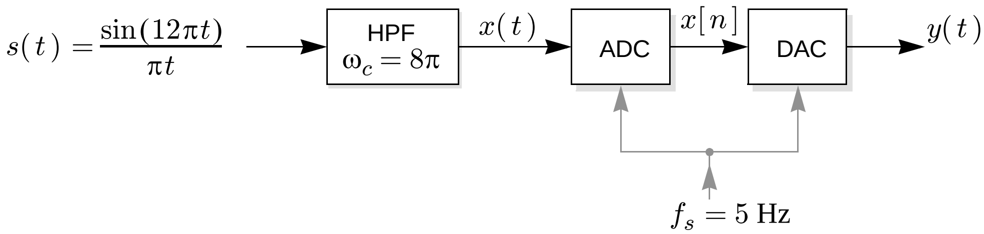


Also shown above is the step response. From this, estimate as close as possible the resistor value:

$R \approx$ ______ Ω.

PROBLEM 7. (10 points)

A sinc function $s(t) = \sin(12\pi t)/(\pi t)$ is passed through an ideal high-pass filter with cutoff 8π (its frequency response is $H(j\omega) = 0$ for $|\omega| < 8\pi$, and $H(j\omega) = 1$ for $|\omega| > 8\pi$). The HPF output $x(t)$ is then passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter, both with sampling rate $f_s = 5$ Hz:

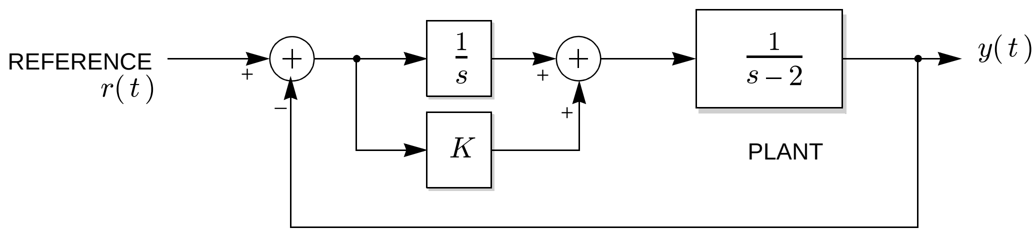


Find an equation for the DAC output $y(t)$, simplified as much as possible:

$y(t) =$ ______ .

PROBLEM 8. (15 points)

Consider the control system shown below:



(a) Sketch the pole-zero plot of the closed-loop system $H(s) = Y(s)/R(s)$ when $K = 0$:

If you find that any of the following are not possible, write "N.P." in the answer box.

(b) The range of values for K that make the closed-loop system *stable* is [ ] .

(c) The closed-loop system is *critically damped* when $K =$ [ ] .

(d) The closed-loop damping ratio is $\zeta = 1/\sqrt{2}$ when $K =$ [ ] .

ECE 3084

FINAL EXAM

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PROBLEM 1. (10 points)

