ECE 3084

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
DECEMBER 13, 2018

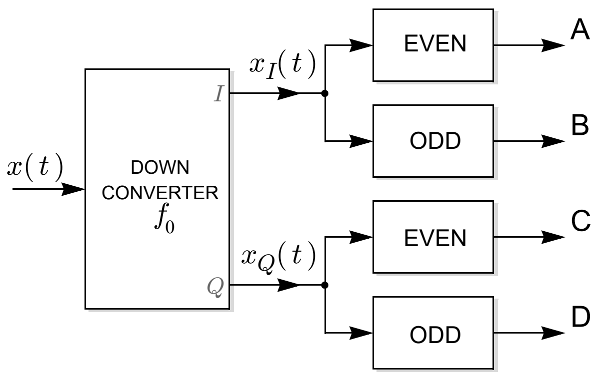
Name: ____

1. The exam is *closed book* & notes, except for three 2-sided sheets of handwritten notes.
2. Silence your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers must be accompanied by concise justifications to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL:	100	

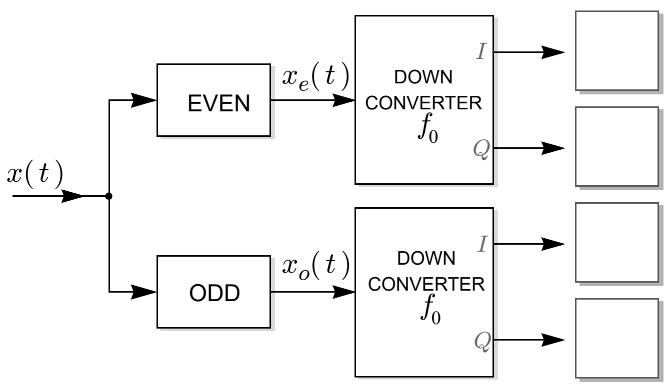
PROBLEM 1.

Let $x(t)$ be a real signal whose FT satisfies $X(j\omega) = 0$ for $|\omega - 2\pi f_0| > \pi f_0$. Define $x_I(t)$ and $x_Q(t)$ as the in-phase and quadrature components of $x(t)$ w.r.t. f_0. Suppose we extract the even and odd parts of these I&Q components, resulting in four signals (labeled A through D), as shown below:



(For example: signal A is the even part of $x_I(t)$, signal B is the odd part of $x_I(t)$, signal C is the even part of $x_Q(t)$, etc.)

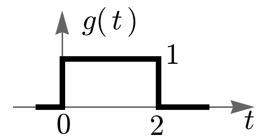
The above system first downconverts, and second extracts the even and odd parts.
The system below does things *in reverse order*: It starts with the same input $x(t)$, but first extracts the even and odd parts, and then downconverts each:



(a) The two systems shown above produce the same set of four outputs:
One of the four outputs of the second system is signal A, another is signal B, etc.
Indicate which is which by writing a letter from $\{$A, B, C, D$\}$ into each answer box.

(b) *Explain!*

PROBLEM 2.



Let $g(t) = u(t) - u(t-2)$ be a rectangle of width 2, and
let $x(t) = g(t) + g(t-\tau)$ for some unspecified delay parameter τ.

(a) The rectangle $g(t)$ has energy $E_g =$ $\boxed{}$ and power $P_g =$ $\boxed{}$.

(b) In the remainder of this problem let $E_x(\tau)$ denote the energy of $x(t)$; its dependence on the delay parameter τ is made explicit.

The smallest possible value for $E_x(\tau)$ is achieved when $\tau =$ $\boxed{}$.

(c) The largest possible value for $E_x(\tau)$ is achieved when $\tau =$ $\boxed{}$.

(d) Find a value of τ for which $E_x(\tau) = 5$: $\tau =$ $\boxed{}$.

PROBLEM 3.

Consider the following 8 signals, all derived from the "sinc-squared" signal $x(t) = 2\left(\frac{\sin(0.5\pi t)}{\pi t}\right)^2$:

(1) $y_1(t) = 4\frac{d}{dt}x(t)$

(2) $y_2(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$

(3) $y_3(t) = x(t - 0.5\pi)$

(4) $y_4(t) = e^{j\pi^2}x(t)$

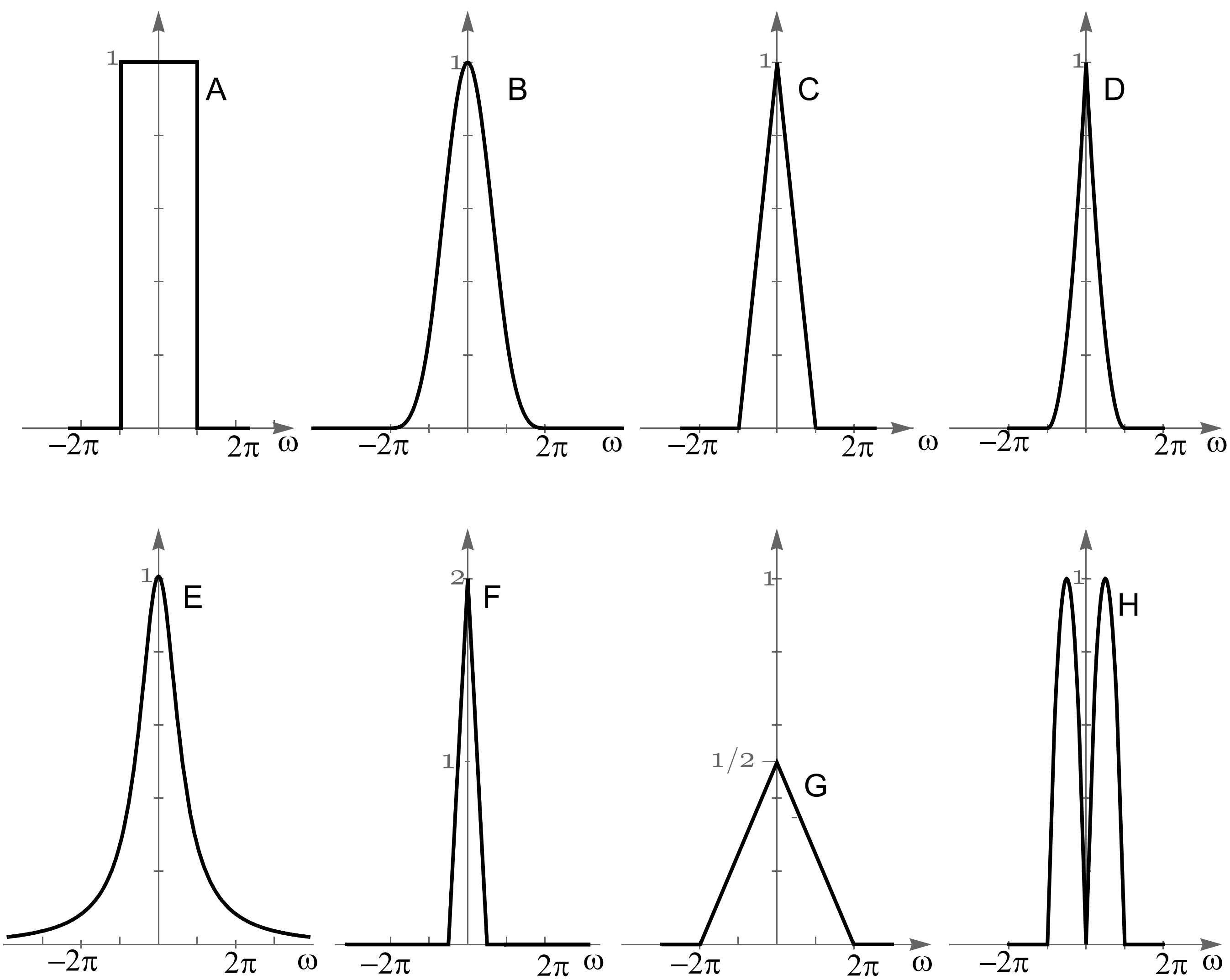
(5) $y_5(t) = x(2t)$

(6) $y_6(t) = 3\pi x^2(t)$

(7) $y_7(t) = x(0.5t)$

(8) $y_8(t) = \int_{-\infty}^{\infty} \frac{\sin(5\pi\tau)}{\pi\tau} x(t-\tau)d\tau$

Shown below are plots of several plots of $|Y(j\omega)|$, the Fourier transform magnitude.
Match each derived signal above with its corresponding magnitude plot.
Indicate your answer by writing a letter (from $\{$A, ... H$\}$) in each answer above above.
(Some plots may be used more than once, others may never be used.)



