ECE 3084

FINAL EXAM

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
DECEMBER 14, 2017

Name: ____

1. The exam is *closed book* & notes, except for three 2-sided sheets of handwritten notes.
2. Silence your phone and put it away. No tablets/laptops/WiFi/etc. No calculators.
3. Final answers must be entered into the answer box.
4. Correct answers must be accompanied by concise justifications to receive full credit.
5. Do not attach additional sheets. If necessary, use the back of the previous page.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL:	100	

PROBLEM 1.

Below are three systems with input $x(t)$ and output $y(t)$. Specify which properties they satisfy by writing a "Y" (for yes) or "N" (for no) into each answer box:

(a) $y(t) = (2 + \sin(t))x(t)$

memoryless	causal	stable	linear	time-invariant	invertible

(b) $y(t) = x(2t)$

memoryless	causal	stable	linear	time-invariant	invertible

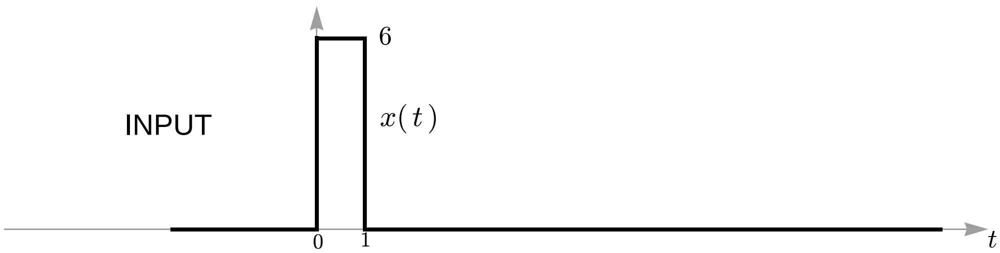
(c) $y(t) = \max\{x(t),\, x(t-1)\}$

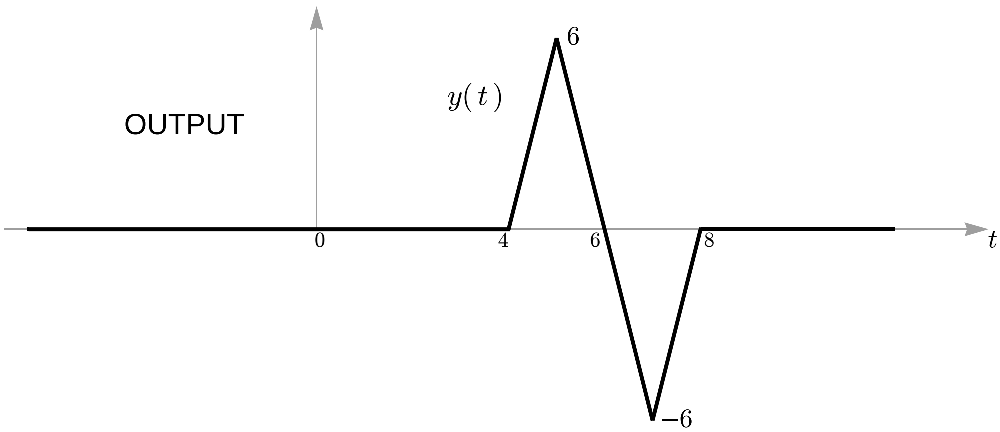
(At each time t, this system compares the inputs at time t and at time $t-1$, and outputs the bigger of the two.)

memoryless	causal	stable	linear	time-invariant	invertible

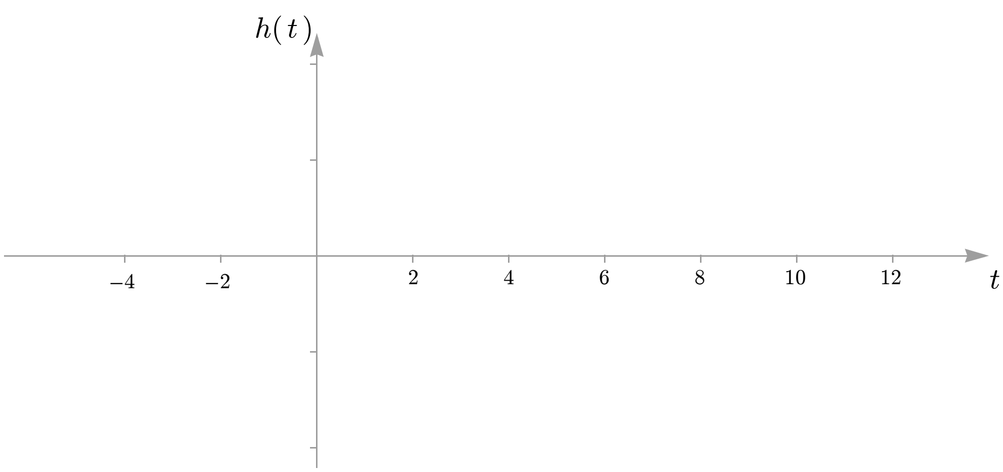
PROBLEM 2.

