ECE 3084

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
APRIL 28, 2015

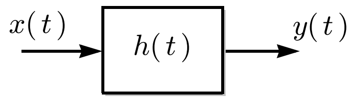
Name: ____

1. The exam is closed book, except for three 2-sided sheets of handwritten notes.
2. Turn off your phone and put it away. No tablets/laptops/WiFi/etc. Calculators are OK.
3. Final answers must be entered into the answer box.
4. Correct answers *must be accompanied by concise justifications* to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	15	
2	10	
3	15	
4	15	
5	15	
6	15	
7	15	
TOTAL:	100	

PROBLEM 1. (15 points)

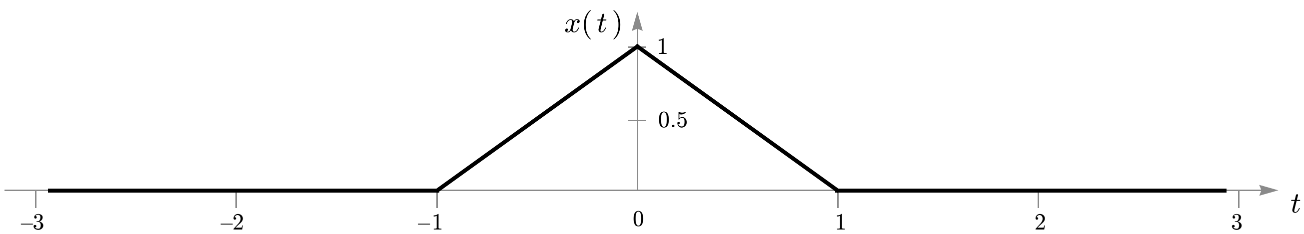
Consider an LTI filter whose impulse response is $h(t) = \delta(t) + 0.5\delta(t-1) + 0.75\delta(t-2)$:



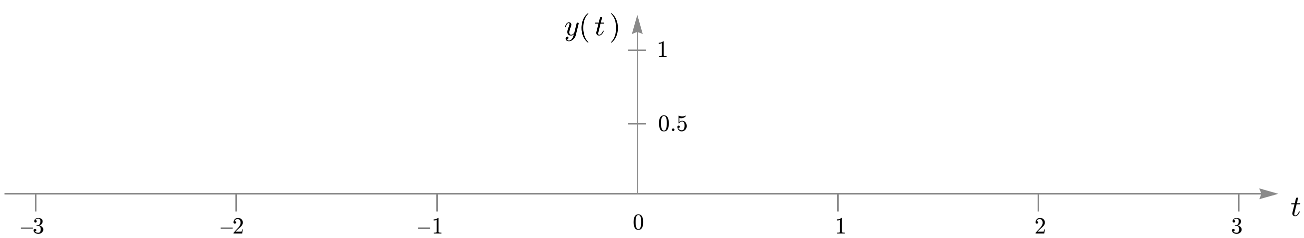
(a) The d.c. gain of the filter is $H_0 =$ $\boxed{}$.

(b) The Laplace transform of $h(t)$ is $H(s) =$ $\boxed{}$.

(c) Suppose the input $x(t)$ is the following triangular pulse:



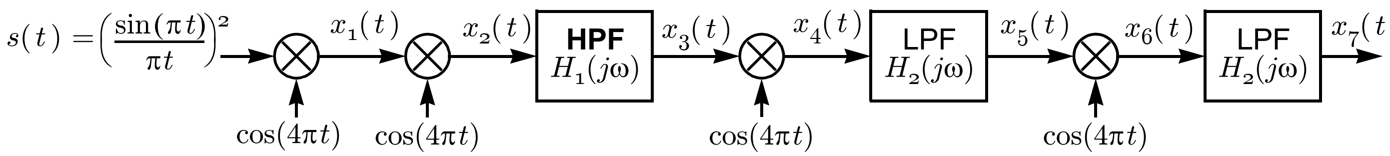
In this case, carefully sketch the output $y(t)$ in the space below:



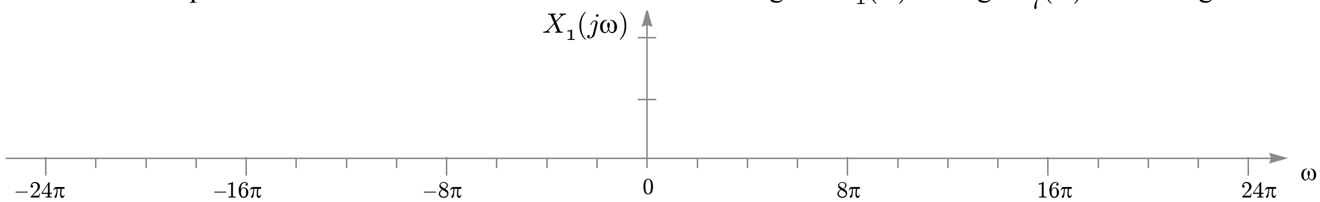
(d) If instead of being triangular, the input $x(t)$ is the 8-Hz sinusoid $x(t) = \cos(16\pi t)$, then the output can be written as $y(t) = A\cos(16\pi t + \theta)$, where:

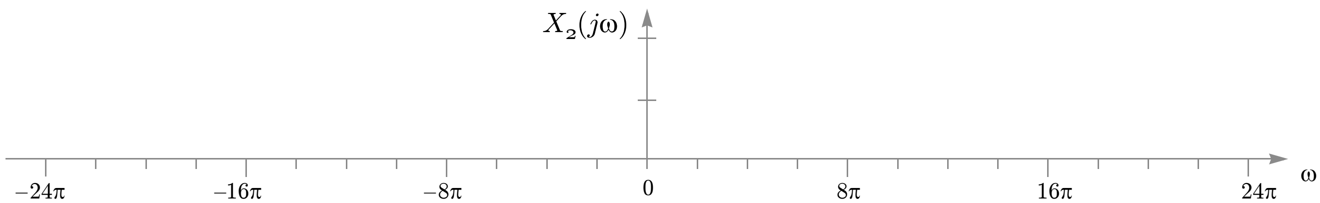
$A =$ $\boxed{}$ and $\theta =$ $\boxed{}$.

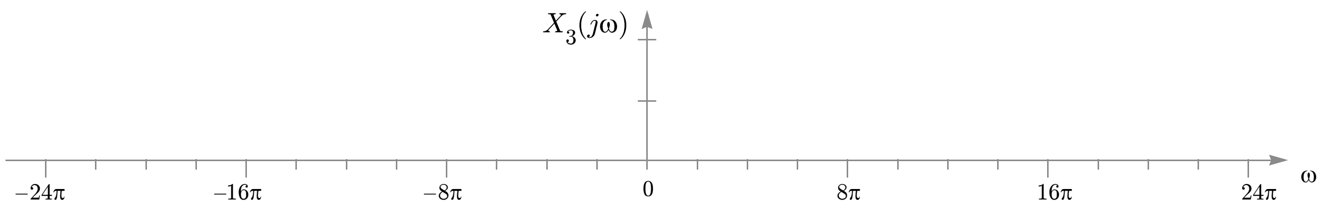
PROBLEM 2. Suppose $s(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$ is fed into the following cascade of oscillators and filters:

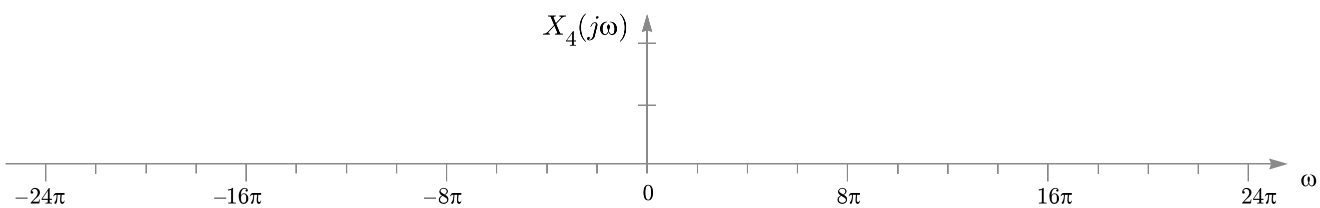


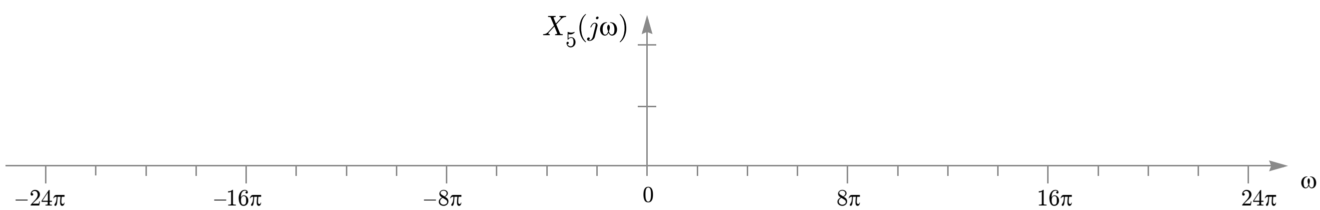
The lone **HPF** is ideal with unity gain and rejects all frequencies below 4π rad/s.
The two LPF's are ideal with unity gain and reject all frequencies *above* 4π rad/s.
Sketch in the space below the Fourier transform of each of the signals $x_1(t)$ through $x_7(t)$ in the diagram:

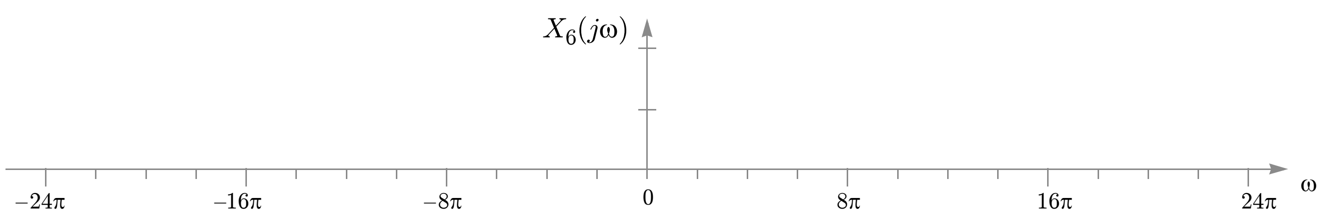


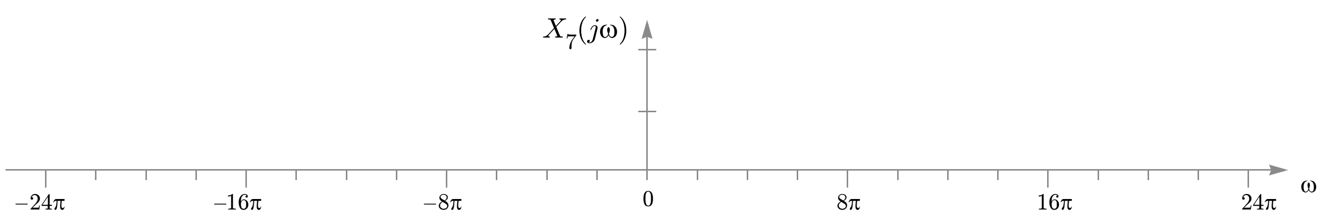










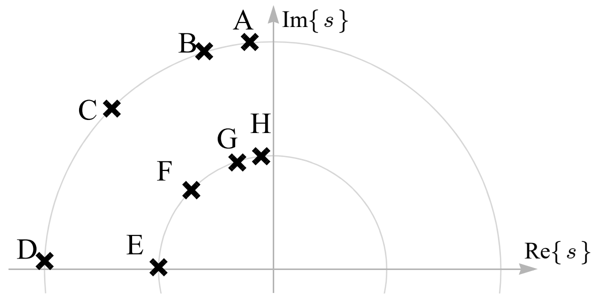


PROBLEM 3. (15 points)

Consider a second-order system with transfer function

$$H(s) = \frac{1}{s^2 + bs + c},$$

where the constants b and c are positive. When $b^2 < 4c$, the two poles will be *complex*:

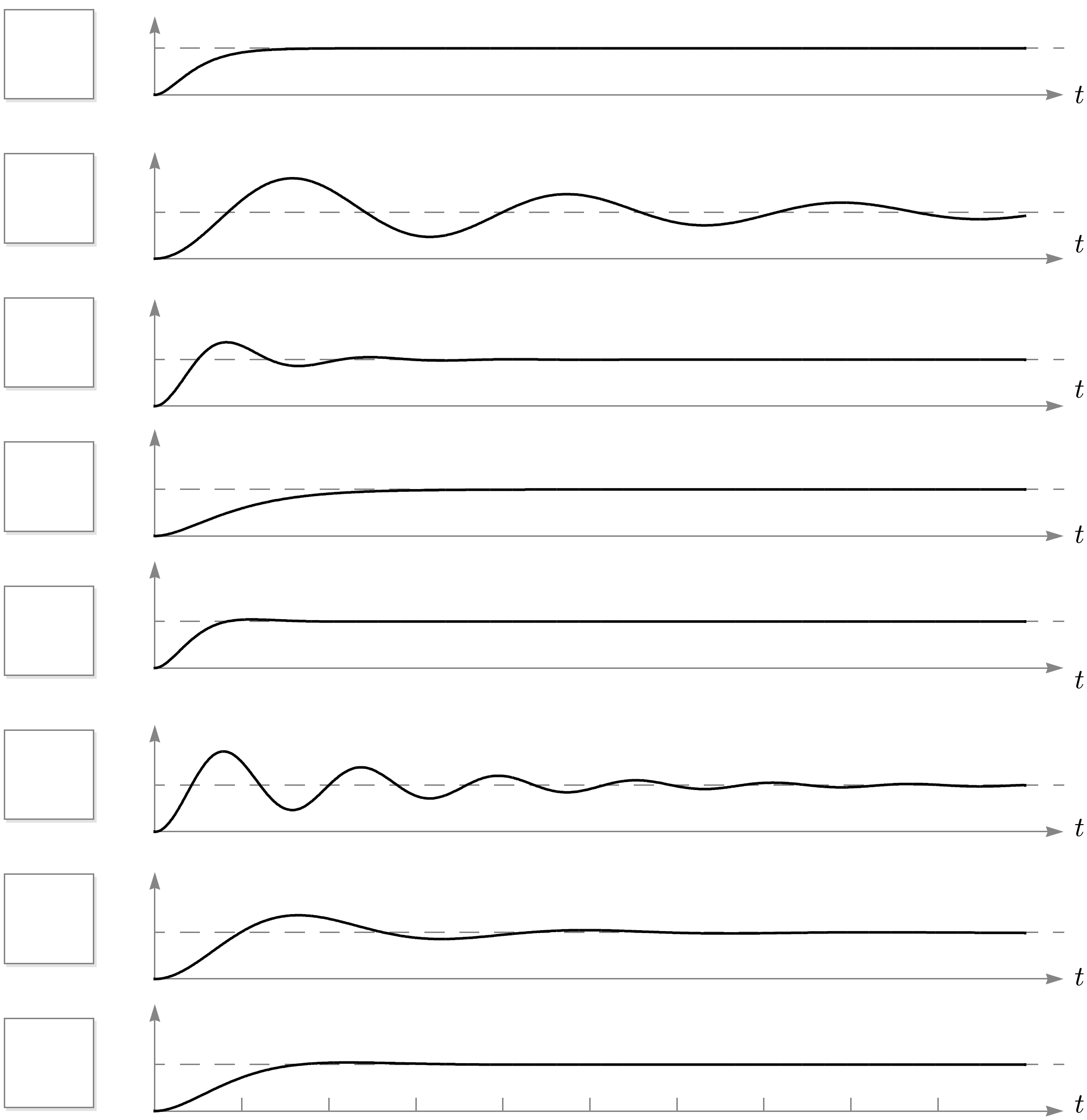


- one pole will be at a location p in the upper half of the s plane;
- the other pole will be at the conjugate location p^* in the lower half of the s plane.