PROBLEM 4.

Below are three systems with input $x(t)$ and output $y(t)$. Specify which properties they satisfy by writing a "Y" (for yes) or "N" (for no) into each answer box:

(a) $y(t) = x(t^2)$

memoryless	causal	stable	linear	time-invariant	invertible

(b) $y(t) = x(t^3)$

memoryless	causal	stable	linear	time-invariant	invertible

(c) $y(t) = 3x(t) + x(t-1)$

memoryless	causal	stable	linear	time-invariant	invertible

PROBLEM 5.

A system that is initially at rest (with zero initial conditions) is described by the following differential equation relating the input $x(t)$ to the output $y(t)$:

$$2\frac{d^2}{dt^2}y(t) = 2\frac{d}{dt}x(t) + 6x(t) - \beta\frac{d}{dt}y(t) - 24y(t).$$

(a) Sketch the pole-zero plot for the system function $H(s) = Y(s)/X(s)$ when $\beta = 16$.

(b) If the **step** response of the system has the form $y(t) = A(1 - e^{-Bt})u(t)$, then it must be that

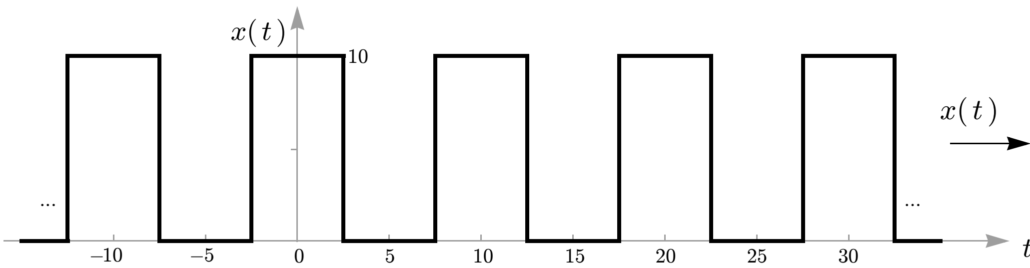
$\beta =$ $\boxed{}$

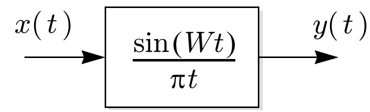
$A =$ $\boxed{}$

$B =$ $\boxed{}$

PROBLEM 6.

Suppose the periodic signal $x(t)$ shown below is fed as an input to an LTI system whose impulse response $h(t) = \frac{\sin(Wt)}{\pi t}$ is a sinc function (the parameter W is positive but otherwise unspecified), producing the output $y(t)$:



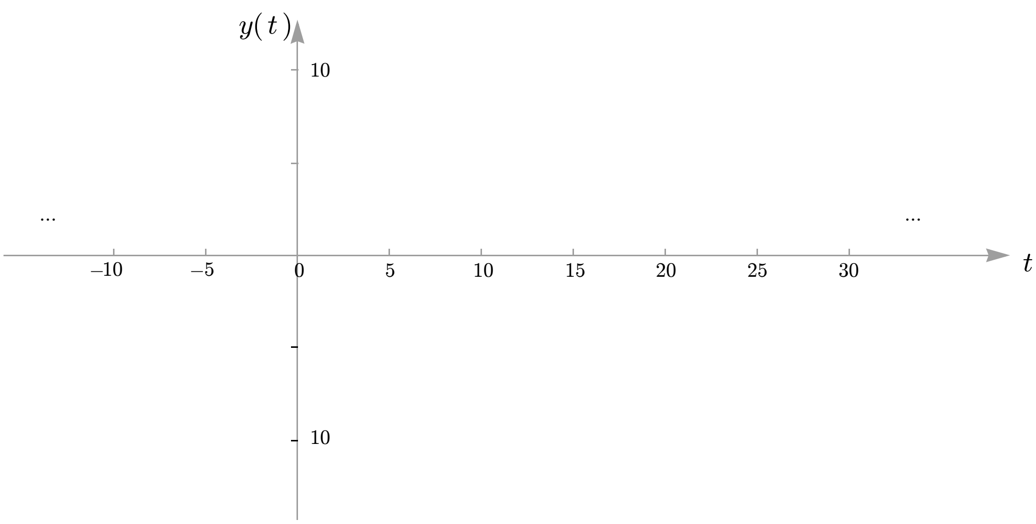


(a) For what range of values for the parameter W will the output be a *constant*, say $y(t) = y_0$ for all t?

$$\boxed{} < W < \boxed{}$$

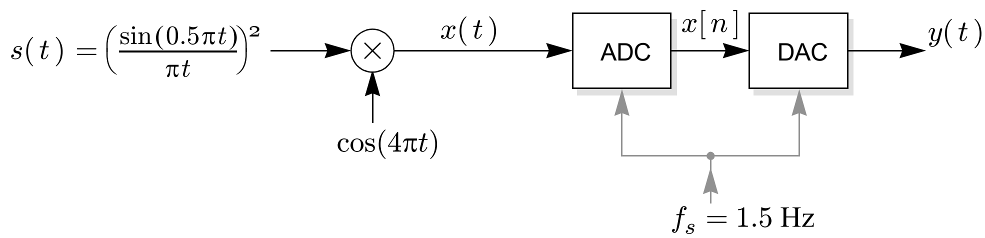
(b) When W is one of the values from part (a), the output constant will be $y_0 = \boxed{}$.

(c) In the space below, *carefully* sketch the output $y(t)$ when $W = \frac{\pi^2}{10}$. (Both axes are already labeled.)



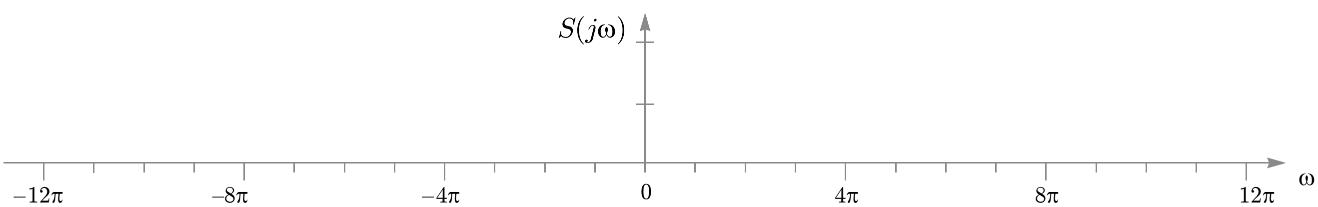
PROBLEM 7.

