ECE 3084

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
MAY 5, 2016

Name: ____

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3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
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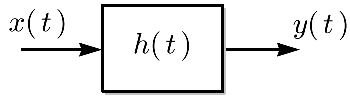
Problem	Points	Score
1	20	
2	15	
3	15	
4	15	
5	15	
6	20	
TOTAL:	100	

PROBLEM 1. (20 points)

Consider an LTI filter with a *rectangular* impulse response is $h(t) = 4(u(t-0.5) - u(t-2.5))$:



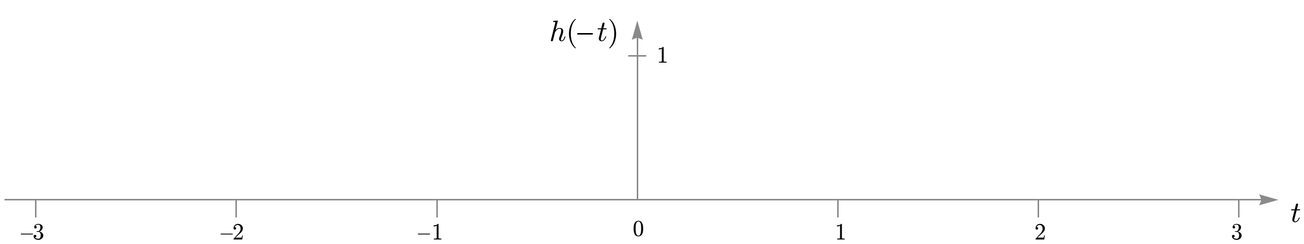
(a) The d.c. gain of the filter is $H_0 =$ ____________ .

(b) The Laplace transform of $h(t)$ is $H(s) =$ ____________ .

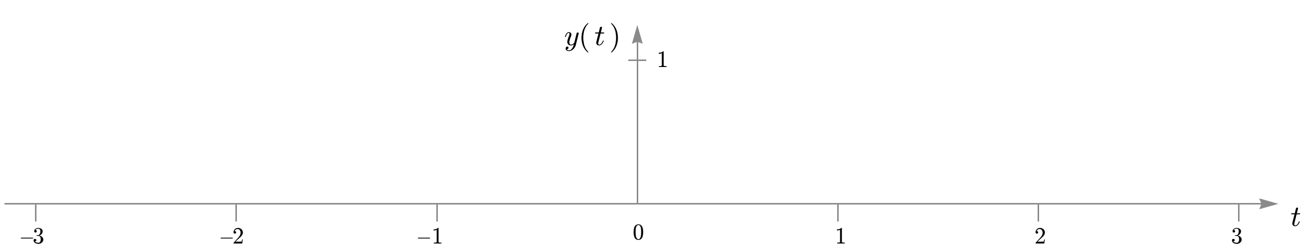
(c) Specify three values of f_0 such that a sinusoidal input of the form $x(t) = \cos(2\pi f_0 t)$ will result in a *zero* output:

$f_0 =$ ____________ , or $f_0 =$ ____________ , or $f_0 =$ ____________ .

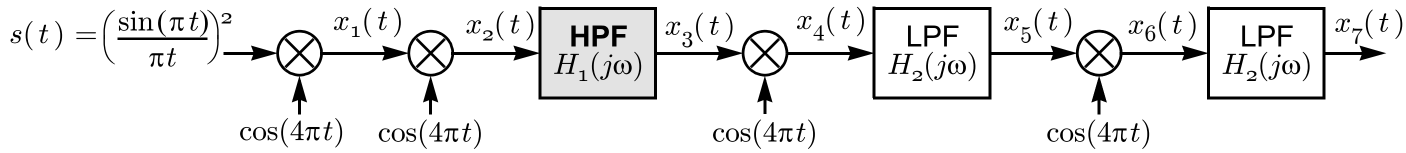
(d) Carefully sketch in the space below the signal $h(-t)$:



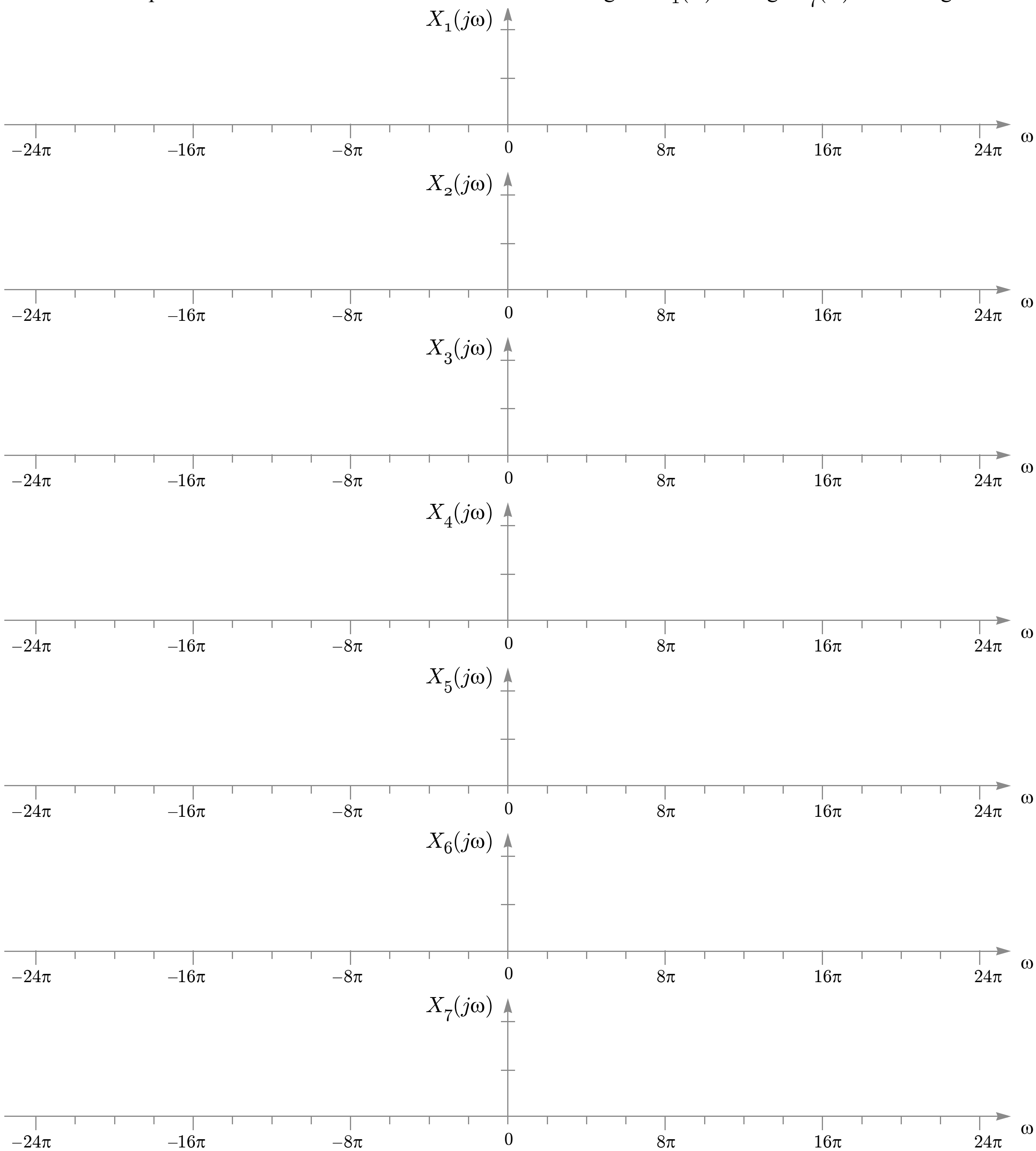
(e) Carefully sketch the output $y(t)$ in the space below for the case when the input is $x(t) = h(-t)$:



PROBLEM 2. Suppose $s(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$ is fed into the following cascade of oscillators and filters:



The lone **HPF** is ideal with unity gain and rejects all frequencies below 4π rad/s.
The two LPF's are ideal with unity gain and reject all frequencies *above* 4π rad/s.
Sketch in the space below the Fourier transform of each of the signals $x_1(t)$ through $x_7(t)$ in the diagram:

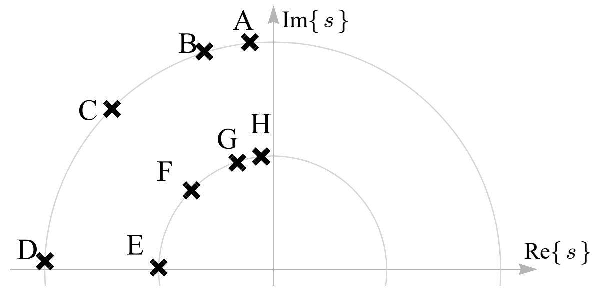


PROBLEM 3. (15 points)

