GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 09-Dec-13 COURSE: ECE 3084A (Prof. Michaels)

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

- Write your name on the front page ONLY. Do not unstaple the test. You should have an additional sheet with Fourier transform tables on one side and Laplace transform tables on the other.
- No calculators, laptops, phones, or other electronic devices allowed. Keep the tables clear of all back backs, books, etc.
- Closed book. However, two pages ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. It's OK to write on both sides.
- Unless stated otherwise, justify your reasoning clearly to receive any partial credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.
- The room is small for the number of students in this section. **BE CAREFUL TO NOT LET YOUR EYES WANDER.** Any sort of communication with your fellow students during this exam is strictly forbidden. Any attempt to read off of your neighbor's exam will result in unpleasant disciplinary action.
- Good luck!

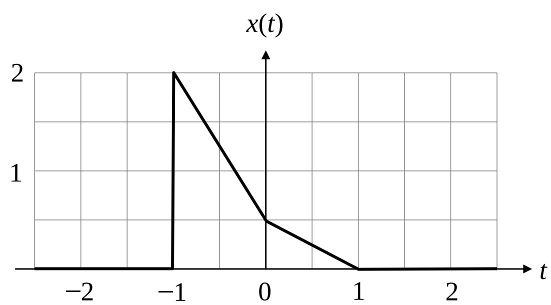
Problem	*Value*	*Score*
1	40	
2	25	
3	20	
4	35	
5	30	
6	25	
7	25	
Total	200	

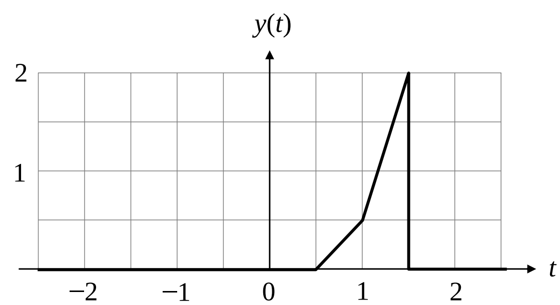
Problem F.1:
Short problem assortment. The six parts of this problem are unrelated to each other.

(a) (5 pts) Consider the system with input $x(t)$ and output $y(t)$ described by the following equation:
$y(t) = \frac{1}{x(t)}$. Circle "yes" or "no" in each row of the table below to indicate whether or not this system is linear, time-invariant, and/or causal.

Linear	yes	no
Time-Invariant	yes	no
Causal	yes	no

(b) (5 pts) The functions $x(t)$ and $y(t)$ are shown below where $y(t) = x(Bt + C)$. Find B and C and write them below the figure.



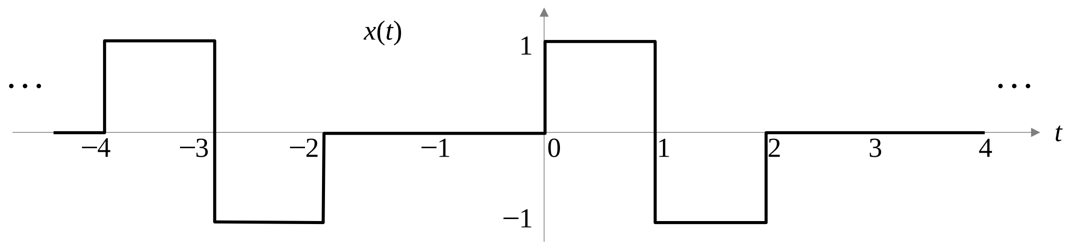


$B =$

$C =$

(c) (5 pts) Simplify the following expression: $\int_0^t \cos(\pi\tau)[\delta(\tau+2) + \delta(\tau-1)]d\tau =$

(d) (10 pts) Consider the Fourier series of the periodic signal $x(t)$ shown below with a period of $T_0 = 4$s. In the table below, circle *exactly one* attribute for each of the $k = 0$, $k = 1$, and $k = 2$ Fourier coefficients; that is, indicate if a_k is either *zero*, *non-zero real*, *imaginary* or *complex* (non-zero real and imaginary parts). You should not need to evaluate any integrals.