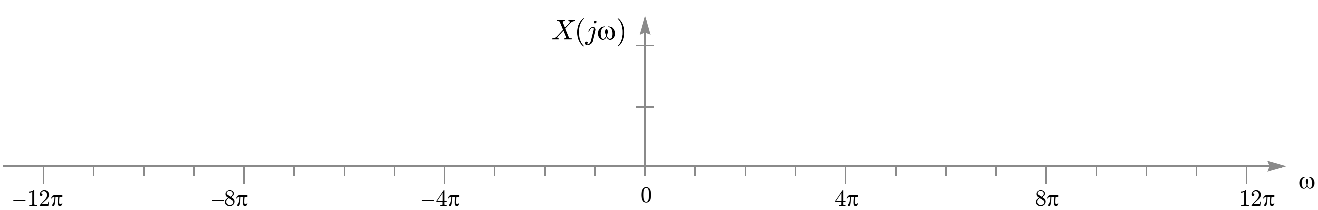
A sinc-squared function $s(t) = \left(\frac{\sin(0.5\pi t)}{\pi t}\right)^2$ is first AM-modulated with carrier 4π rad/s, producing $x(t)$, which is then fed to a back-to-back connection of an ideal analog-to-digital converter (ADC) and an ideal digital-analog (DAC) converter, both with sampling rate $f_s = 1.5$ Hz:

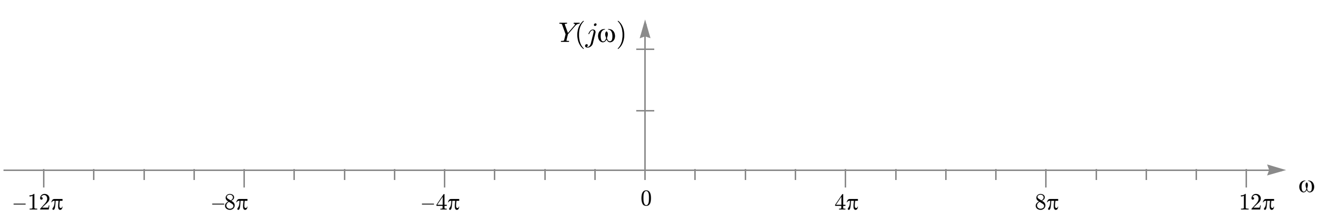


(a) The zero-th ADC sample is $x[0] =$ ______ .

(b) Sketch in the space below the Fourier transform of the three signals $s(t)$, $x(t)$ and $y(t)$:







(c) Label the *y*-axis in all three sketches of part (b).

(d) The DAC output $y(t)$ can be written in the form $y(t) = A\frac{\sin(Bt)}{t} - C\left(\frac{\sin(Dt)}{t}\right)^2$, where

$A =$ $\boxed{}$

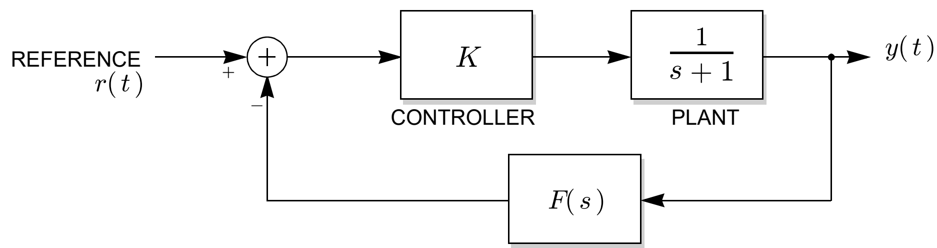
$B =$ $\boxed{}$

$C =$ $\boxed{}$

$D =$ $\boxed{}$

PROBLEM 8.

Consider the control system shown below:



(a) When $F(s) = 1$,
the range of values for K that make the closed-loop system *stable* is

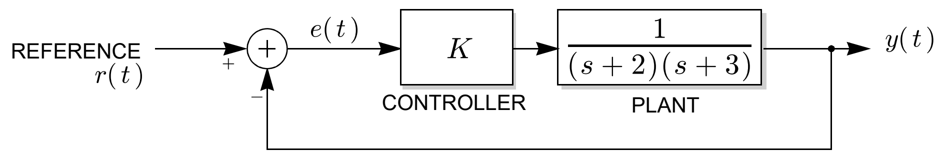
(b) *Delay in the Feedback Path* — Let's consider what happens when the feedback path has **delay**, so that the error signal is not $e(t) = r(t) - y(t)$, but instead is $e(t) = r(t) - y(t-\tau)$, where τ is the feedback delay. In principle we can model this delay by choosing the feedback transfer function shown in the above diagram to be $F(s) = e^{-s\tau}$, but this would lead to an irrational transfer function that prevents us from thinking about poles and zeros. Therefore, let us instead adopt the *Padé* approximation, namely $e^{-s\tau} \approx \frac{1 - s\tau/2}{1 + s\tau/2}$, and set the feedback transfer function to:

$$F(s) = \frac{1 - s\tau/2}{1 + s\tau/2}.$$

With this choice for $F(s)$, and with a delay of $\tau = 2$,
the range of values for K that make the closed-loop system *stable* is

PROBLEM 9.

Consider P-control for a second-order plant with transfer function $G_p(s) = \dfrac{1}{(s+2)(s+3)}$, as shown below:



(a) If the reference is a unit step, $r(t) = u(t)$, the *steady-state error* (expressed as a function of the controller gain K) is

$$e_{ss} = \lim_{t\to\infty} e(t) = \boxed{}.$$

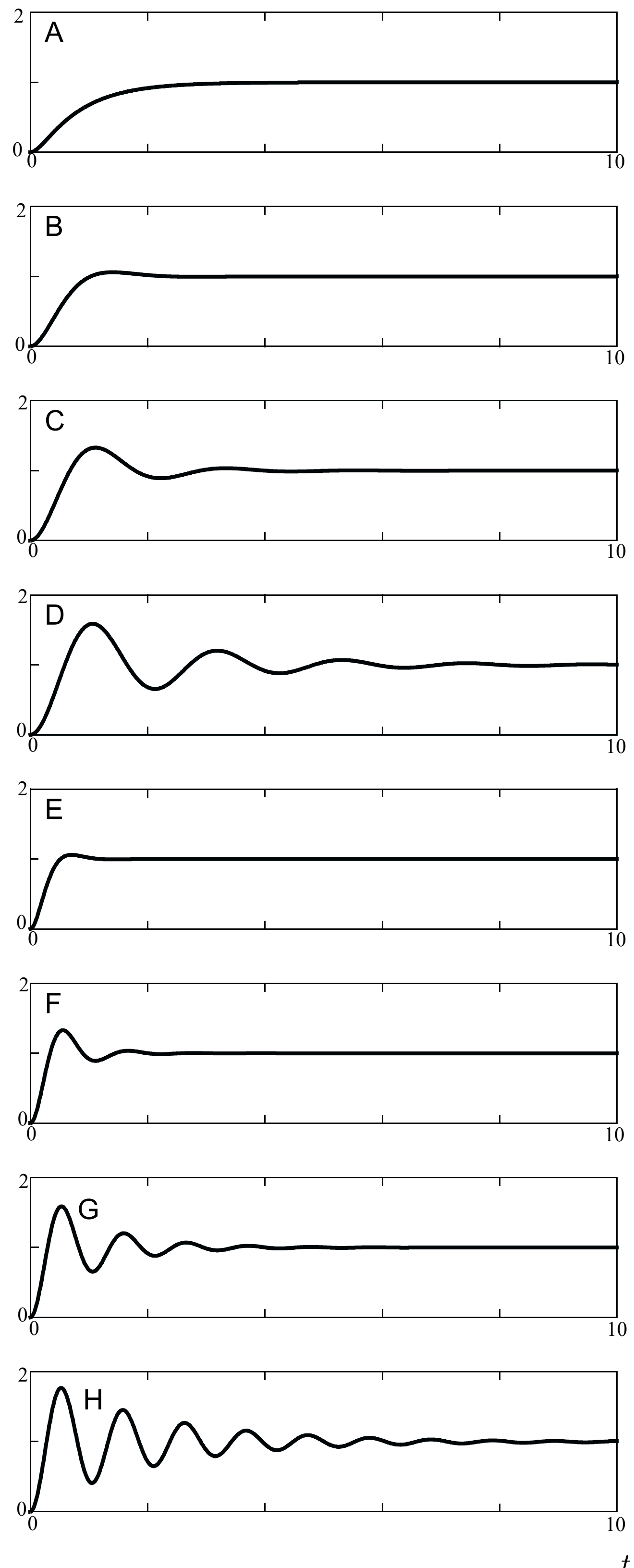
(b) Sketch a pole-zero plot for the closed-loop transfer function $H(s) = Y(s)/R(s)$ when $K = 6$:

(c) The closed-loop system is *critically damped* when $K = \boxed{}$.

(d) To achieve a damping coefficient of $\zeta = \dfrac{1}{\sqrt{2}}$, the gain must be $K = \boxed{}$.

PROBLEM 10.

Match each transfer function below with its corresponding *step response* shown to the right, by writing a letter from {A ... H} into each answer box:



(a) $H(s) = \dfrac{36}{s^2 + s + 36}$

(b) $H(s) = \dfrac{36}{s^2 + 2s + 36}$

(c) $H(s) = \dfrac{36}{s^2 + 4s + 36}$

(d) $H(s) = \dfrac{36}{s^2 + 8s + 36}$

(e) $H(s) = \dfrac{9}{s^2 + 8s + 9}$

(f) $H(s) = \dfrac{9}{s^2 + 4s + 9}$

(g) $H(s) = \dfrac{9}{s^2 + 2s + 9}$

(h) $H(s) = \dfrac{9}{s^2 + s + 9}$

Table of Fourier Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Frequency-Domain:* $X(j\omega)$
Right-sided exponential	$e^{-at}u(t) \quad (a>0)$	$\dfrac{1}{a+j\omega}$
Left-sided exponential	$e^{bt}u(-t) \quad (b>0)$	$\dfrac{1}{b-j\omega}$
Square pulse	$[u(t+T/2)-u(t-T/2)]$	$\dfrac{\sin(\omega T/2)}{\omega/2}$
"sinc" function	$\dfrac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega+\omega_0)-u(\omega-\omega_0)]$
Impulse	$\delta(t)$	1
Shifted impulse	$\delta(t-t_0)$	$e^{-j\omega t_0}$
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
General cosine	$A\cos(\omega_0 t+\phi)$	$\pi A e^{j\phi}\delta(\omega-\omega_0)+\pi A e^{-j\phi}\delta(\omega+\omega_0)$
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega-\omega_0)+j\pi\delta(\omega+\omega_0)$
General periodic signal	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega-k\omega_0)$
Impulse train	$\sum_{n=-\infty}^{\infty} \delta(t-nT)$	$\dfrac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega-2\pi k/T)$

Table of Fourier Transform Properties		
Property Name	*Time-Domain* $x(t)$	*Frequency-Domain* $X(j\omega)$
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}X(j(\omega/a))$
Delay	$x(t-t_d)$	$e^{-j\omega t_d}X(j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega-\omega_0))$
Modulation	$x(t)\cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega-\omega_0))+\frac{1}{2}X(j(\omega+\omega_0))$
Differentiation	$\dfrac{d^k x(t)}{dt^k}$	$(j\omega)^k X(j\omega)$
Convolution	$x(t)*h(t)$	$X(j\omega)H(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega)*P(j\omega)$

Date 14-Sept-2013

Table of Laplace Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Laplace-Domain:* $X(s)$
Impulse	$\delta(t)$	1
Delayed Impulse	$\delta(t-t_0), \quad t_0 \geq 0$	e^{-st_0}
Step	$u(t)$	$\frac{1}{s}$
Rectangular Pulse	$u(t)-u(t-T), \quad T>0$	$\frac{1-e^{-sT}}{s}$
Ramp	$tu(t)$	$\frac{1}{s^2}$
Polynomial	$t^k u(t), \quad k \geq 0$	$\frac{k!}{s^{k+1}}$
Exponential	$e^{-at}u(t)$	$\frac{1}{s+a}$
Polynomial × Exponential	$t^k e^{-at}u(t)$	$\frac{k!}{(s+a)^{k+1}}$
Cosine	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$
Sine	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
Exponential × Cosine	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
Exponential × Sine	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

Table of Laplace Transform Properties		
Property Name	*Time-Domain:* $x(t)$	*Laplace-Domain:* $X(s)$
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Right-Shift	$x(t-t_0), \quad t_0 \geq 0$	$e^{-st_0}X(s)$
Time Scaling	$x(at), \quad a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
First Derivative	$\dot{x}(t)=\frac{d}{dt}x(t)$	$sX(s)-x(0)$
Second Derivative	$\ddot{x}(t)=\frac{d^2}{dt^2}x(t)$	$s^2X(s)-sx(0)-\dot{x}(0)$
Integration	$\int_0^t x(\tau)d\tau$	$\frac{X(s)}{s}$
Modulation	$x(t)e^{at}$	$X(s-a)$
Convolution	$x(t)*h(t)$	$X(s)H(s)$
Final Value Theorem	$\lim_{t\to\infty} x(t)$, (if limit exists)	$\lim_{s\to 0} sX(s)$
Initial Value Theorem	$\lim_{t\to 0} x(t)$, (if limit exists)	$\lim_{s\to\infty} sX(s)$

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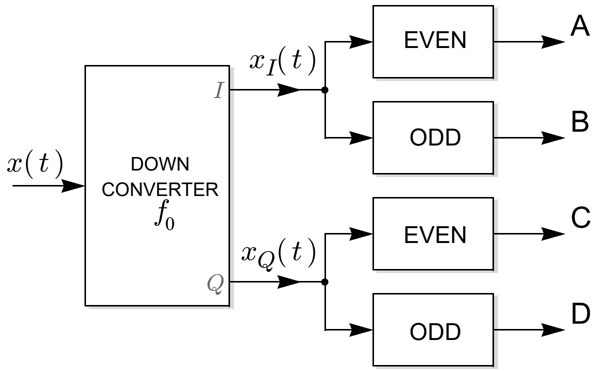
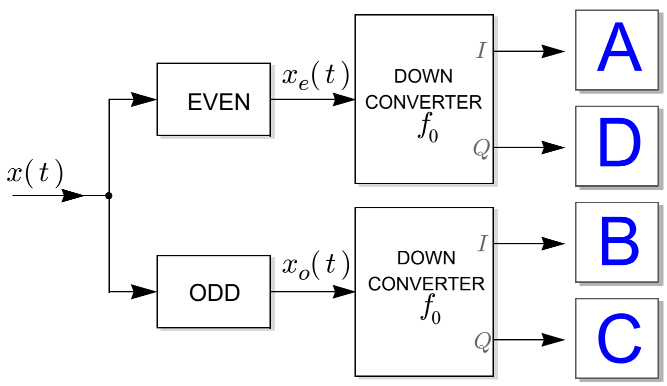


(For example: signal A is the even part of $x_I(t)$, signal B is the odd part of $x_I(t)$, signal C is the even part of $x_Q(t)$, etc.)