Consider a 2nd-order system with transfer function

$$H(s) = \frac{1}{s^2 + bs + c},$$

where the constants b and c are positive. When $b^2 < 4c$, the two poles will be *complex*:

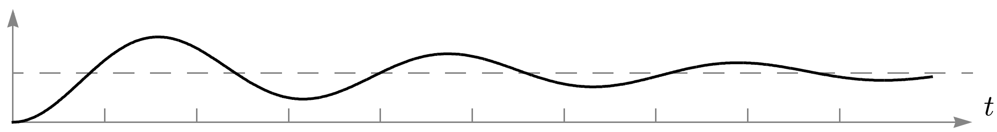


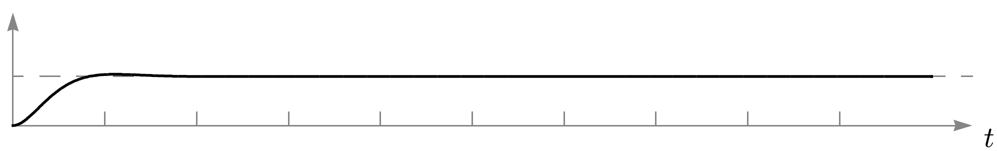
- one pole will be at a location p in the upper half of the s plane;
- the other pole will be at the conjugate location p^* in the lower half of the s plane.

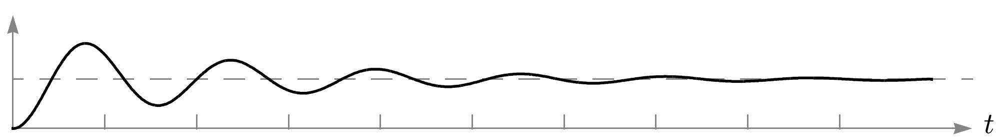
Shown above are eight possible locations for the upper pole p. (The lower pole at p^* is not shown.)

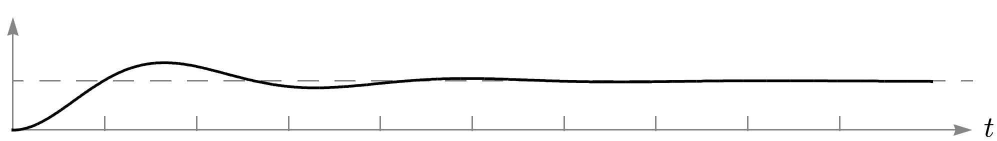
Match each location p above with its corresponding step response below by writing a letter from {A, B, C, D, E, F, G, H} into each answer box below:

(a) .