$k = 0$	zero	non-zero real	imaginary	complex
$k = 1$	zero	non-zero real	imaginary	complex
$k = 2$	zero	non-zero real	imaginary	complex

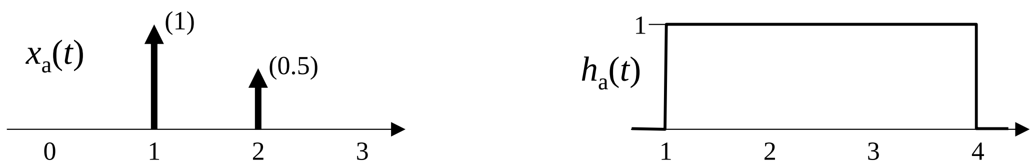
(e) (5 pts) Find $X(j\omega)$, the Fourier transform of the signal $x(t) = \sum_{n=0}^{\infty} e^{-at}\delta(t - nT)$

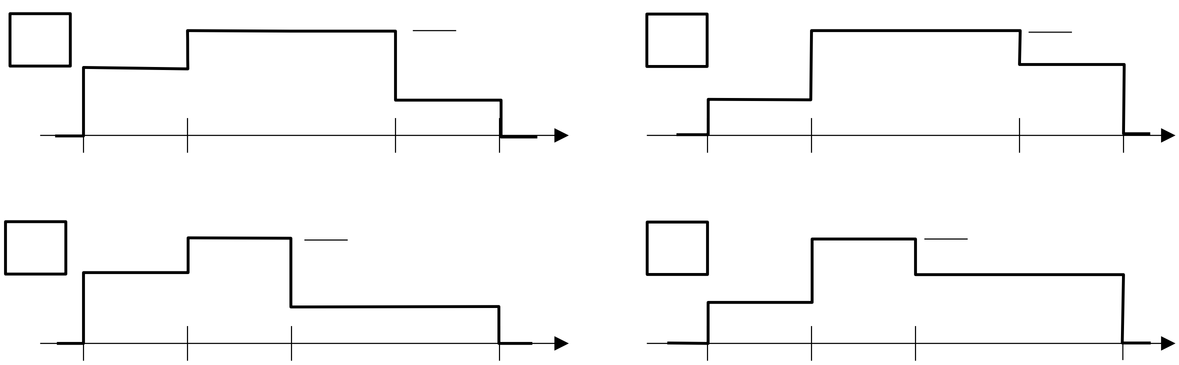
(f) (10 pts) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = u(\omega + 2) - u(\omega + 1) + u(\omega - 1) - u(\omega - 2)$. You may find it helpful to sketch $X(j\omega)$ first.

Problem F.2:

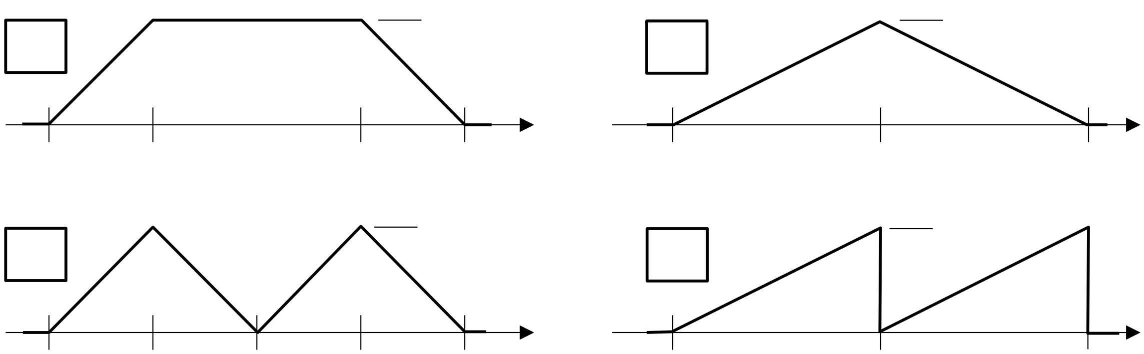
For each part of this problem, consider the convolution $y(t) = x(t) * h(t)$. For each part, $x(t)$ and $h(t)$ are different, and four choices for $y(t)$ are shown. Indicate your choice of $y(t)$ by putting an "X" inside the box adjacent to your selected plot. For each chosen $y(t)$, label its horizontal axis at all tic marks shown, and also label the vertical axis to show the maximum value.