Suppose the rectangular signal $x(t)$ shown below is fed as the input to a linear time-invariant system, resulting in the output signal $y(t)$ shown below:



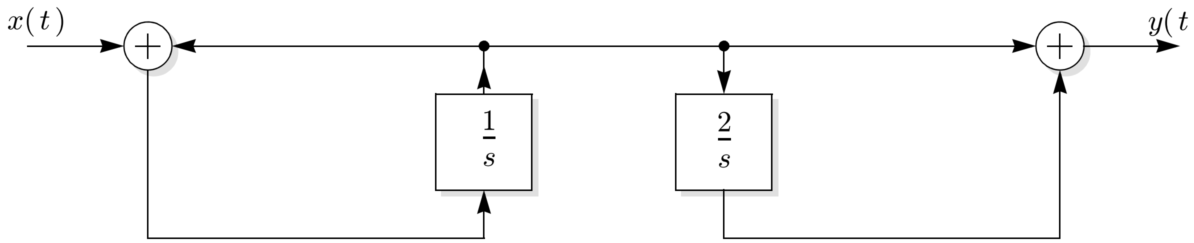


In the space below, carefully sketch the impulse response $h(t)$ of the system, labeling both axes:



PROBLEM 3.

Consider the LTI system shown below:



(a) The impulse response of the overall system (with input $x(t)$ and output $y(t)$) can be written as:

$$h(t) = (A + (B + Ct)e^{-Dt})u(t),$$

where $A =$ ______, $B =$ ______, $C =$ ______, $D =$ ______.

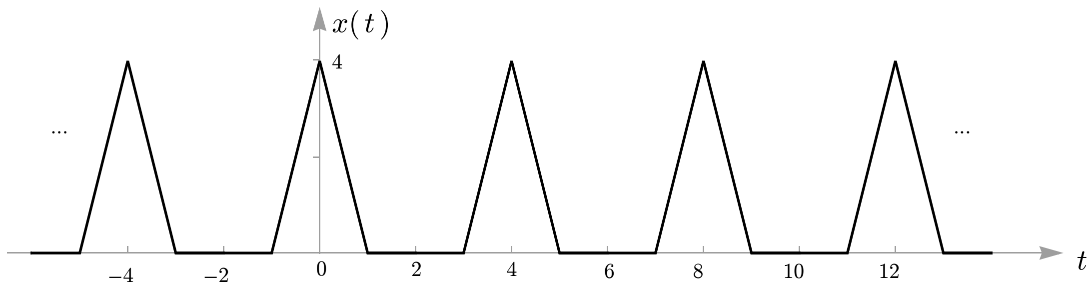
(b) The differential equation relating the input $x(t)$ to the output $y(t)$ can be written as:

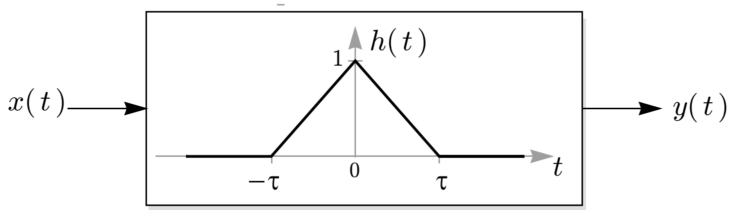
$$\frac{d^2}{dt^2}y(t) + a_1\frac{d}{dt}y(t) + a_0y(t) = b_2\frac{d^2}{dt^2}x(t) + b_1\frac{d}{dt}x(t) + b_0x(t),$$

where $b_2 =$ ______, $b_1 =$ ______, $b_0 =$ ______.

and $a_1 =$ ______, $a_0 =$ ______.

PROBLEM 4. Consider the periodic signal $x(t)$ shown below:





Suppose $x(t)$ is fed as an input to an LTI system with impulse response $h(t)$, producing the output $y(t)$. The impulse response $h(t)$ is triangular, centered at zero, with width 2τ and height 1.

(a) The fundamental frequency of the periodic signal $x(t)$ is $f_0 =$ $\boxed{}$ Hz.

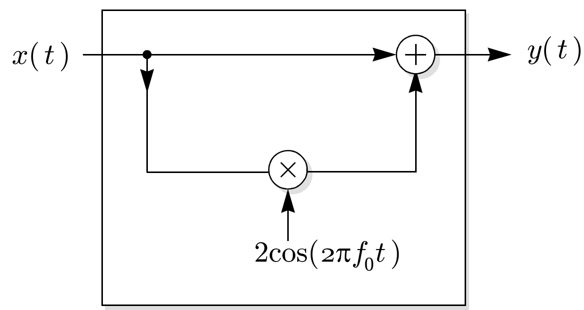
(b) In the FS representation $x(t) = \sum_k a_k e^{jk2\pi f_0 t}$, the zero-th coefficient is $a_0 =$ $\boxed{}$.

(c) If the output is a constant, $y(t) = y_0$ for all t, then it must be that:

$\tau =$ $\boxed{}$ and $y_0 =$ $\boxed{}$.

PROBLEM 5.