Shown above are eight possible locations for the upper pole p. (The lower pole at p^* is not shown.)

Match each location p above with its corresponding step response below by writing a letter from {A, B, C, D, E, F, G, H} into each answer box below:

(a) .



PROBLEM 4. (15 points)

Consider an LTI system with input $x(t)$, output $y(t)$, and transfer function $H(s) = \frac{2s}{s+8}$.

YES NO

(a) ☐ ☐ The system is BIBO stable.

(b) The system acts as a [LPF][BPF][HPF]. (Circle one.)

(c) Write a differential equation relating the input $x(t)$ to the output $y(t)$ of this system:

(d) Find an equation for the "ramp response" of this system;
i.e., find the output $y(t)$ when the input is the unit ramp $x(t) = tu(t)$:

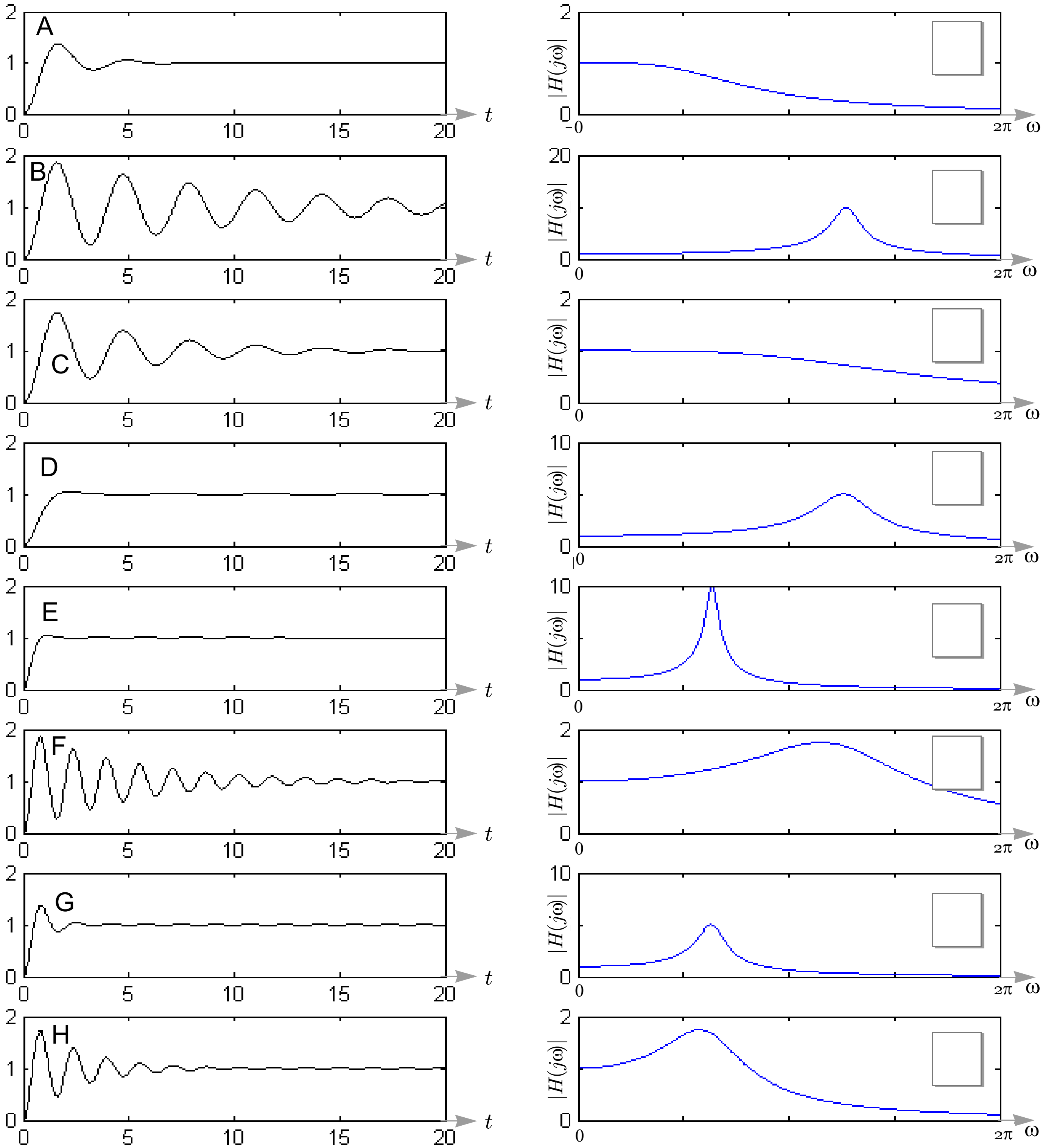
$y(t) =$ ☐

(e) Use the final value theorem to determine the steady-state value $y(\infty)$ of the ramp response:

$y(\infty) =$ ☐

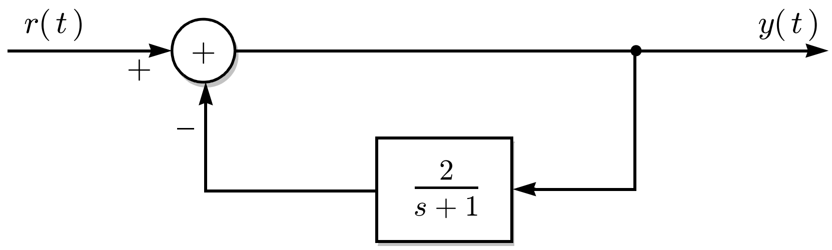
(Sanity check: your answers to parts (d) and (e) should agree.)