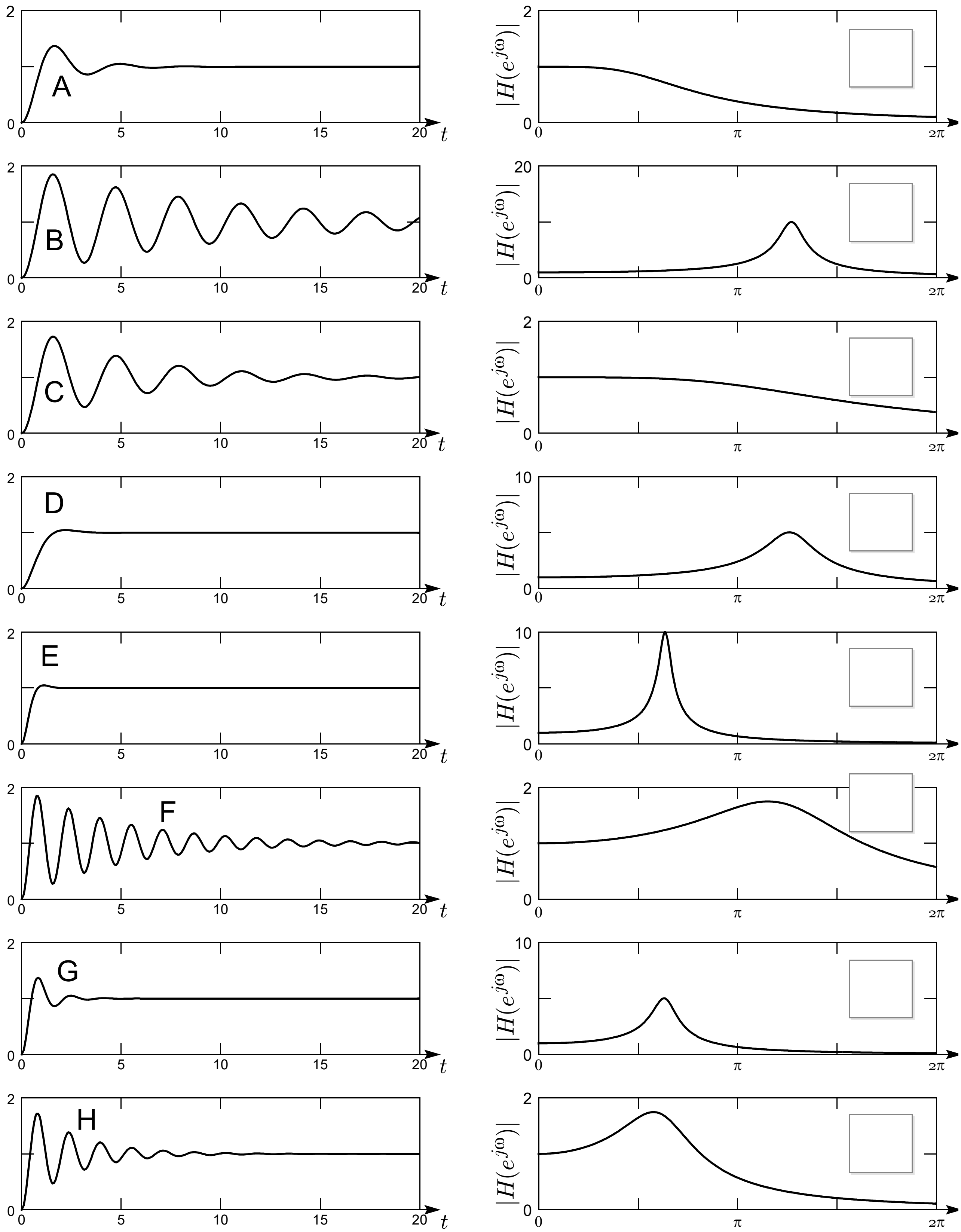






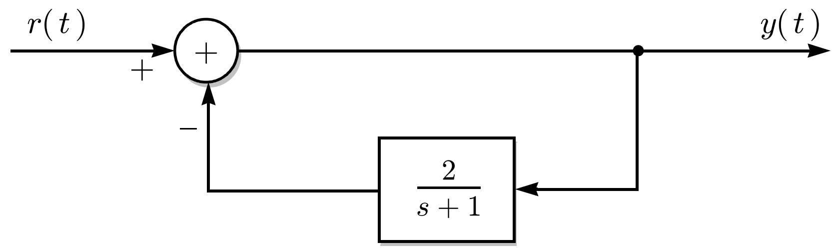


PROBLEM 4. Shown on the left are the step responses of eight second-order low-pass filters, labeled A through H. Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magnitude response to its corresponding step response by writing a letter (A through H) in each answer box.



PROBLEM 5. (15 points)

Consider the following closed-loop system with zero initial conditions:



When the input is the decaying exponential $r(t) = e^{-t}u(t)$, the output can be written as:

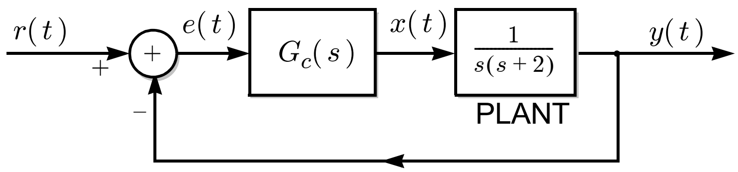
$$y(t) = (Ae^{-t} + Be^{-Ct})u(t),$$

where

$A =$ ☐ $B =$ ☐ $C =$ ☐ .