Consider the system shown below, whose output $y(t)$ is the sum of the input $x(t)$ and the input *multiplied by a sinusoid* $2\cos(2\pi f_0 t)$:



Find the sinusoid frequency f_0 such that a sinc-squared input of the form $x(t) = \left(\frac{\sin(2\pi t)}{\pi t}\right)^2$

results in an output that can be written as the product of two *different* sinc functions:

$$y(t) = A\left(\frac{\sin(Bt)}{t}\right)\left(\frac{\sin(Ct)}{t}\right), \qquad \text{where } B \neq C.$$

$\Rightarrow$ $f_0 = \boxed{}$ Hz.

(You need not specify the constants A, B, and C.)

PROBLEM 6.

(a) TRUE FALSE

If $x_e(t)$ and $x_o(t)$ are the even and odd parts of a signal $x(t)$, then they must have the same energy: $\int_{-\infty}^{\infty} x_e^2(t)dt = \int_{-\infty}^{\infty} x_o^2(t)dt$.

Explain!

(b) TRUE FALSE

The power of a signal is never larger that its energy.

Explain!

(c) TRUE FALSE

There exists a nonzero signal that is both even and odd.

Explain!

(d) TRUE FALSE

A second-order system that exhibits a resonance peak in its magnitude response cannot be underdamped.

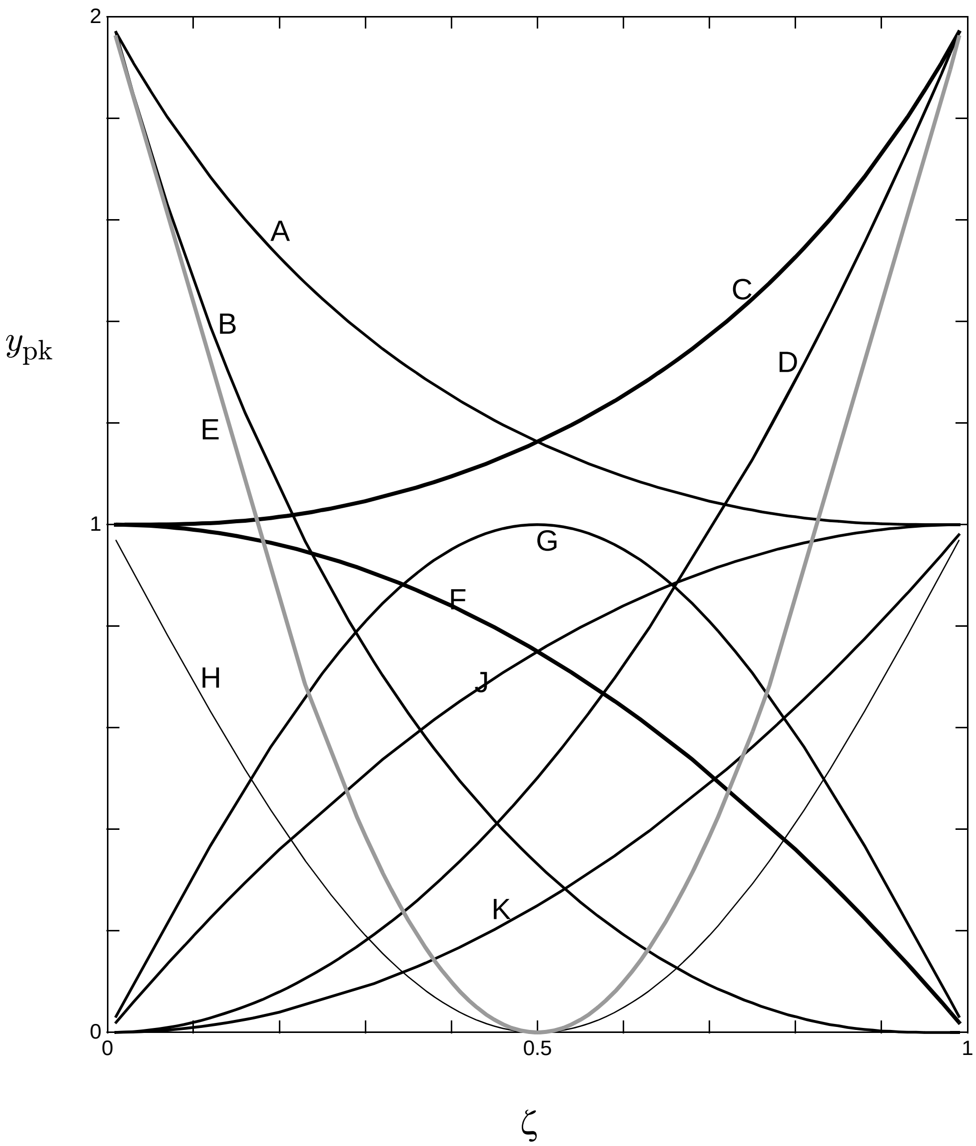
Explain!

PROBLEM 7.

Let $y(t)$ be the step response of the second-order system $H(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.
Let $y_{\rm pk} = \max\{y(t)\colon -\infty < t < \infty\}$ denote its peak value.