PROBLEM 5. Shown on the left are the step responses of eight second-order filters, labeled A through H. Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magnitude response to its corresponding step response by writing a letter (A through H) in each answer box.



PROBLEM 6. (15 points)

Consider the following closed-loop system with zero initial conditions:



When the input is the decaying exponential $r(t) = e^{-t}u(t)$, the output can be written as:

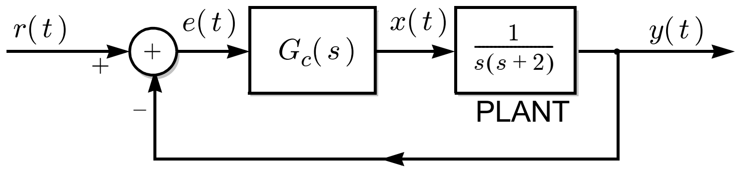
$$y(t) = (Ae^{-t} + Be^{-Ct})u(t),$$

where

$A =$ $\boxed{}$ $B =$ $\boxed{}$ $C =$ $\boxed{}$.

PROBLEM 7. (15 points)

The diagram below shows feedback control of a plant with transfer function $G_p(s) = \dfrac{1}{s(s+2)}$:



The closed-loop transfer function is $H(s) = Y(s)/R(s)$. The controller can be expressed as:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s.$$

NOTATION:	K_p	K_i	K_d
P:	nonzero	0	0
PI:	nonzero	nonzero	0
PD:	nonzero	0	nonzero
PID:	nonzero	nonzero	nonzero

(a) $H(s)$ is a *third*-order system when using [P][PI][PD][PID] control. (Circle all that apply.)

(b) $H(s)$ is a *second*-order system when using [P][PI][PD][PID] control. (Circle all that apply.)

(c) The d.c. gain is $H(0) = 1$ when using [P][PI][PD][PID] control. (Circle all that apply.)

(d) With P control, oscillations in the closed-loop step response can be avoided by choosing $K_p <$ $\boxed{}$.

(e) With P control, a damping ratio of $\zeta = 1/\sqrt{2}$ can be achieved by choosing $K_p =$ $\boxed{}$.

(f) Using PD control with $K_p = 25$, the closed-loop step response can be made to achieve its maximum value at time $t_{\max} = 2$ seconds by choosing $K_d =$ $\boxed{}$.

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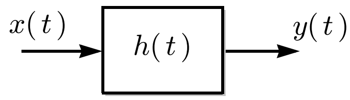
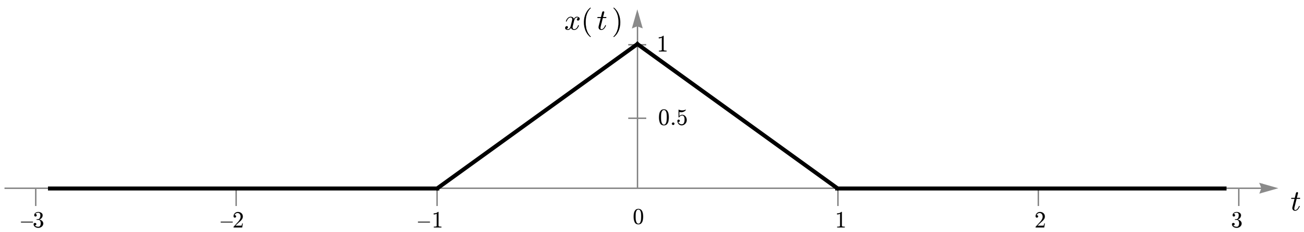


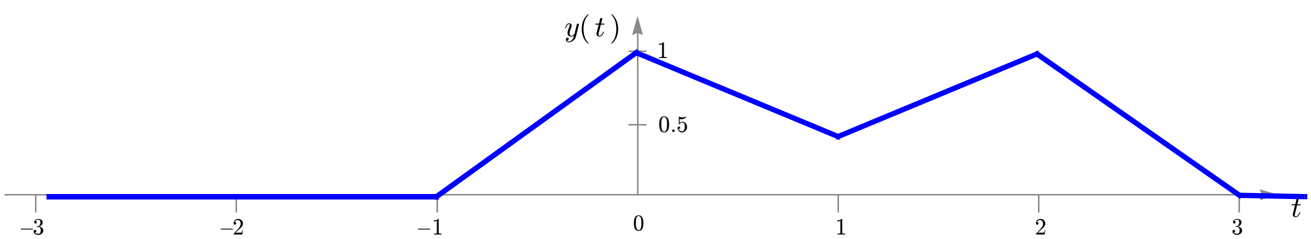
(a) The d.c. gain of the filter is $H_0 = \boxed{2.25}$.

(b) The Laplace transform of $h(t)$ is $H(s) = \boxed{1 + 0.5e^{-s} + 0.75e^{-2s}}$.