The above system first downconverts, and second extracts the even and odd parts. The system below does things *in reverse order*: It starts with the same input $x(t)$, but first extracts the even and odd parts, and then downconverts each:



(a) The two systems shown above produce the same set of four outputs: One of the four outputs of the second system is signal A, another is signal B, etc. Indicate which is which by writing a letter from {A, B, C, D} into each answer box.

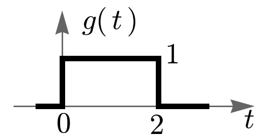
(b) *Explain!* In I&Q form: $x(t) = x_I(t)\cos(\cdot) - x_Q(t)\sin(\cdot)$

$$\Rightarrow x(-t) = x_I(-t)\cos(\cdot) + x_Q(-t)\sin(\cdot)$$

Add $\Rightarrow x_e(t) = \underbrace{\mathcal{E}\{x_I(t)\}}_{A}\cos(\cdot) - \underbrace{\mathcal{O}\{x_Q(t)\}}_{D}\sin(\cdot)$

Subtract $\Rightarrow x_o(t) = \underbrace{\mathcal{O}\{x_I(t)\}}_{B}\cos(\cdot) - \underbrace{\mathcal{E}\{x_Q(t)\}}_{C}\sin(\cdot)$

PROBLEM 2.



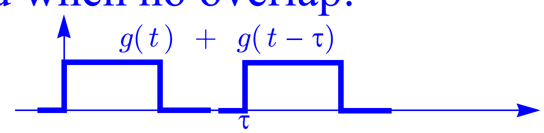
Let $g(t) = u(t) - u(t-2)$ be a rectangle of width 2, and
let $x(t) = g(t) + g(t-\tau)$ for some unspecified delay parameter τ.

(a) The rectangle $g(t)$ has energy $E_g = \boxed{2}$ and power $P_g = \boxed{0}$.

(b) In the remainder of this problem let $E_x(\tau)$ denote the energy of $x(t)$; its dependence on the delay parameter τ is made explicit.

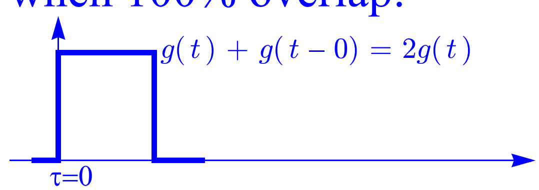
The smallest possible value for $E_x(\tau)$ is achieved when $\tau = \boxed{\text{any } |\tau| > 2}$.

Energy minimized when no overlap:



(c) The largest possible value for $E_x(\tau)$ is achieved when $\tau = \boxed{0}$.

Energy maximized when 100% overlap:



(d) Find a value of τ for which $E_x(\tau) = 5$: $\tau = \boxed{1.5}$.

$$E_x = \int_{-\infty}^{\infty} (g(\tau) + g(t-\tau))^2 dt = 2 + 2 + 2\int_{-\infty}^{\infty} g(\tau) g(t-\tau) dt$$

$$= 4 + 2(2-\tau)$$

$$= 8 - 2\tau \qquad = 5 \text{ when } \tau = 1.5$$

PROBLEM 3.

Consider the following 8 signals, all derived from the "sinc-squared" signal $x(t) = 2\left(\frac{\sin(0.5\pi t)}{\pi t}\right)^2$:

H	(1) $y_1(t) = 4\frac{d}{dt}x(t)$	G	(5) $y_5(t) = x(2t)$
D	(2) $y_2(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$	B	(6) $y_6(t) = 3\pi x^2(t)$
C	(3) $y_3(t) = x(t-0.5\pi)$	F	(7) $y_7(t) = x(0.5t)$
C	(4) $y_4(t) = e^{j\pi^2}x(t)$	C	(8) $y_8(t) = \int_{-\infty}^{\infty} \frac{\sin(5\pi\tau)}{\pi\tau}x(t-\tau)d\tau$

Shown below are plots of several plots of $|Y(j\omega)|$, the Fourier transform magnitude.
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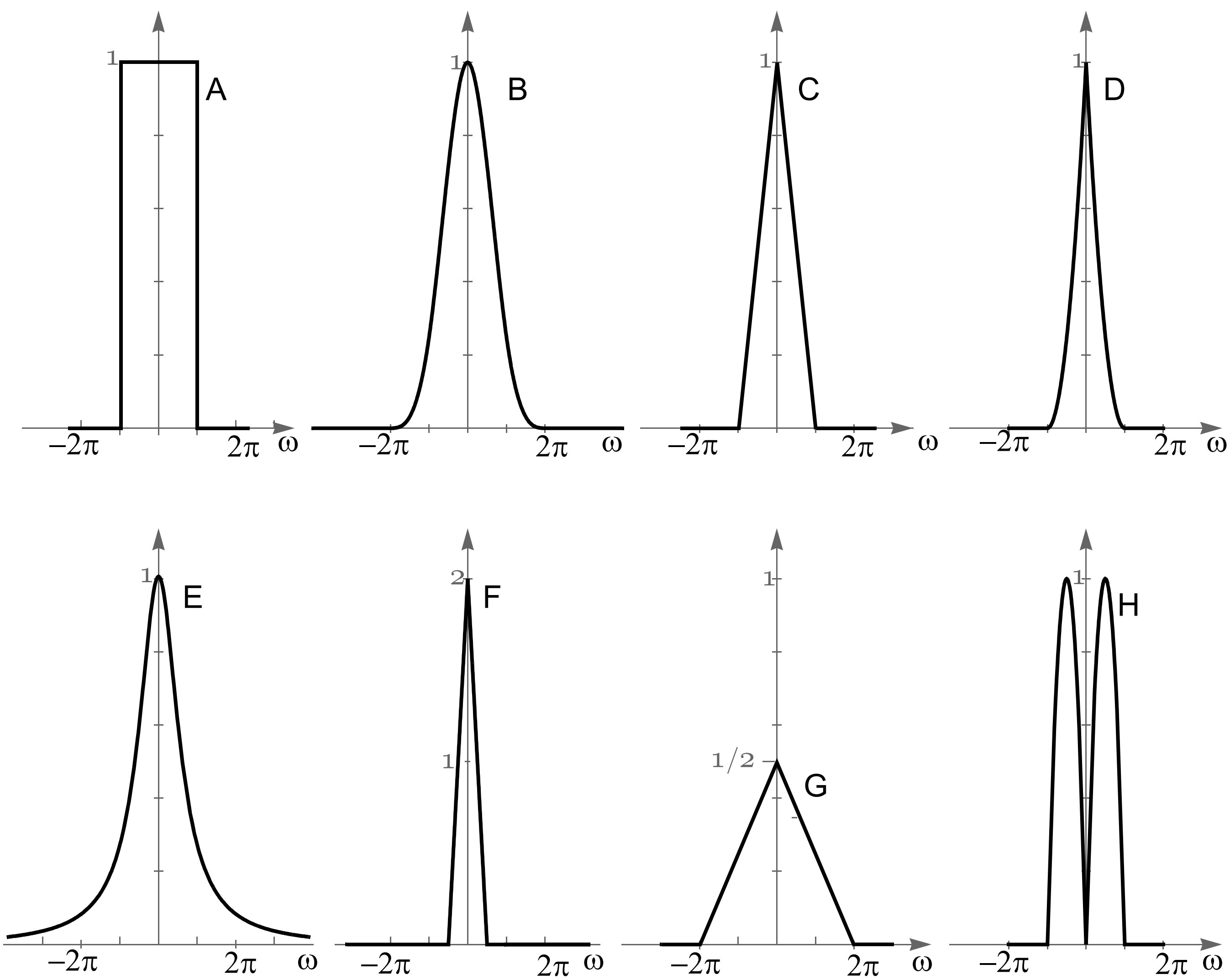

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Below are three systems with input $x(t)$ and output $y(t)$. Specify which properties they satisfy by writing a "Y" (for yes) or "N" (for no) into each answer box:

	memoryless	causal	stable	linear	time-invariant	invertible
(a) $y(t) = x(t^2)$			Y	Y		
(b) $y(t) = x(t^3)$			Y	Y		Y
(c) $y(t) = 3x(t) + x(t-1)$		Y	Y	Y	Y	Y

PROBLEM 5.