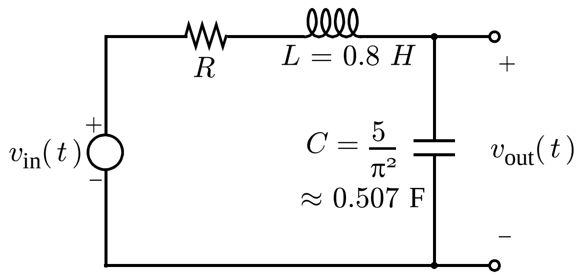
Identify which of the following curves is a plot of $y_{\rm pk}$ versus ζ
by writing the corresponding letter (A through K) in the answer box: $\boxed{}$.

Explain!



PROBLEM 8.

Shown below is a circuit diagram of a filter, with a $L = 0.8$ H inductor and a $C = 5/\pi^2 \approx 0.507$ F capacitor:



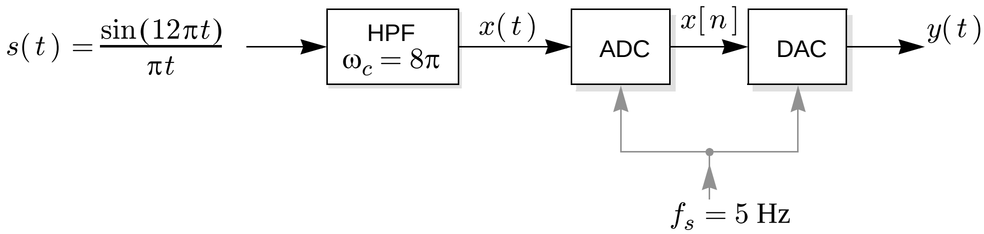
(a) The natural frequency of this filter is $\omega_n =$ $\boxed{}$ rad/s.

(b) The filter is a [LPF][BPF][HPF] (circle one).

(c) For what range of values resistor values will the magnitude response of this filter exhibit a resonance peak in the frequency domain? $R <$ $\boxed{}$ Ω.

PROBLEM 9.

A sinc function $s(t) = \sin(12\pi t)/(\pi t)$ is passed through an ideal high-pass filter with cutoff 8π (its frequency response is $H(j\omega) = 0$ for $|\omega| < 8\pi$, and $H(j\omega) = 1$ for $|\omega| > 8\pi$). The HPF output $x(t)$ is then passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter, both with sampling rate $f_s = 5$ Hz:

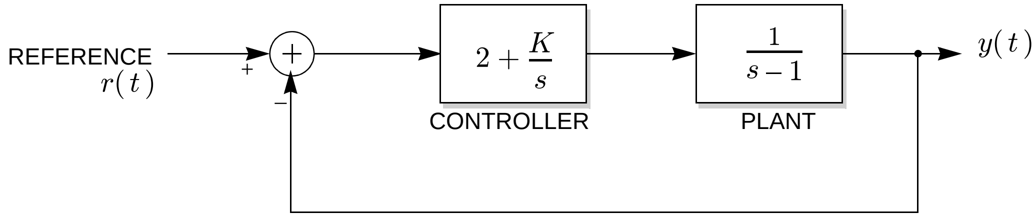


Find an equation for the DAC output $y(t)$, simplified as much as possible:

$y(t) = \boxed{\quad}$.

PROBLEM 10.

Consider the control system shown below:



(a) When $K \neq 0$, this is best described as [P][PI][PD][PID] control (circle one).

(b) Is the plant stable? YES NO

(c) Sketch the pole-zero plot of the closed-loop system $H(s) = Y(s)/R(s)$ when $K = 0$:

If you find that any of the following are not possible, write "N.P." in the answer box.

(d) The range of values for K that make the closed-loop system *stable* is $\boxed{}$.

(e) The closed-loop system is *critically damped* when $K = \boxed{}$.

(f) The closed-loop damping ratio is $\zeta = 1/\sqrt{2}$ when $K = \boxed{}$.

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PROBLEM 1.

Below are three systems with input $x(t)$ and output $y(t)$. Specify which properties they satisfy by writing a “Y” (for yes) or “N” (for no) into each answer box:

(a) $y(t) = (2 + \sin(t))x(t)$

memoryless	causal	stable	linear	time-invariant	invertible
			N		

(others are yes)