(a) (5 points)

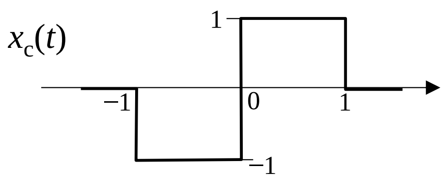


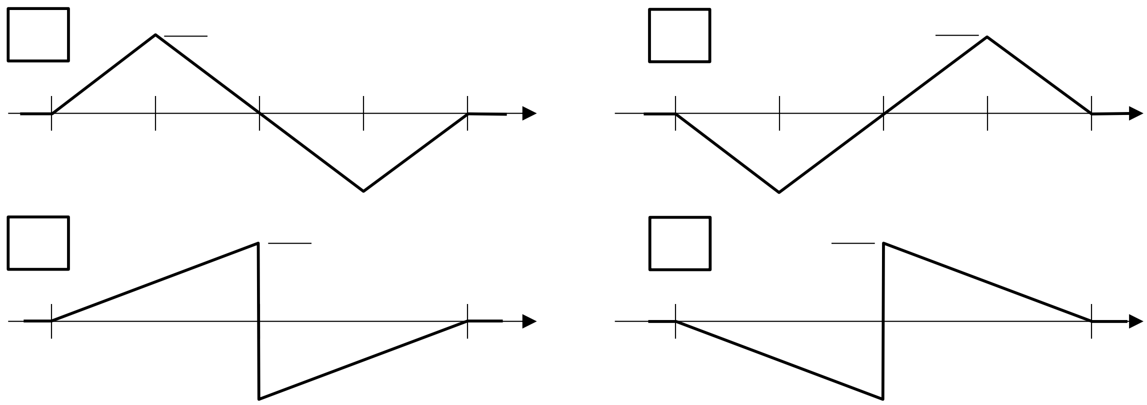


(b) (5 points)

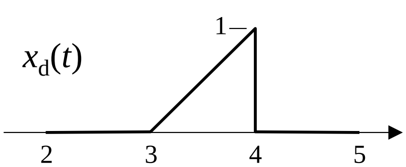


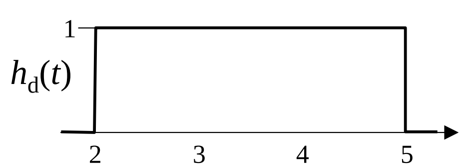
(c) (5 points)

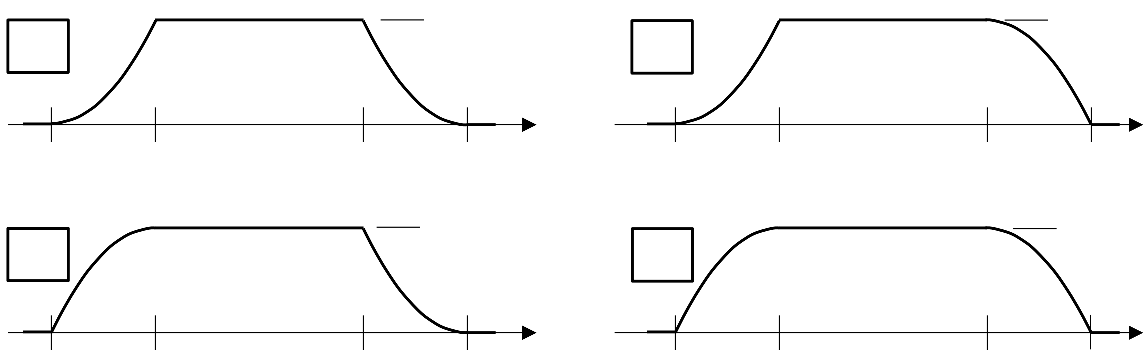




(d) (5 points)