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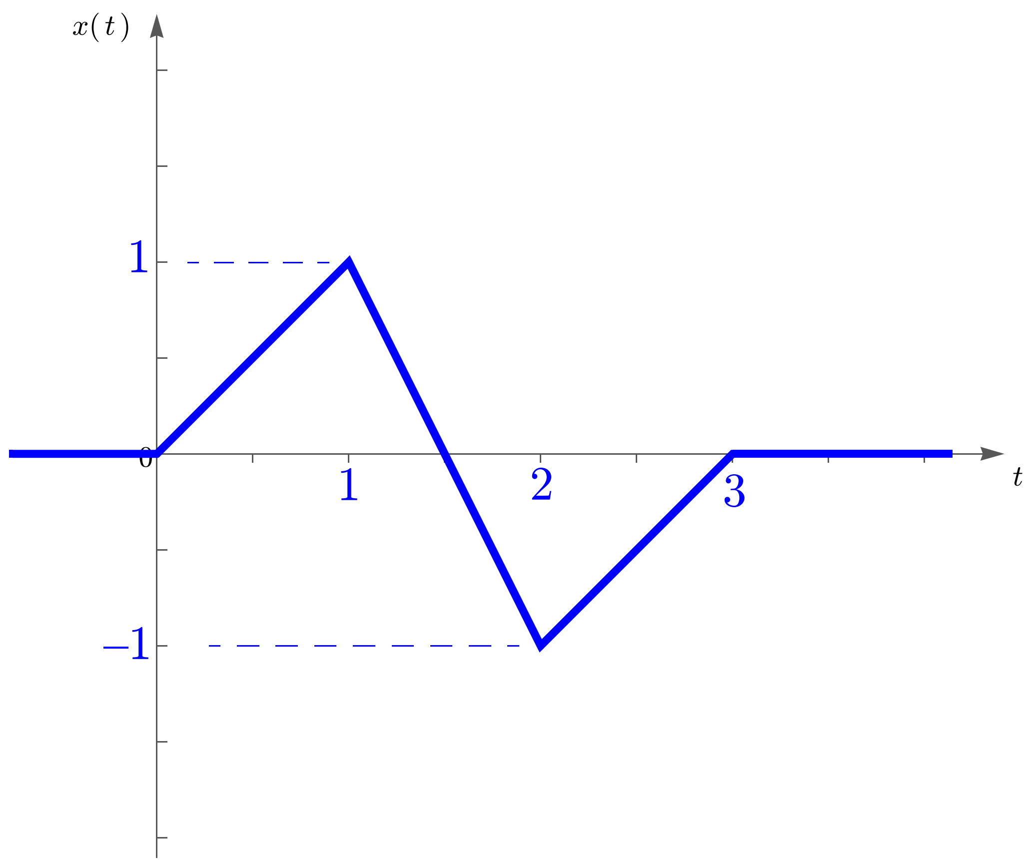
(a) In terms of $r(t) = tu(t)$, the inverse Laplace transform of $X(s)$ can be written as: $x(t) = Ar(t) + Br(t - C) + Dr(t - E) + Fr(t - G)$,

where (*hint:* all answers are integers!):

$A =$ 1 , $B =$ –3 , $C =$ 1 , $D =$ 3 , $E =$ 2 , $F =$ –1 , $G =$ 3

(b) Provide a sketch of $x(t)$ in the space below, *taking care to carefully label both axes*:



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(a) $y(t) = 2 + x(t)$

memoryless	causal	stable	linear	time-invariant	invertible
			N		

(others are yes)

(b) $y(t) = (\pi - e^{-|t|})x(t)$

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				N	

(others are yes)

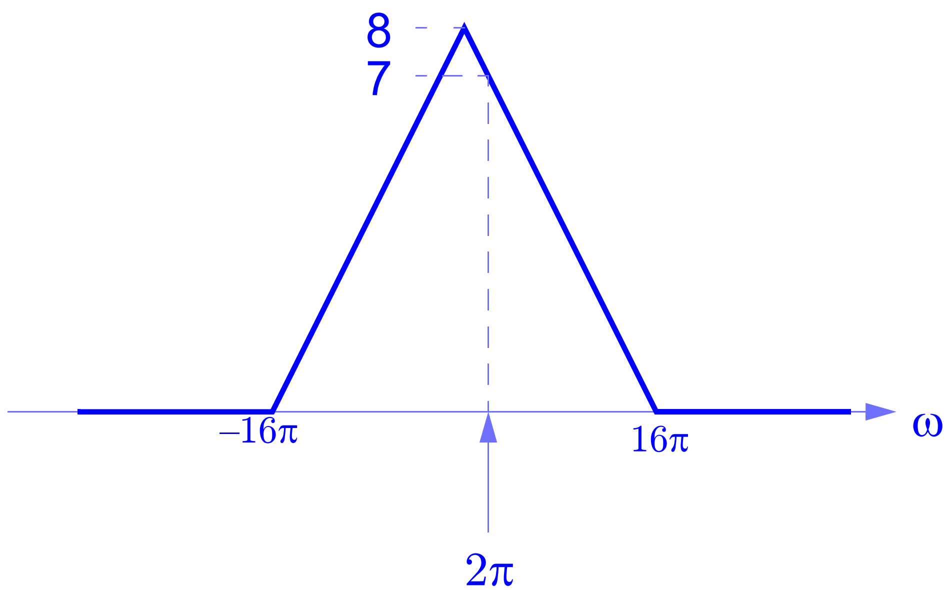
(c) $y(t) = \int_{-\infty}^{t} x(\tau)e^{-(t-\tau)}d\tau$