A system that is initially at rest (with zero initial conditions) is described by the following differential equation relating the input $x(t)$ to the output $y(t)$:

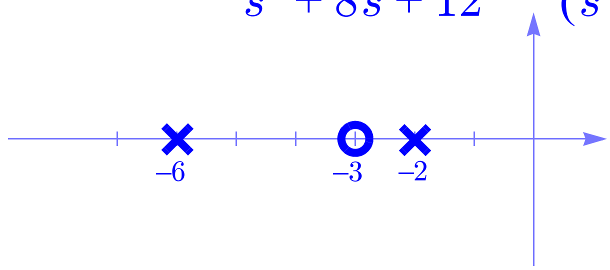
$$2\frac{d^2}{dt^2}y(t) = 2\frac{d}{dt}x(t) + 6x(t) - \beta\frac{d}{dt}y(t) - 24y(t).$$

(a) Sketch the pole-zero plot for the system function $H(s) = Y(s)/X(s)$ when $\beta = 16$.

Take LT of both sides, solve for $Y(s)/X(s)$

$$\Rightarrow H(s) = \frac{2s+6}{2s^2+\beta s+24} = \frac{s+3}{s^2+\frac{\beta}{2}s+12}$$

$$\beta = 16 \Rightarrow H(s) = \frac{s+3}{s^2+8s+12} = \frac{(s+3)}{(s+2)(s+6)}$$



(b) If the **step** response of the system has the form $y(t) = A(1 - e^{-Bt})u(t)$, then it must be that

This is the step resp of a first-order system

$\Rightarrow$ we must have pole-zero cancellation:

$$H(s) = \frac{s+3}{s^2+\frac{\beta}{2}s+12} = \frac{\cancel{s+3}}{(\cancel{s+3})(s+4)} = \frac{1}{s+4}$$

$\beta = 14$

B

$\Rightarrow$ dc gain is $H(0) = A = 1/4$

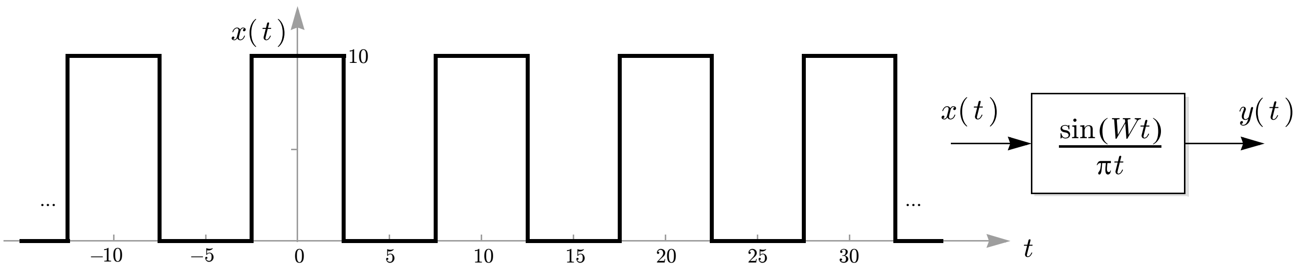
$\beta = \boxed{14}$

$A = \boxed{\frac{1}{4}}$

$B = \boxed{4}$

PROBLEM 6.

Suppose the periodic signal $x(t)$ shown below is fed as an input to an LTI system whose impulse response $h(t) = \frac{\sin(Wt)}{\pi t}$ is a sinc function (the parameter W is positive but otherwise unspecified), producing the output $y(t)$:



(a) For what range of values for the parameter W will the output be a *constant*, say $y(t) = y_0$ for all t?

$$\boxed{0} < W < \boxed{0.2\pi}$$

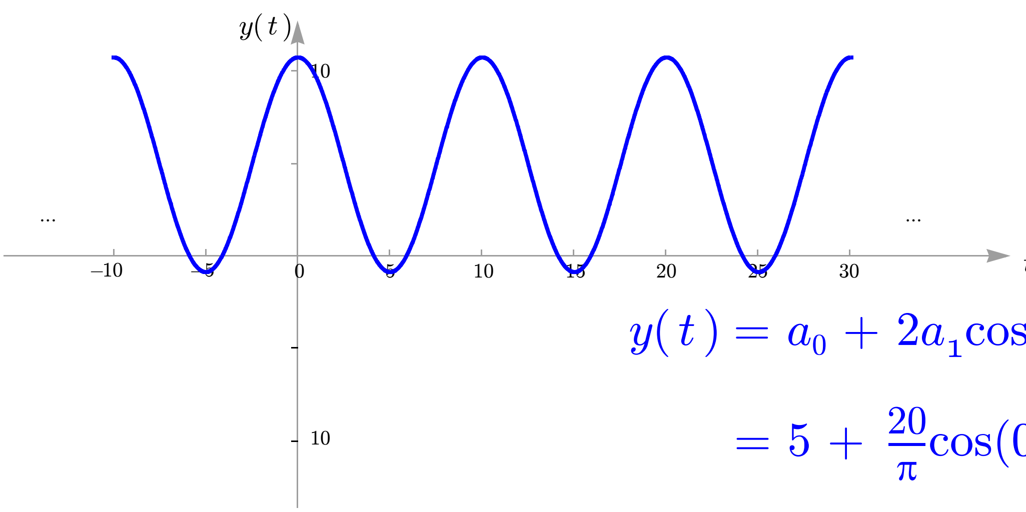
The cutoff frequency must be less than the fundamental

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{10}$$

(b) When W is one of the values from part (a), the output constant will be $y_0 = \boxed{5}$.

The output will be the dc component a_0 of input, scaled by the dc gain $H(0) = 1$ of the filter $\Rightarrow y_0 = a_0 = 5$

(c) In the space below, *carefully* sketch the output $y(t)$ when $W = \frac{\pi^2}{10}$. (Both axes are already labeled.)