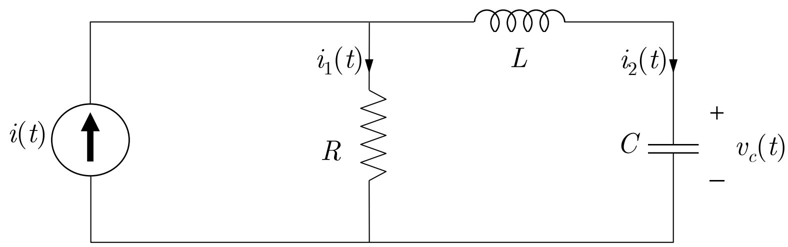






Problem F.3:

Consider the circuit shown below with input current source $i(t)$:



(a) (10 pts) Draw this circuit in the s-domain assuming zero initial conditions. Properly label all quantities indicated on the original circuit for full credit.

(b) (10 pts) Determine the transfer function relating the input current source to the voltage across the capacitor, $H(s) = V_c(s)/I(s)$, by analyzing it in the s-domain. Simplify $H(s)$ so that it is expressed as a ratio of polynomials in s.

(c) (5 pts) Let $i(t) = I_0 u(t)$. Use the Final Value Theorem to find the steady state output voltage.

Problem F.4:

Consider a system with input $x(t)$ and output $y(t)$ whose impulse response is $h(t) = \delta(t) + \delta(t - t_d)$. This system is a reasonable model for a signal with an echo (often called "multipath") where t_d, the time delay of the echo, is unknown.

(a) (10 pts) Find $H(j\omega)$ in terms of t_d. Simplify it as much as possible, which will help you with part (c).

(b) (10 pts) Find $y(t)$ for $t_d = 4$ and $x(t) = 5 + u(t) - u(t-1) + 2\cos(\frac{\pi}{2}t)$.

(c) (10 pts) Accurately sketch $|H(j\omega)|$ for $|\omega| < 6\pi/t_d$.

(d) (5 pts) One strategy for finding t_d is to excite the system with a delta function; i.e., $x(t) = \delta(t)$, and analyze the response $y(t)$ in the frequency domain by finding its nulls (frequencies where $Y(j\omega) = 0$). Express t_d in terms of ω_{null}, the first null of $Y(j\omega)$, for $x(t) = \delta(t)$. (This should be very straightforward if part (c) is correct).

Problem F.5:

(5 pts each) The table below gives transfer functions of six second order LTI systems. For each system, find the pole locations and decide whether the system is stable or unstable. If the system is stable, circle the attributes regarding filter type, damping and resonance that apply to that system. The attribute "Resonant" means that there is a peak in the frequency response for a low-pass or high-pass filter. If the system is unstable, give an example of one bounded input $x(t)$ starting at $t = 0$ that results in an unbounded output $y(t)$.

(a) $H(s) = \dfrac{2}{s^2 + 6s + 5}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$
(b) $H(s) = \dfrac{2s^2}{s^2 + 2s + 10}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$
(c) $H(s) = \dfrac{36}{s^2 + 36}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$

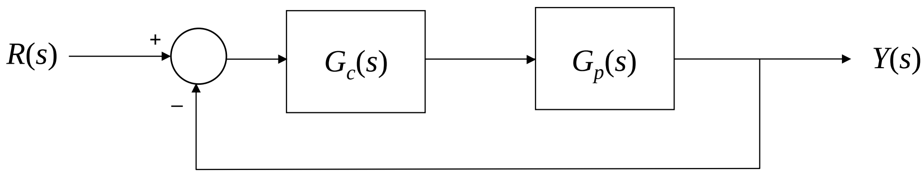
(d) $H(s) = \dfrac{20s}{s^2 + 10s + 25}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$

(e) $H(s) = \dfrac{s^2}{s^2 + 4s - 5}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$

(f) $H(s) = \dfrac{100}{s^2 + 8s + 25}$, Poles: Circle One: Stable Unstable
If stable, circle the appropriate attributes associated with $H(s)$:   Low-pass Filter Band-pass Filter High-pass Filter   Underdamped Critically Damped Overdamped Resonant
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output $x(t) =$

Problem F.6:

This problem considers velocity control of a plant $G_p(s)$ using a controller $G_c(s)$ where $G_c(s) = K_p$ (proportional control) and $G_p(s) = \dfrac{1}{s(s+a)}$.