PROBLEM 6. (20 points)

The diagram below shows feedback control of a plant with transfer function $G_p(s) = \dfrac{1}{s(s+2)}$:



The closed-loop transfer function is $H(s) = Y(s)/R(s)$. The controller can be expressed as:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s.$$

NOTATION:	K_p	K_i	K_d
P:	nonzero	0	0
PI:	nonzero	nonzero	0
PD:	nonzero	0	nonzero
PID:	nonzero	nonzero	nonzero

(a) $H(s)$ is a *third*-order system when using [P][PI][PD][PID] control. (Circle all that apply.)

(b) $H(s)$ is a *second*-order system when using [P][PI][PD][PID] control. (Circle all that apply.)

(c) The d.c. gain is $H(0) = 1$ when using [P][PI][PD][PID] control. (Circle all that apply.)

(d) With P control, oscillations in the closed-loop step response can be avoided by choosing $K_p <$ [ ].

(e) With P control, a damping ratio of $\zeta = 1/\sqrt{2}$ can be achieved by choosing $K_p =$ [ ].

(f) Consider the special case of PD control wheresay $K_p = 4K$ and $K_d = K$.
In this case we can get $\zeta = 1/\sqrt{2}$ by choosing $K =$ [ ].

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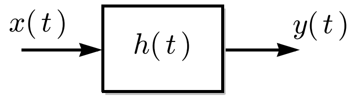


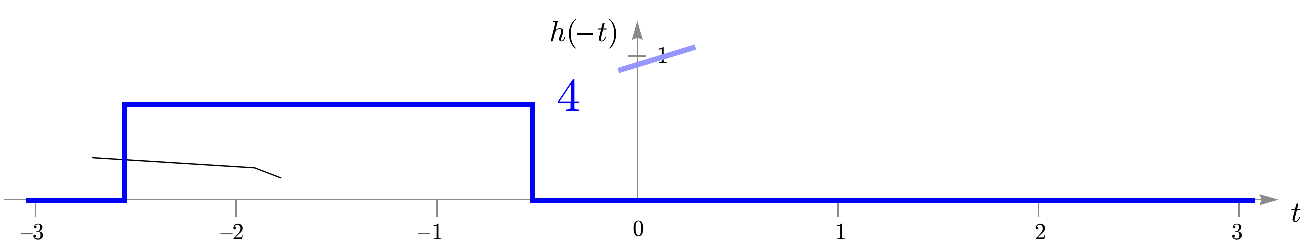
(a) The d.c. gain of the filter is $H_0 =$ [8].

(b) The Laplace transform of $h(t)$ is $H(s) =$ [$\frac{4}{s}(e^{-0.5s} - e^{-2.5s})$].

(c) Specify three values of f_0 such that a sinusoidal input of the form $x(t) = \cos(2\pi f_0 t)$ will result in a *zero* output:

$f_0 =$ [0.5], or $f_0 =$ [1], or $f_0 =$ [1.5].

(d) Carefully sketch in the space below the signal $h(-t)$:



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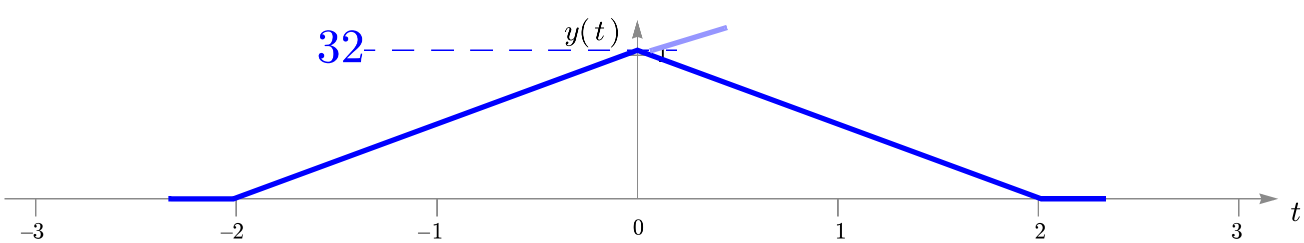

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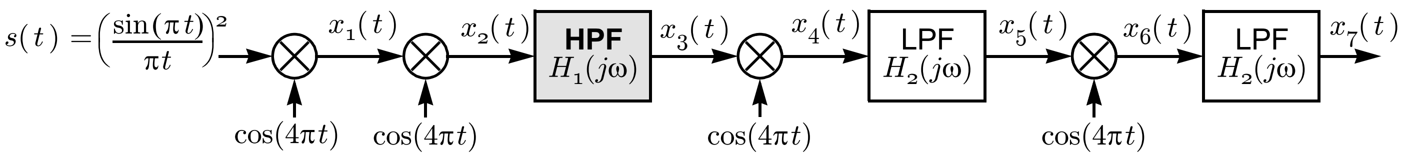