memoryless	causal	stable	linear	time-invariant	invertible
N					

(others are yes)

PROBLEM 3. (15 points)

Find an equation for the convolution of $x(t) = \left(\frac{\sin(8\pi t)}{\pi t}\right)^2$ with $h(t) = \cos(2\pi t)$, simplified as much as possible (no integrals!):



$y(t) = x(t) * h(t) =$ $7\cos(2\pi t)$.

PROBLEM 4. (10 points)

(a) In "I & Q", the "I" stands for [in-phase], and "Q" stands for [quadrature].

(b) In twelve words or less, what is "pulse compression"?

shortening of pulse "width" via matched filter

(c) TRUE [ ] FALSE [X] If $x_e(t)$ and $x_o(t)$ are the even and odd parts of a signal $x(t)$, then they must have the same energy: $\int_{-\infty}^{\infty} x_e^2(t)dt = \int_{-\infty}^{\infty} x_o^2(t)dt$.

Explain!

For an *even* signal:
the first integral is nonzero, while the second integral is zero.

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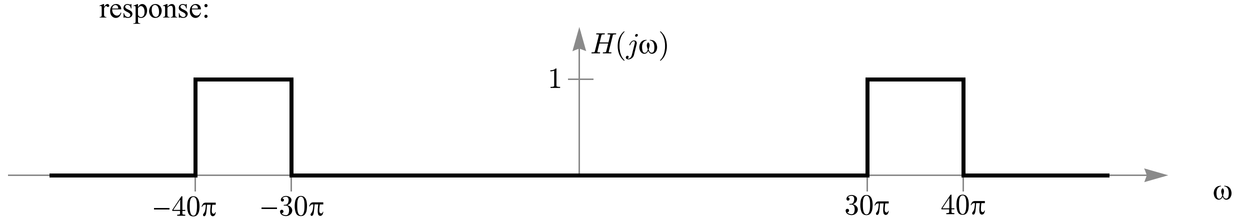
You are an engineer charged with constructing three bandpass filters having the following frequency response:



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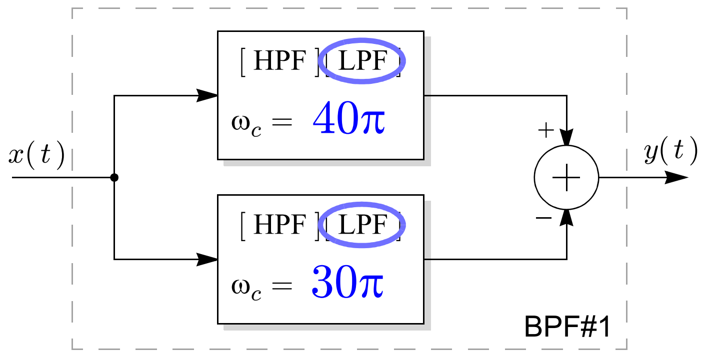