(b) $y(t) = x(2t)$

memoryless	causal	stable	linear	time-invariant	invertible
N	N			N	

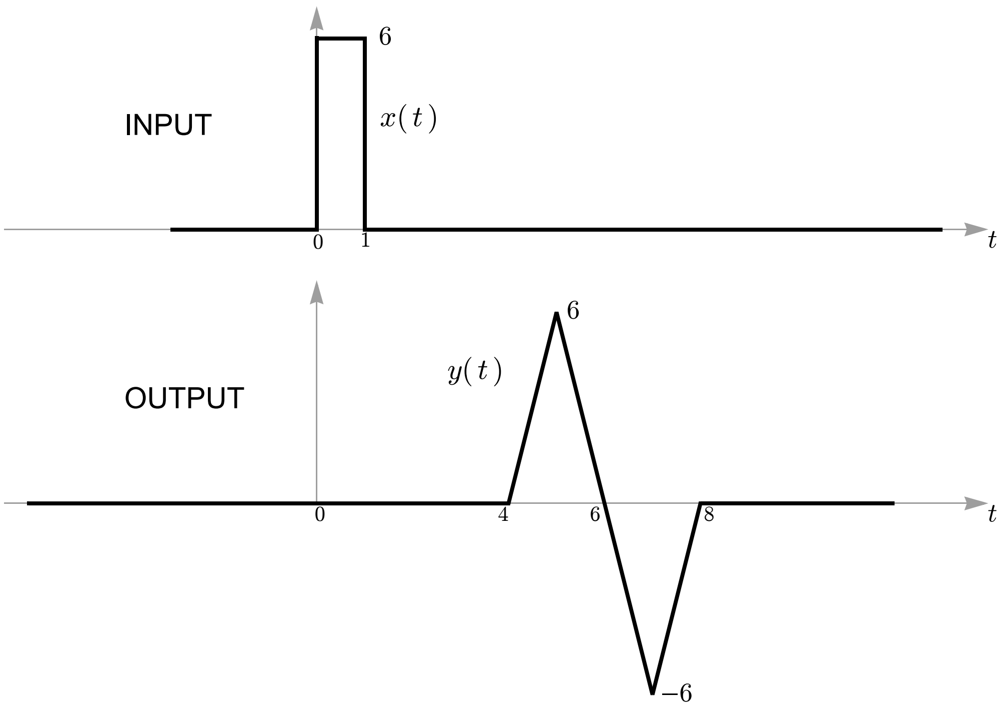
(c) $y(t) = \max\{x(t), x(t-1)\}$

(At each time t, this system compares the inputs at time t and at time $t-1$, and outputs the bigger of the two.)

memoryless	causal	stable	linear	time-invariant	invertible
N			N		N

PROBLEM 2.

Suppose the rectangular signal $x(t)$ shown below is fed as the input to a linear time-invariant system, resulting in the output signal $y(t)$ shown below:



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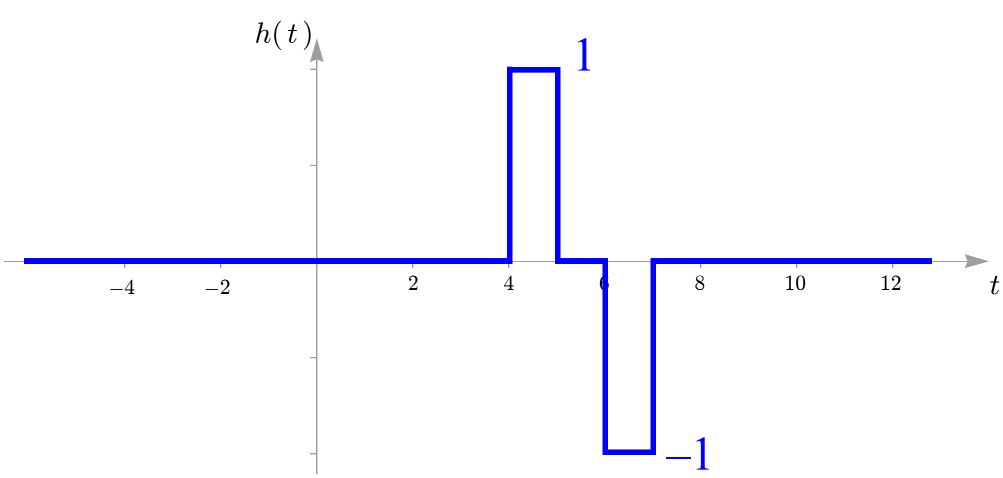

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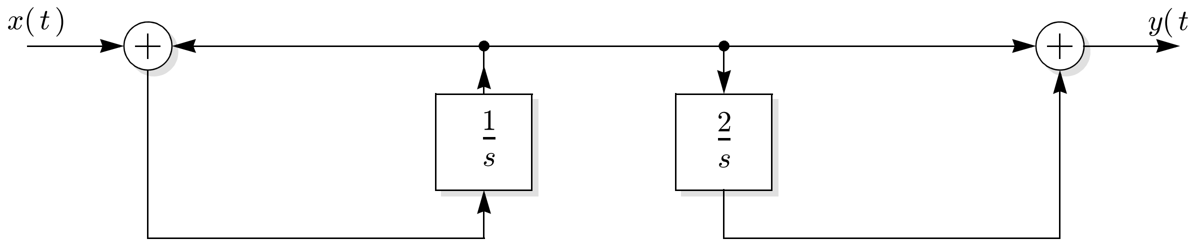


(a) The impulse response of the overall system (with input $x(t)$ and output $y(t)$) can be written as:

$$h(t) = (A + (B + Ct)e^{-Dt})u(t),$$

where $A =$ $\boxed{-2}$, $B =$ $\boxed{3}$, $C =$ $\boxed{0}$, $D =$ $\boxed{-1}$.