(c) Suppose the input $x(t)$ is the following triangular pulse:



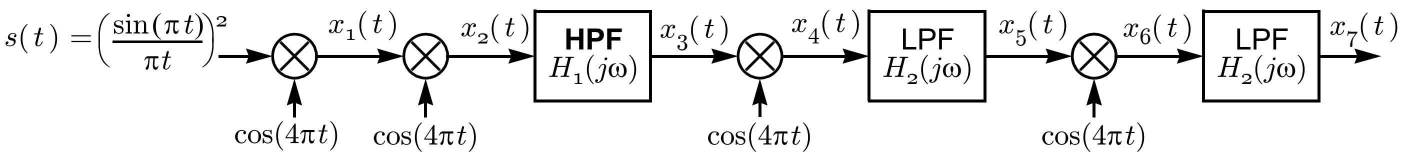
In this case, carefully sketch the output $y(t)$ in the space below:



(d) If instead of being triangular, the input $x(t)$ is the 8-Hz sinusoid $x(t) = \cos(16\pi t)$, then the output can be written as $y(t) = A\cos(16\pi t + \theta)$, where:

$A = \boxed{2.25}$ and $\theta = \boxed{0}$.

PROBLEM 2. Suppose $s(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$ is fed into the following cascade of oscillators and filters:



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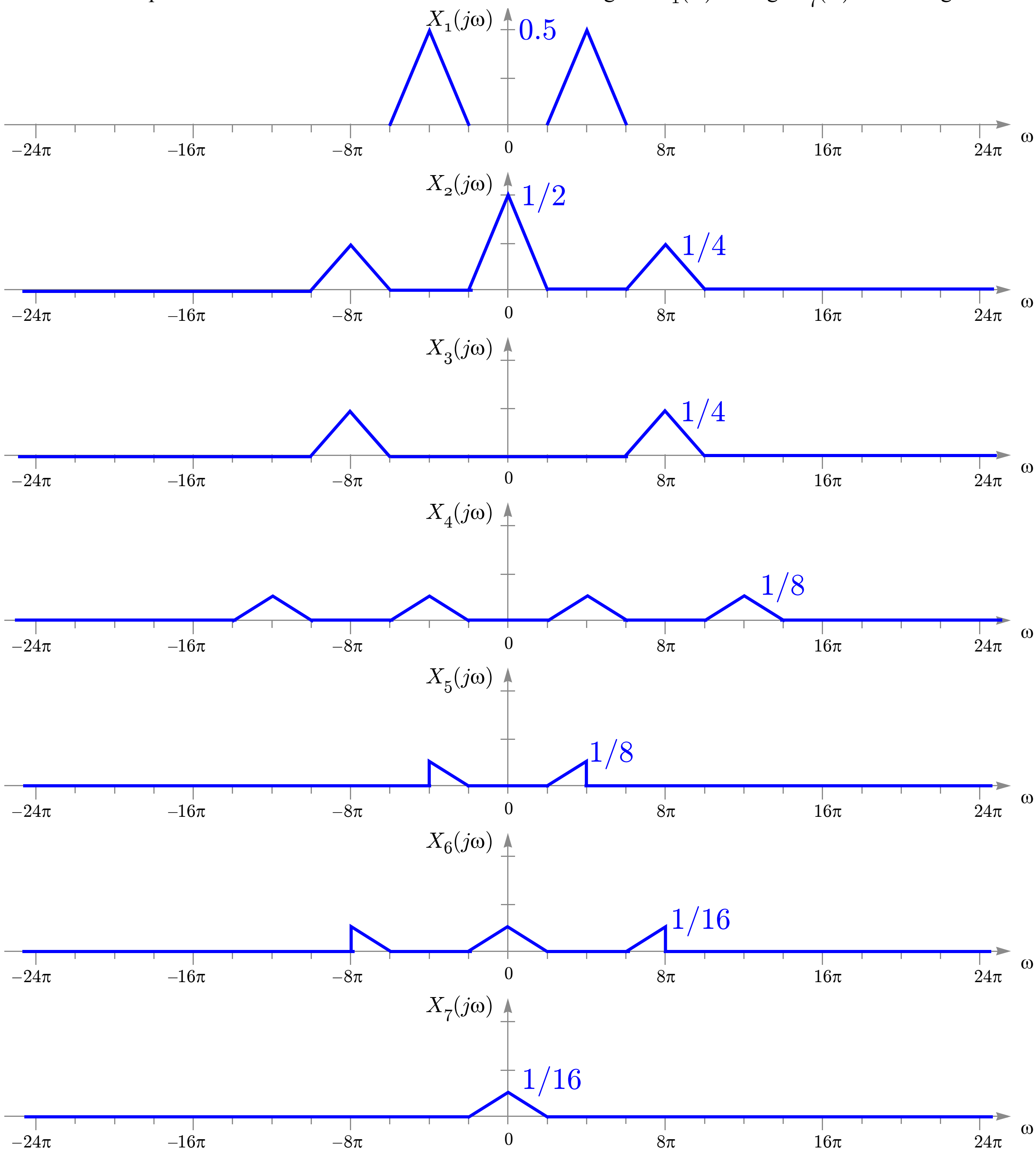

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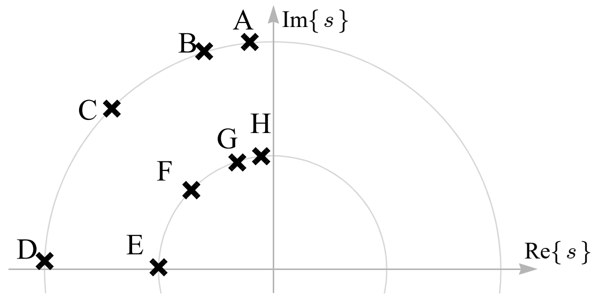

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where the constants b and c are positive.
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Shown above are eight possible locations for the upper pole p. (The lower pole at p^* is not shown.)