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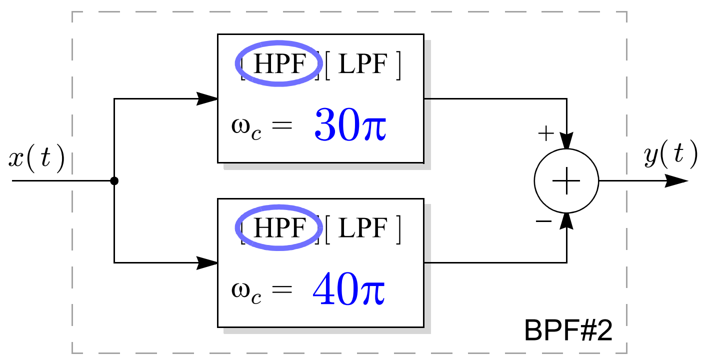

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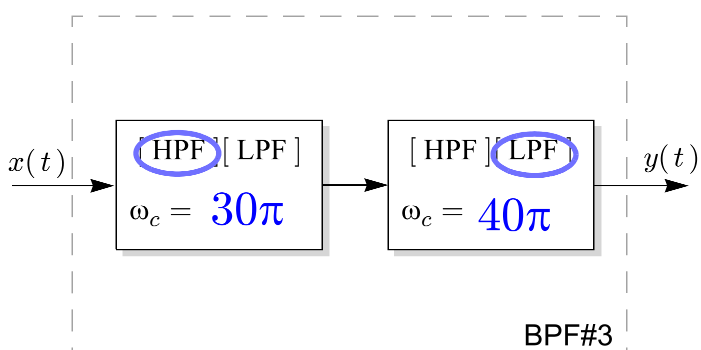

1. The ideal LPF rejects all frequencies *above* its ω_c, while the ideal HPF rejects all frequencies *below* its ω_c.

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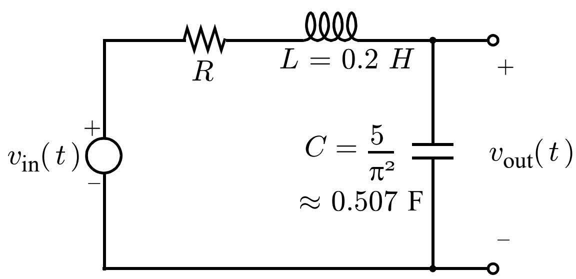
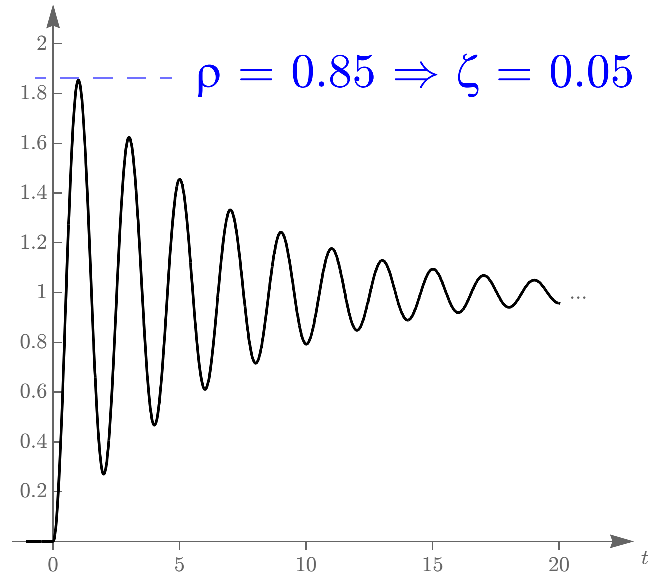




Also shown above is the step response. From this, estimate as close as possible the resistor value:

$R \approx$ 0.063 Ω.