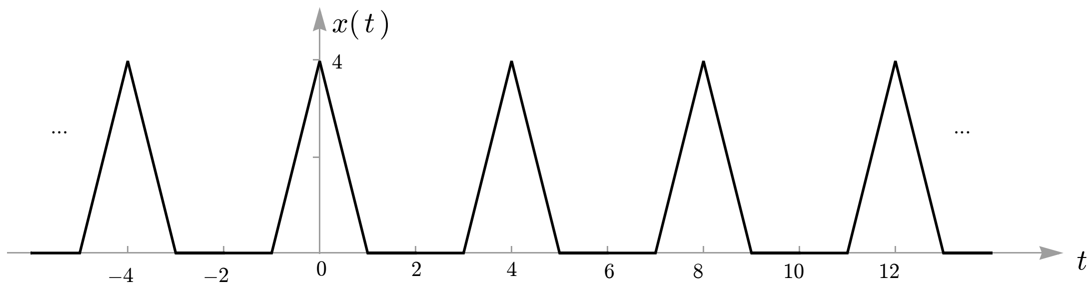
(b) The differential equation relating the input $x(t)$ to the output $y(t)$ can be written as:

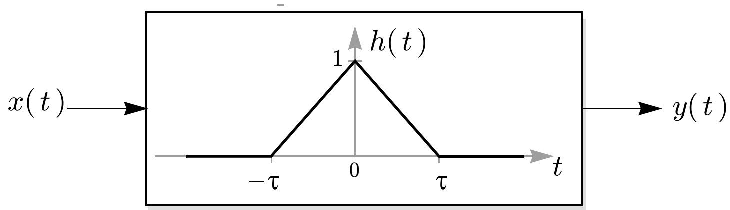
$$\frac{d^2}{dt^2}y(t) + a_1\frac{d}{dt}y(t) + a_0 y(t) = b_2\frac{d^2}{dt^2}x(t) + b_1\frac{d}{dt}x(t) + b_0 x(t),$$

where $b_2 =$ $\boxed{0}$, $b_1 =$ $\boxed{1}$, $b_0 =$ $\boxed{2}$.

and $a_1 =$ $\boxed{-1}$, $a_0 =$ $\boxed{0}$.

PROBLEM 4. Consider the periodic signal $x(t)$ shown below:





Suppose $x(t)$ is fed as an input to an LTI system with impulse response $h(t)$, producing the output $y(t)$. The impulse response $h(t)$ is triangular, centered at zero, with width 2τ and height 1.

(a) The fundamental frequency of the periodic signal $x(t)$ is $f_0 =$ $\boxed{0.25}$ Hz.

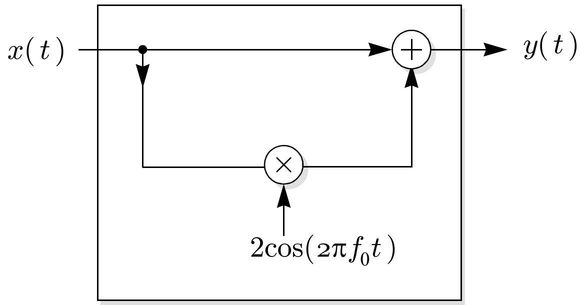
(b) In the FS representation $x(t) = \sum_k a_k e^{jk2\pi f_0 t}$, the zero-th coefficient is $a_0 =$ $\boxed{1}$.

(c) If the output is a constant, $y(t) = y_0$ for all t, then it must be that:

$\tau =$ $\boxed{4}$ and $y_0 =$ $\boxed{4}$.

PROBLEM 5.

Consider the system shown below, whose output $y(t)$ is the sum of the input $x(t)$ and the input *multiplied by a sinusoid* $2\cos(2\pi f_0 t)$:



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results in an output that can be written as the product of two *different* sinc functions:

$$y(t) = A\left(\frac{\sin(Bt)}{t}\right)\left(\frac{\sin(Ct)}{t}\right), \qquad \text{where } B \neq C.$$

$$\Rightarrow \qquad f_0 = \boxed{2} \text{ Hz.}$$

(You need not specify the constants A, B, and C.)

PROBLEM 6.

(a) TRUE ☐ FALSE ☒ If $x_e(t)$ and $x_o(t)$ are the even and odd parts of a signal $x(t)$, then they must have the same energy: $\int_{-\infty}^{\infty} x_e^2(t)dt = \int_{-\infty}^{\infty} x_o^2(t)dt$.

Explain!

When signal is even, one integral is zero, the other is not.

(b) TRUE ☒ FALSE ☐ The power of a signal is never larger that its energy.

$$P = \frac{E}{\infty}$$

Explain!

(c) TRUE ☐ FALSE ☒ There exists a nonzero signal that is both even and odd.

Explain!

(d) TRUE ☐ FALSE ☒ A second-order system that exhibits a resonance peak in its magnitude response cannot be underdamped.

Explain!