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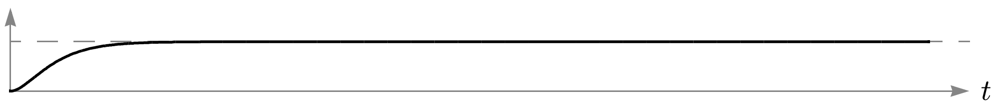
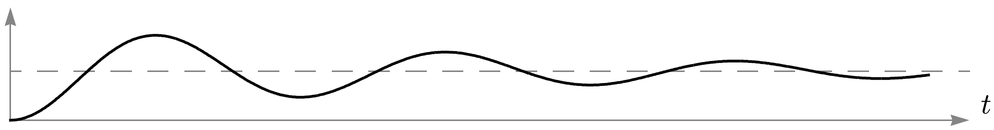
(a) .

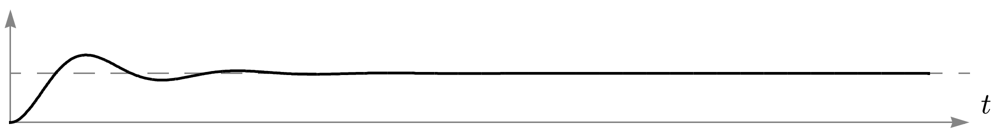
D



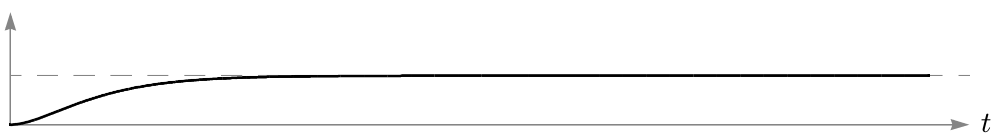
H



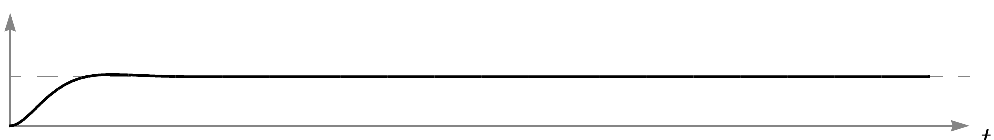
B



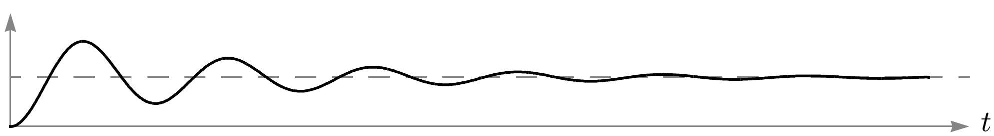
E



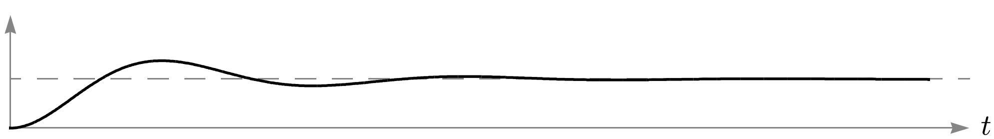
C



A



G



F

PROBLEM 4. (15 points)

Consider an LTI system with input $x(t)$, output $y(t)$, and transfer function $H(s) = \frac{2s}{s+8}$.

(a) YES [X] NO [ ] The system is BIBO stable.

(b) The system acts as a [LPF][BPF][HPF]. (Circle one.)

(c) Write a differential equation relating the input $x(t)$ to the output $y(t)$ of this system:

$$\frac{d}{dt}y(t) + 8y(t) = 2\frac{d}{dt}x(t)$$

(d) Find an equation for the "ramp response" of this system;
i.e., find the output $y(t)$ when the input is the unit ramp $x(t) = tu(t)$:

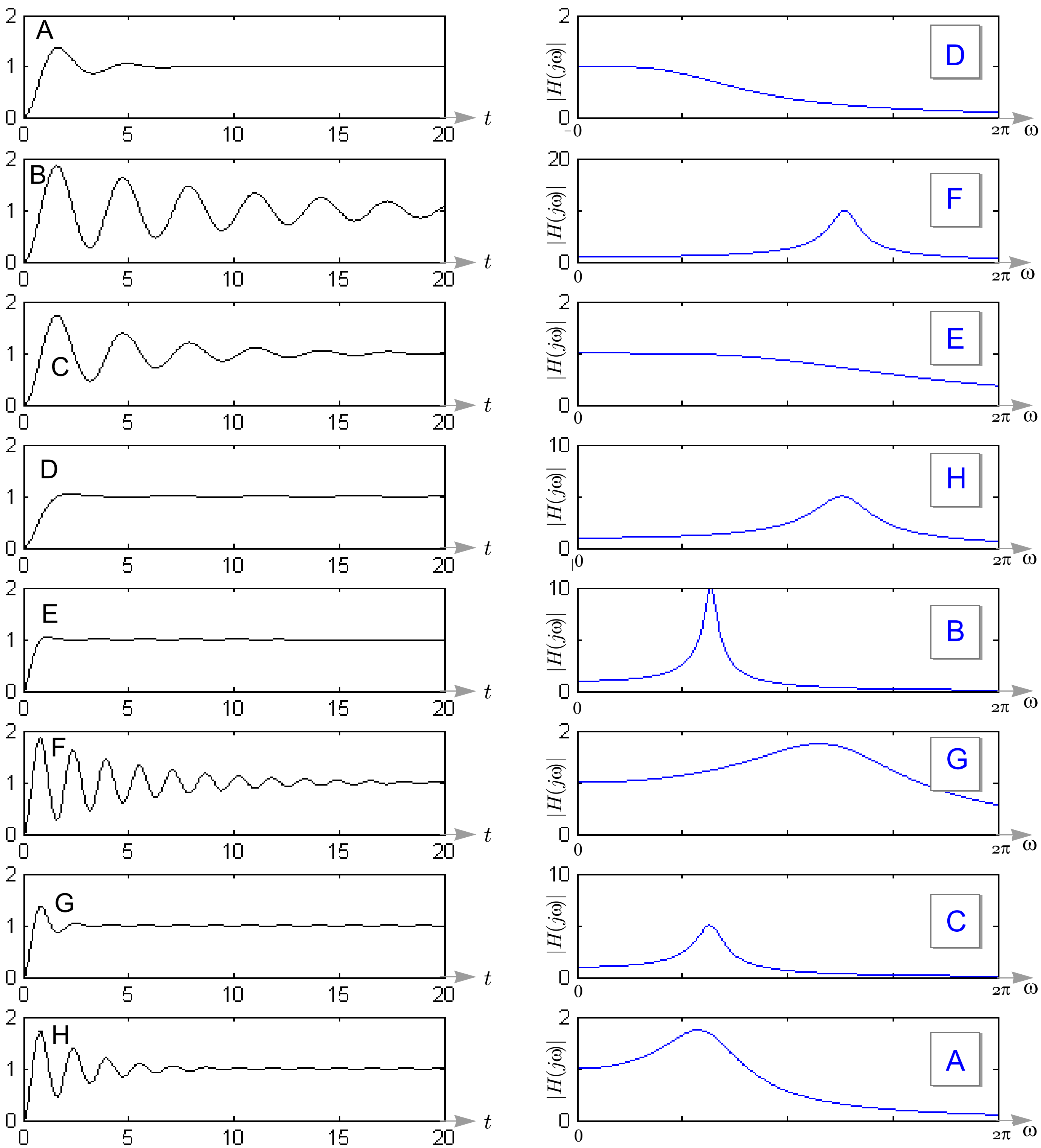
$$y(t) = \boxed{\tfrac{1}{4}(1 - e^{-8t})u(t)}$$

(e) Use the final value theorem to determine the steady-state value $y(\infty)$ of the ramp response:

$$y(\infty) = \boxed{\frac{1}{4}}$$

(Sanity check: your answers to parts (d) and (e) should agree.)

PROBLEM 5. Shown on the left are the step responses of eight second-order filters, labeled A through H. Shown on the right are the magnitude responses for these filters, but in a scrambled order. Match each magnitude response to its corresponding step response by writing a letter (A through H) in each answer box.



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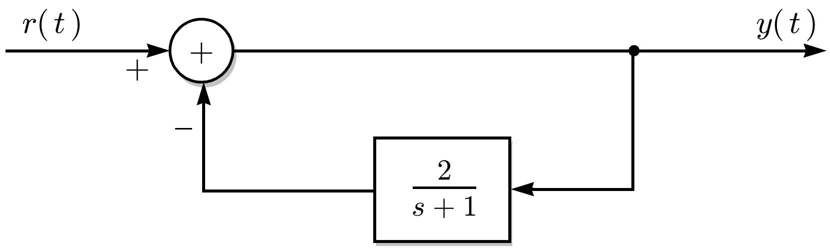

When the input is the decaying exponential $r(t) = e^{-t}u(t)$, the output can be written as:

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where

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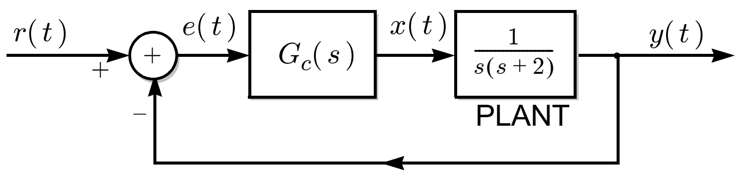

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(d) With P control, oscillations in the closed-loop step response can be avoided by choosing $K_p <$ $\boxed{1}$.

(e) With P control, a damping ratio of $\zeta = 1/\sqrt{2}$ can be achieved by choosing $K_p =$ $\boxed{2}$.

(f) Using PD control with $K_p = 25$, the closed-loop step response can be made to achieve its maximum value at time $t_{\max} = 2$ seconds by choosing $K_d =$ $\boxed{7.49}$.