$$H(s) = \frac{1/(sC)}{1/(sC) + R + sL}$$

$$= \frac{1/(LC)}{s^2 + \frac{R}{L}s + 1/(LC)}$$

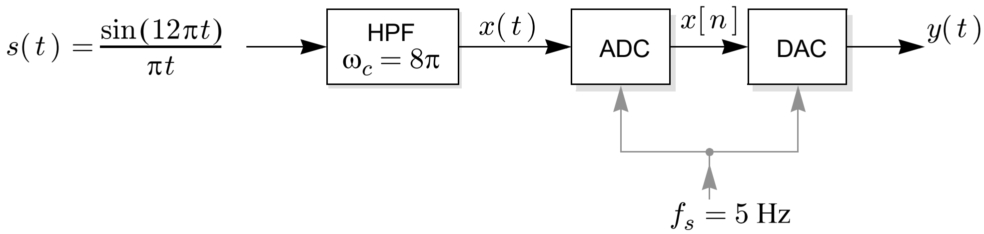
$$= \frac{\pi}{s^2 + 5Rs + \pi^2}$$

$$\Rightarrow 5R = 2\zeta\omega_n = 2\pi\zeta = 0.1\pi$$

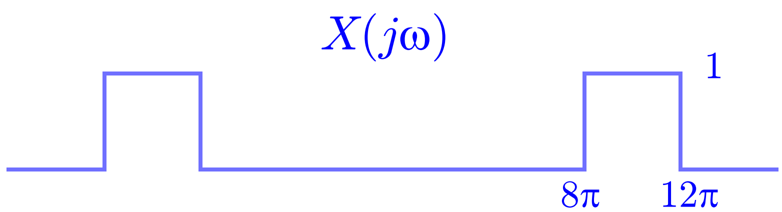
$$\Rightarrow R = 0.02\pi = 0.063.$$

PROBLEM 7. (10 points)

A sinc function $s(t) = \sin(12\pi t)/(\pi t)$ is passed through an ideal high-pass filter with cutoff 8π (its frequency response is $H(j\omega) = 0$ for $|\omega| < 8\pi$, and $H(j\omega) = 1$ for $|\omega| > 8\pi$). The HPF output $x(t)$ is then passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter, both with sampling rate $f_s = 5$ Hz:



Find an equation for the DAC output $y(t)$, simplified as much as possible:



$$\Rightarrow x(t) = 2g(t)\cos(10\pi t)$$

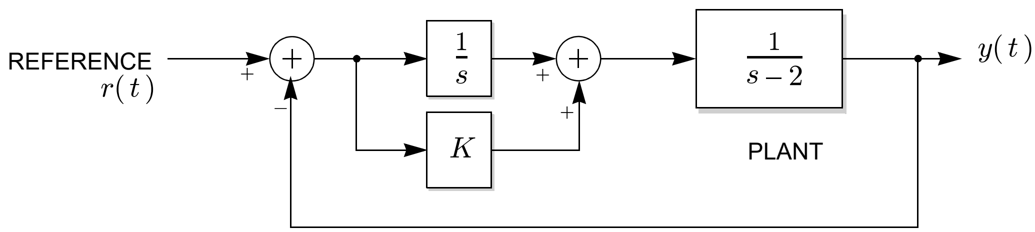
$$\Rightarrow x[n] = 2g(\tfrac{n}{5})\cos(10\pi \tfrac{n}{5})$$

$$= 2g(\tfrac{n}{f_s}) \qquad \Rightarrow y(t) = 2g(t)$$

$y(t) =$ $\boxed{2\dfrac{\sin(2\pi t)}{\pi t}}$.

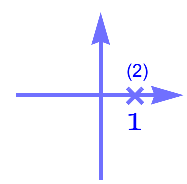
PROBLEM 8. (15 points)

Consider the control system shown below:



(a) Sketch the pole-zero plot of the closed-loop system $H(s) = Y(s)/R(s)$ when $K = 0$:

$$H(s) = \frac{GG}{1+GG} = \frac{K(s+1/K)}{s^2+(K-2)s+1} \overset{K=0}{=} \frac{1}{(s-1)^2}$$



If you find that any of the following are not possible, write "N.P." in the answer box.

(b) The range of values for K that make the closed-loop system *stable* is $K > 2$.

(c) The closed-loop system is *critically damped* when $K =$ 4.

(d) The closed-loop damping ratio is $\zeta = 1/\sqrt{2}$ when $K =$ $2+\sqrt{2}$ ≈ 3.414