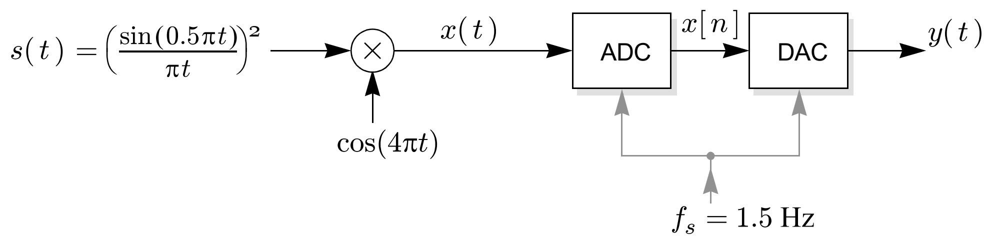


$$y(t) = a_0 + 2a_1\cos(0.2\pi t)$$

$$= 5 + \frac{20}{\pi}\cos(0.2\pi t)$$

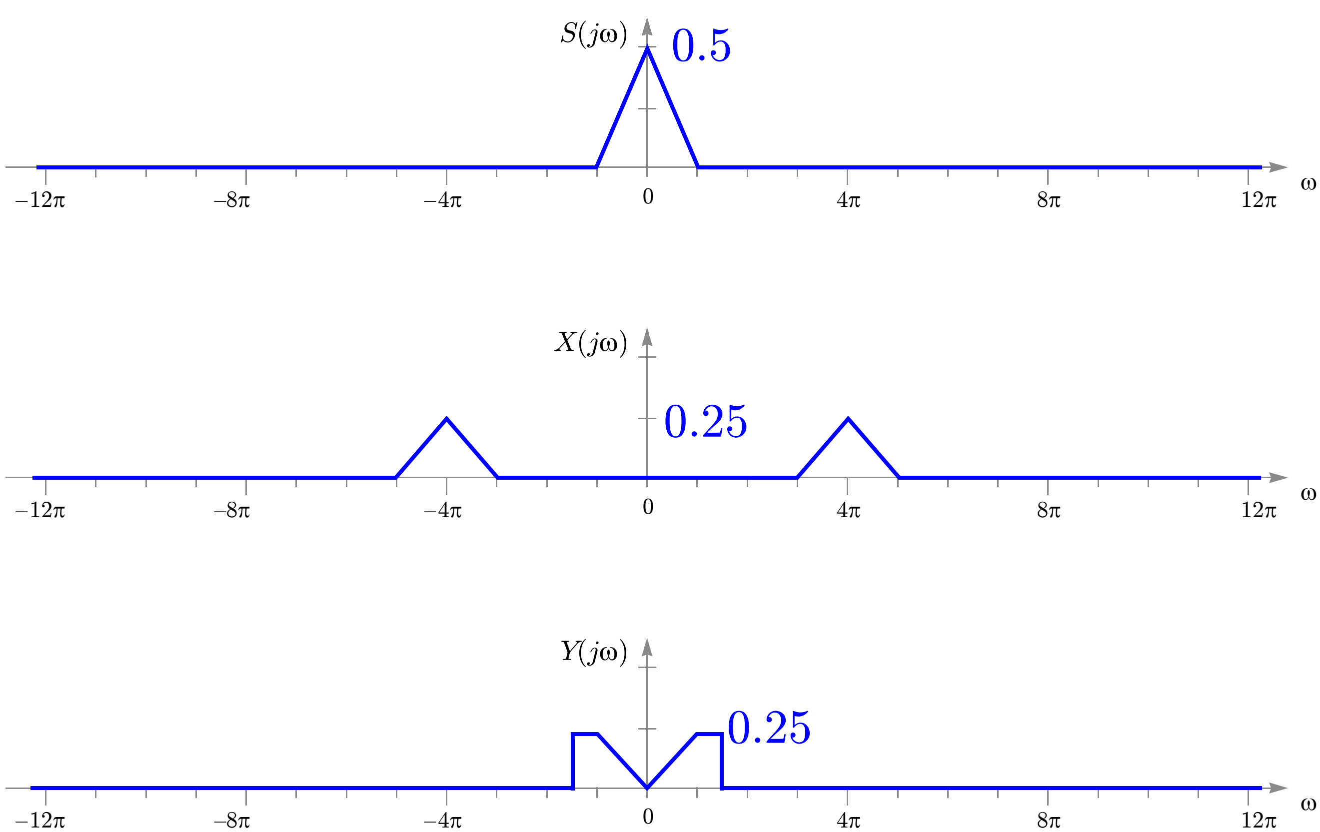
PROBLEM 7.

A sinc-squared function $s(t) = \left(\frac{\sin(0.5\pi t)}{\pi t}\right)^2$ is first AM-modulated with carrier 4π rad/s, producing $x(t)$, which is then fed to a back-to-back connection of an ideal analog-to-digital converter (ADC) and an ideal digital-analog (DAC) converter, both with sampling rate $f_s = 1.5$ Hz:



(a) The zero-th ADC sample is $x[0] = \boxed{0.25}$.

(b) Sketch in the space below the Fourier transform of the three signals $s(t)$, $x(t)$ and $y(t)$:



(c) Label the *y*-axis in all three sketches of part (b).

(d) The DAC output $y(t)$ can be written in the form $y(t) = A\dfrac{\sin(Bt)}{t} - C\left(\dfrac{\sin(Dt)}{t}\right)^2$, where

$A = \boxed{\dfrac{1}{4\pi}}$

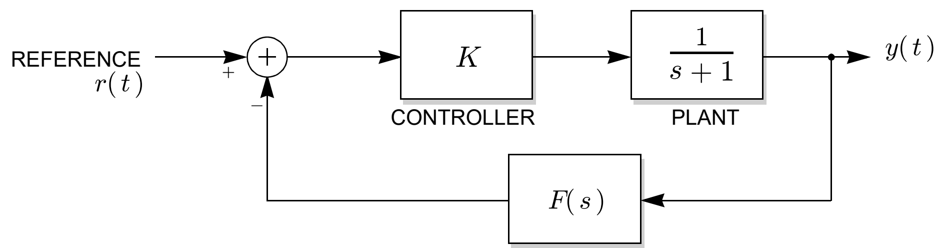
$B = \boxed{1.5\pi}$

$C = \boxed{\dfrac{1}{2\pi^2}}$

$D = \boxed{0.5\pi}$

PROBLEM 8.

Consider the control system shown below:



(a) When $F(s) = 1$,

the range of values for K that make the closed-loop system *stable* is $\boxed{K > -1}$.

Closed-loop transfer function is

$$H(s) = \frac{K}{s + K + 1} \quad \Rightarrow \text{pole at } -K-1\text{, in left plane when}$$

(b) *Delay in the Feedback Path* — Let's consider what happens when the feedback path has **delay**, so that the error signal is not $e(t) = r(t) - y(t)$, but instead is $e(t) = r(t) - y(t - \tau)$, where τ is the feedback delay. In principle we can model this delay by choosing the feedback transfer function shown in the above diagram to be $F(s) = e^{-s\tau}$, but this would lead to an irrational transfer function that prevents us from thinking about poles and zeros. Therefore, let us instead adopt the *Padé* approximation, namely $e^{-s\tau} \approx \frac{1 - s\tau/2}{1 + s\tau/2}$, and set the feedback transfer function to:

$$F(s) = \frac{1 - s\tau/2}{1 + s\tau/2} = \frac{1 - s}{1 + s}$$

With this choice for $F(s)$, and with a delay of $\tau = 2$,

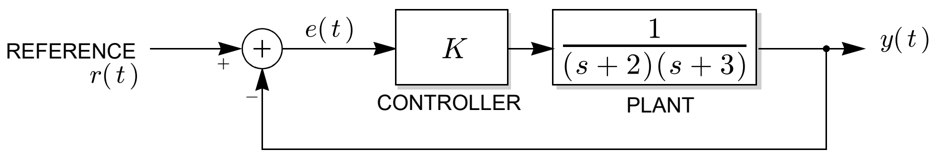
the range of values for K that make the closed-loop system *stable* is $\boxed{-1 < K < 2}$.

$$\text{Closed-loop } H(s) = \frac{Ks + K}{s^2 + \underbrace{(2 - K)}s + \underbrace{K + 1}}$$

these must be positive

PROBLEM 9.