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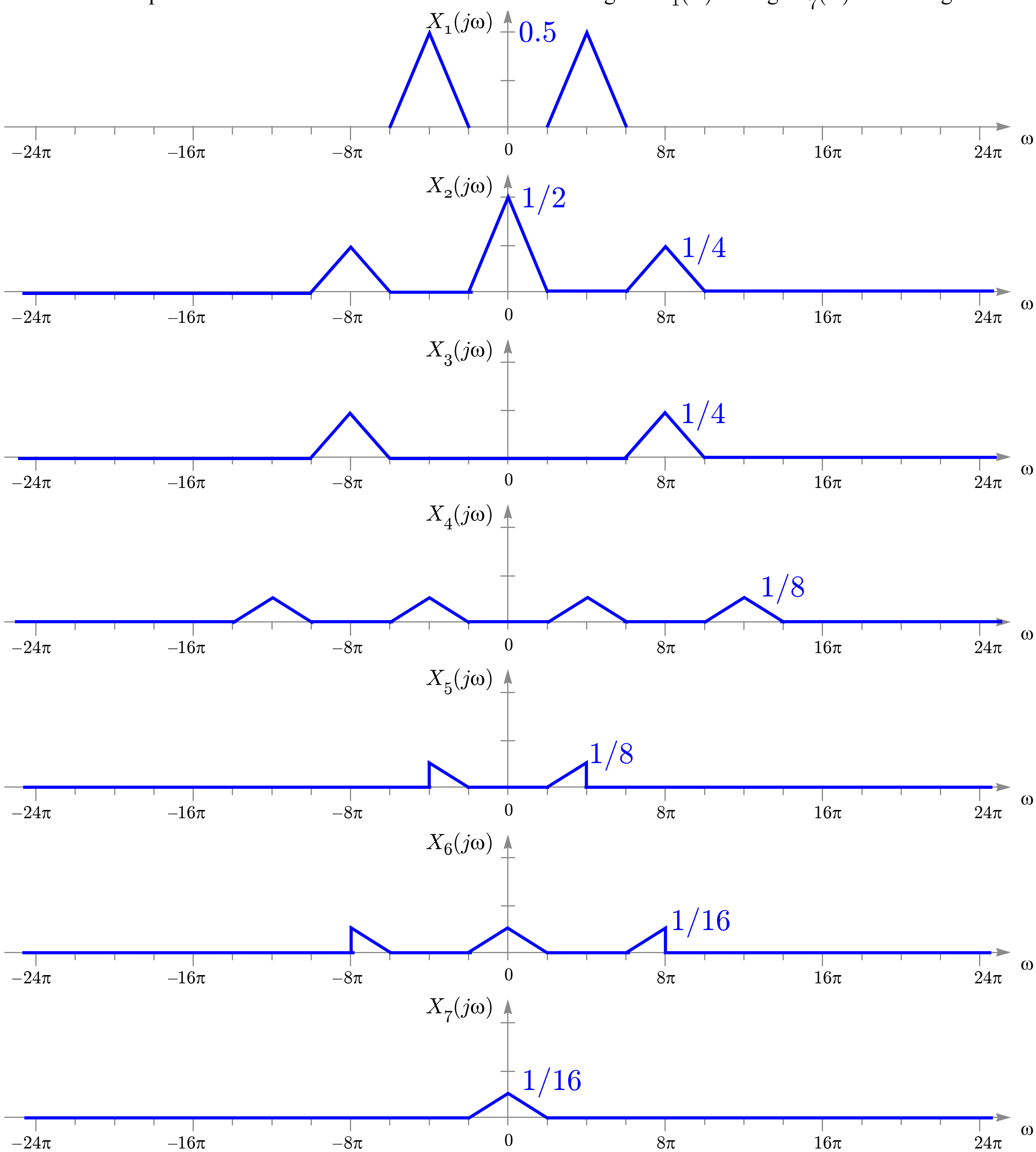