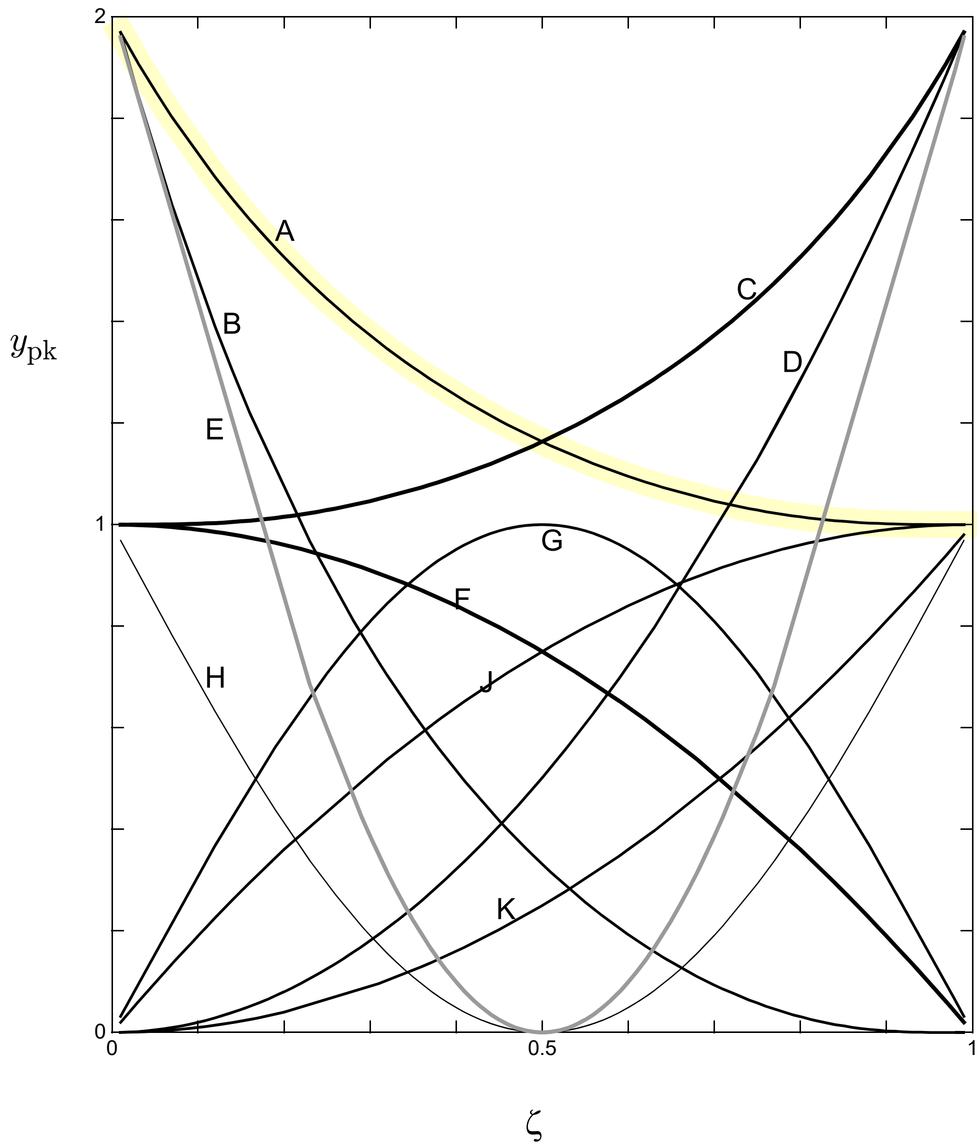
PROBLEM 7.

Let $y(t)$ be the step response of the second-order system $H(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.
Let $y_{\text{pk}} = \max\{y(t): -\infty < t < \infty\}$ denote its peak value.

Identify which of the following curves is a plot of y_{pk} versus ζ
by writing the corresponding letter (A through K) in the answer box: $\boxed{\text{A}}$.

Explain!

There will be lots of overshoot when ζ is small. As ζ increases, there will be less and less. At $\zeta = 1$, there will be no overshoot.
Only curve A starts out big and ends at $y_{\text{pk}} = 1$.



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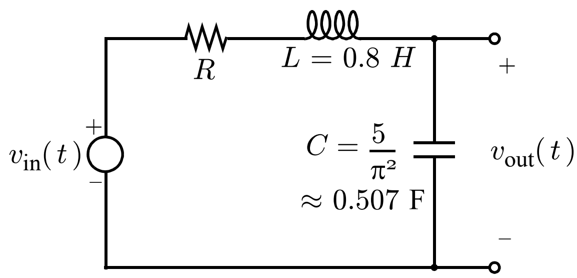


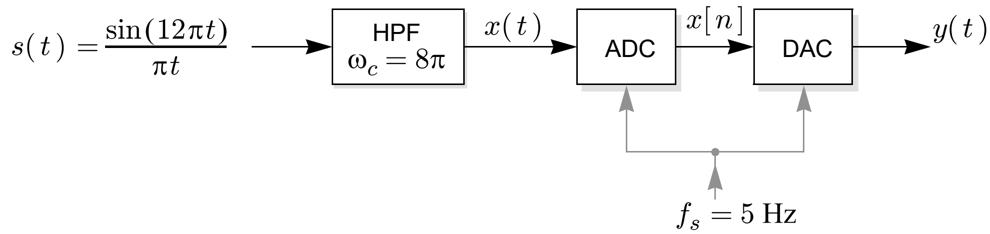
(a) The natural frequency of this filter is $\omega_n = \boxed{\frac{\pi}{2}}$ rad/s.