Consider P-control for a second-order plant with transfer function $G_p(s) = \dfrac{1}{(s+2)(s+3)}$, as shown below:



(a) If the reference is a unit step, $r(t) = u(t)$, the *steady-state error* (expressed as a function of the controller gain K) is

$$e_{ss} = \lim_{t\to\infty} e(t) = \boxed{\frac{6}{K+6}}.$$

Closed-loop transfer function is

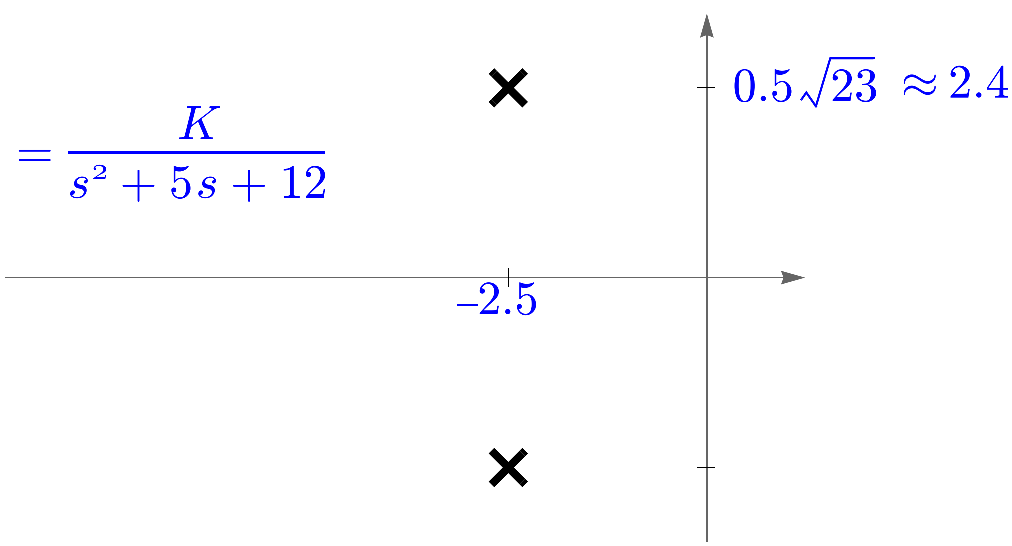
$$H(s) = \frac{K}{s^2 + 5s + 6 + K}$$

$\Rightarrow y(t)$ converges to dc gain is $H(0) = \dfrac{K}{6+K}$

$\Rightarrow e(t)$ converges to $1 - H(0)$

(b) Sketch a pole-zero plot for the closed-loop transfer function $H(s) = Y(s)/R(s)$ when $K = 6$:

$$H(s) = \frac{K}{s^2 + 5s + 12}$$



(c) The closed-loop system is *critically damped* when $K = \boxed{0.25}$.

Choose K so that both poles have same location:

$$H(s) = \frac{K}{s^2 + 5s + 6 + K} = \frac{K}{(s+a)^2}$$

$$\Rightarrow a = 2.5$$

$$\Rightarrow K = a^2 - 6 = 1/4$$

(d) To achieve a damping coefficient of $\zeta = \frac{1}{\sqrt{2}}$, the gain must be $K = \boxed{6.5}$.

Looking at the denominator of $H(s)$,

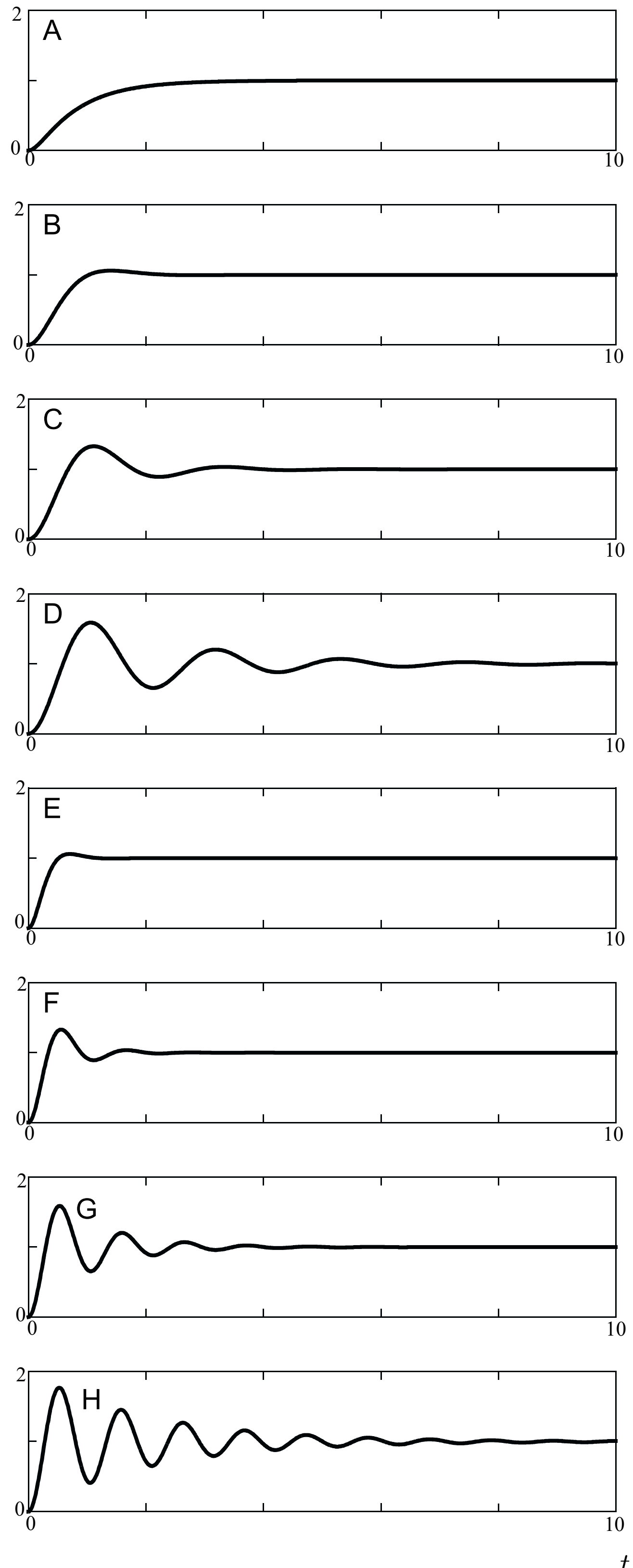
the coeff of s is $2\zeta\omega_n$, while the constant term is ω_n^2

$$\Rightarrow 5 = 2\zeta\omega_n = 2\zeta\sqrt{6+K}$$

$$\Rightarrow 6 + K = \frac{25}{4\zeta^2} = 6.5$$

PROBLEM 10.

Match each transfer function below with its corresponding *step response* shown to the right, by writing a letter from {A ... H} into each answer box:



(a) H $H(s) = \dfrac{36}{s^2 + s + 36}$

(b) G $H(s) = \dfrac{36}{s^2 + 2s + 36}$

(c) F $H(s) = \dfrac{36}{s^2 + 4s + 36}$

(d) E $H(s) = \dfrac{36}{s^2 + 8s + 36}$

(e) A $H(s) = \dfrac{9}{s^2 + 8s + 9}$

(f) B $H(s) = \dfrac{9}{s^2 + 4s + 9}$

(g) C $H(s) = \dfrac{9}{s^2 + 2s + 9}$

(h) D $H(s) = \dfrac{9}{s^2 + s + 9}$