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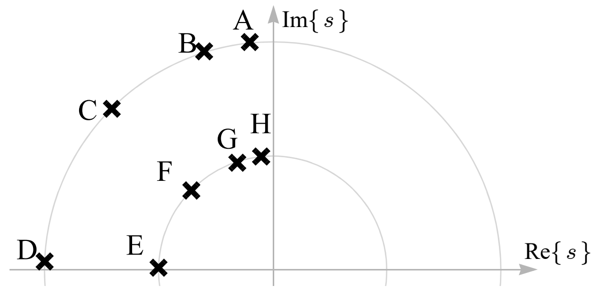

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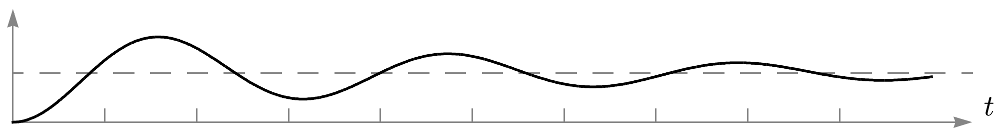
Match each location p above with its corresponding step response below by writing a letter from {A, B, C, D, E, F, G, H} into each answer box below:

(a) .

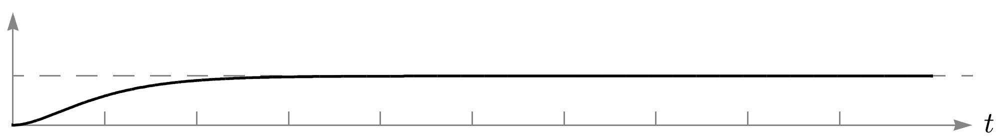
D

H

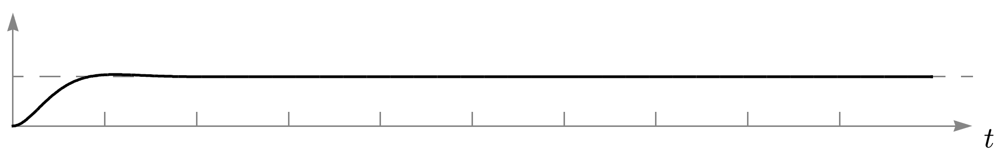


B

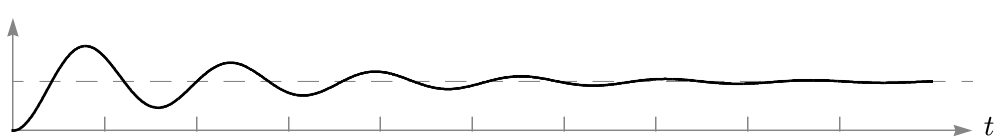
E



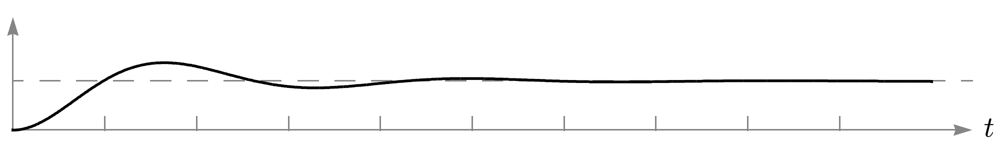
C



A



G



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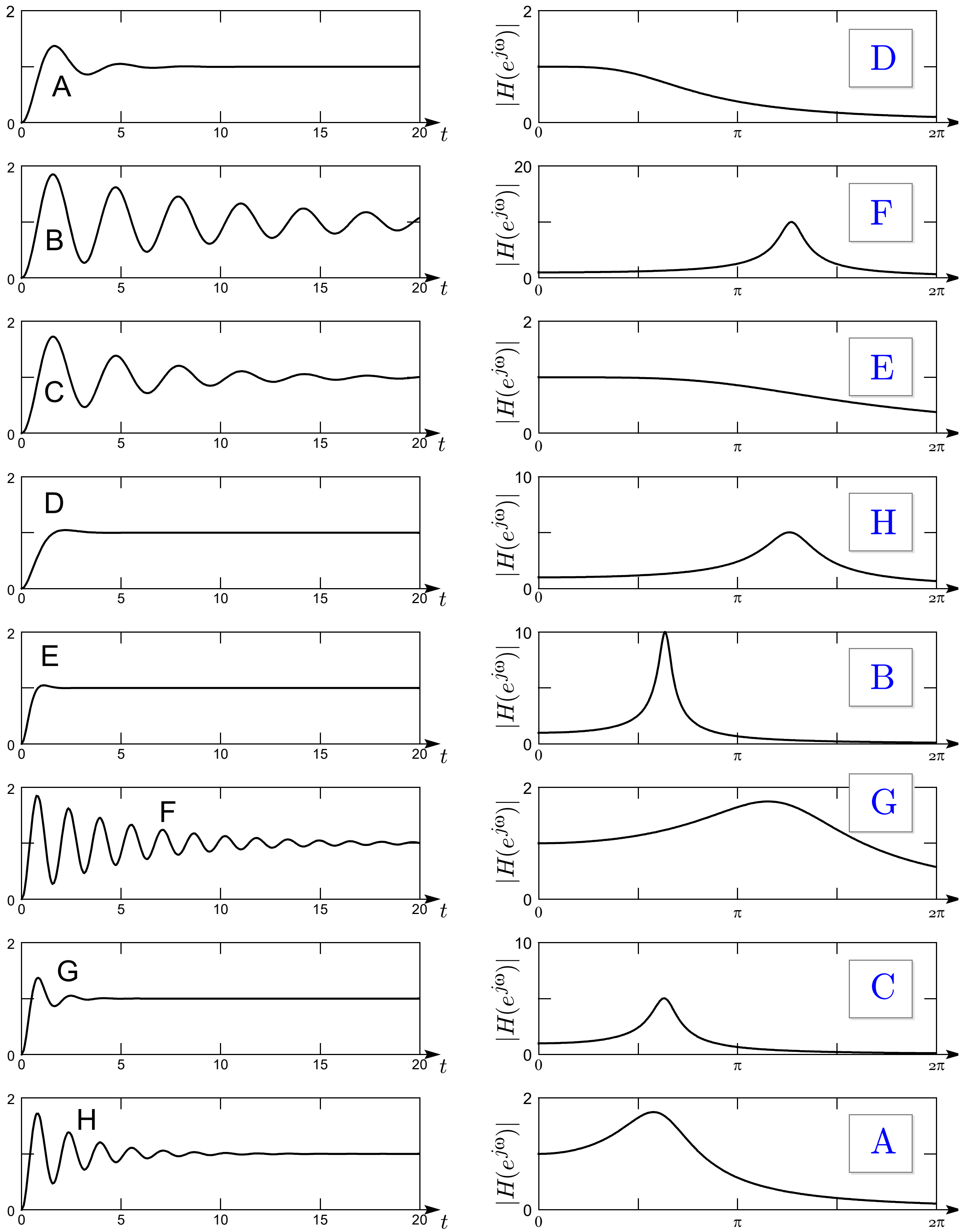