(b) The filter is a [LPF][BPF][HPF] (circle one).

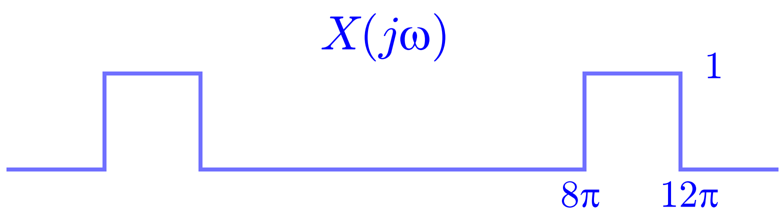
(c) For what range of values resistor values will the magnitude response of this filter exhibit a resonance peak in the frequency domain? $R < \boxed{1.78}$ Ω.

PROBLEM 9.

A sinc function $s(t) = \sin(12\pi t)/(\pi t)$ is passed through an ideal high-pass filter with cutoff 8π (its frequency response is $H(j\omega) = 0$ for $|\omega| < 8\pi$, and $H(j\omega) = 1$ for $|\omega| > 8\pi$). The HPF output $x(t)$ is then passed as an input into a back-to-back connection of an ideal analog-to-digital converter (ADC) followed by an ideal digital-to-analog (DAC) converter, both with sampling rate $f_s = 5$ Hz:



Find an equation for the DAC output $y(t)$, simplified as much as possible:



$$\Rightarrow x(t) = 2g(t)\cos(10\pi t)$$

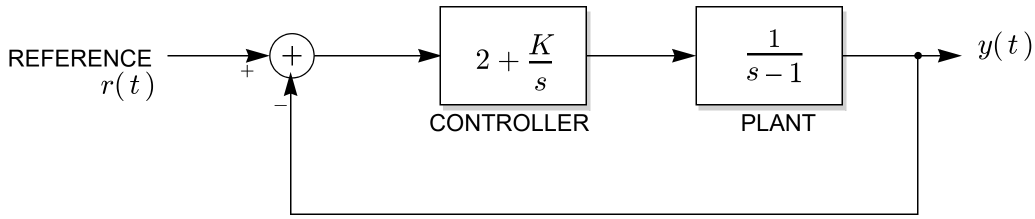
$$\Rightarrow x[n] = 2g\left(\frac{n}{5}\right)\cos\left(10\pi\frac{n}{5}\right)$$

$$= 2g\left(\frac{n}{f_s}\right) \qquad \Rightarrow y(t) = 2g(t)$$

$y(t) = \boxed{2\dfrac{\sin(2\pi t)}{\pi t}}$.

PROBLEM 10.

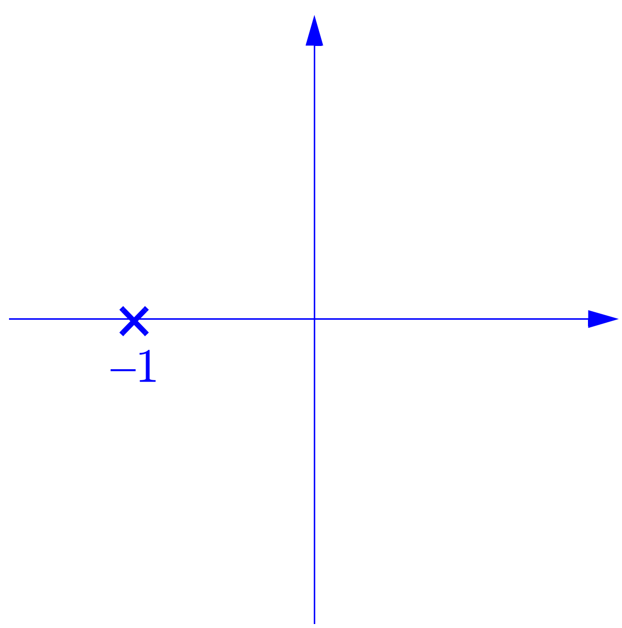
Consider the control system shown below:



(a) When $K \neq 0$, this is best described as [P][PI][PD][PID] control (circle one).

(b) Is the plant stable? YES ☐ NO ☒

(c) Sketch the pole-zero plot of the closed-loop system $H(s) = Y(s)/R(s)$ when $K = 0$:



If you find that any of the following are not possible, write "N.P." in the answer box.

(d) The range of values for K that make the closed-loop system *stable* is $\boxed{K \in [0,\ \infty)}$.

(e) The closed-loop system is *critically damped* when $K = \boxed{0.25}$.

(f) The closed-loop damping ratio is $\zeta = 1/\sqrt{2}$ when $K = \boxed{0.5}$