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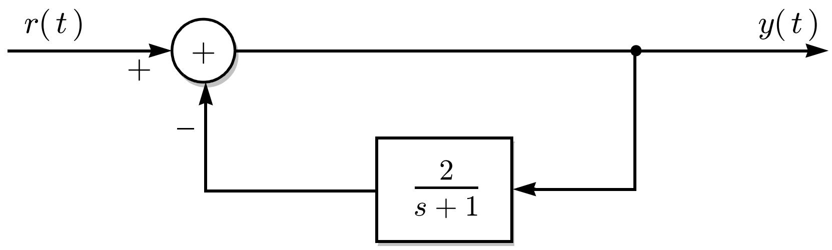

When the input is the decaying exponential $r(t) = e^{-t}u(t)$, the output can be written as:

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where

$A =$ 0 $\qquad B =$ 1 $\qquad C =$ 3 .

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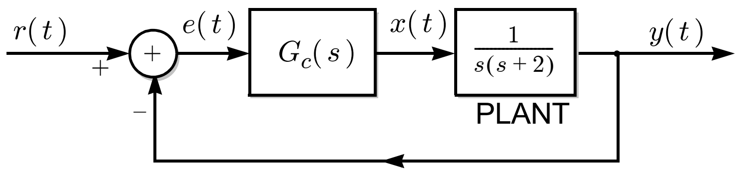

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(e) With P control, a damping ratio of $\zeta = 1/\sqrt{2}$ can be achieved by choosing $K_p =$ [2].

(f) Consider the special case of PD control where $K_p = 4K$ and $K_d = K$. In this case we can get $\zeta = 1/\sqrt{2}$ by choosing $K =$ [2].