(a) (5 pts) Find the transfer function $H(s)$ of the closed loop system in terms of a, the plant pole location, and K_p. Express your answer as a ratio of polynomials with all like terms combined.

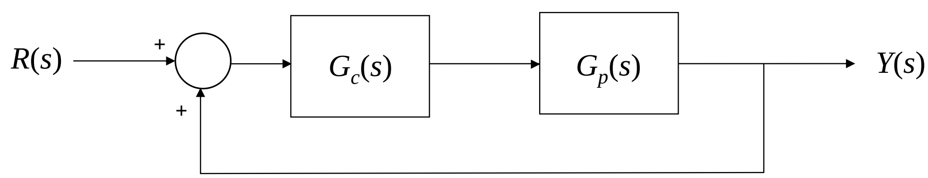
(b) (10 pts) For $K_p > 0$ and $a > 0$, indicate whether the following statements about the closed loop system are true or false. Select "Can't Tell" if there is insufficient information to determine if it is true or false.

The system is stable for all $K_p > 0$ and $a > 0$	True	False	Can't Tell
The system perfectly tracks a step function input	True	False	Can't Tell
The system is overdamped	True	False	Can't Tell
The natural frequency ω_n depends only on K_p	True	False	Can't Tell
The damping ratio ζ depends only on a	True	False	Can't Tell

(c) (10 pts) Consider the case where $a = 10$. Find K_p such that $\zeta = 5/6$. Where are the poles of the resulting closed loop system?

Problem F.7:

Consider the feedback system shown below, which differs from that of the previous problem by having *positive* feedback (see the positive sign where $y(t)$ feeds into the summing junction). The transfer function of a closed loop system with positive feedback is $H_+(s) = \dfrac{G_c(s)G_p(s)}{1 - G_c(s)G_p(s)}$. Positive feedback is usually undesirable, but it is often quite easy to accidentally hook up a system incorrectly such that the feedback is positive rather than negative (Prof. Michaels knows this from personal experience). As for the previous problem, consider the situation for which the plant output, $y(t)$, is the velocity of a mechanical system, and the goal is to control velocity.



(a) (5 pts) Find the transfer function $H_+(s)$ of the closed loop system with positive feedback for $G_c(s) = K_p$ (proportional control) and $G_p(s) = \dfrac{1}{s(s+a)}$. Express your answer as a ratio of polynomials with all like terms combined.

(b) (10 pts) For $a = 9$ and $K_p = 10$, find the complete step response; that is, find $y(t)$ for $r(t) = u(t)$.

(c) (10 pts) Keeping in mind that $y(t)$ is the velocity of a mechanical system, consider the case whereby the systems engineer immediately observes the undesirable step response from part (b) and turns off the input as soon as possible; i.e., $r(t) = u(t) - u(t - t_d)$. What is $y(t)$ for $t >> t_d$? Does it return to zero (i.e., eventually stops moving), does it stabilize to a specific value (i.e., runs at a constant velocity), or does it keep increasing (until something limits the velocity)?

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- Good luck!

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7	25	
Total	200	

Problem F.1:

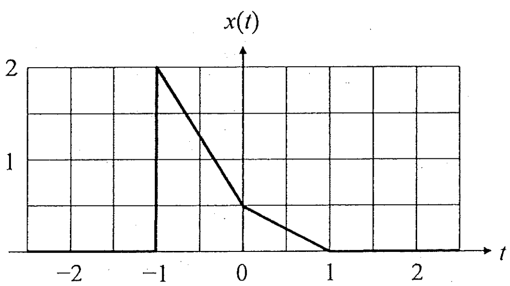
Short problem assortment. The six parts of this problem are unrelated to each other.

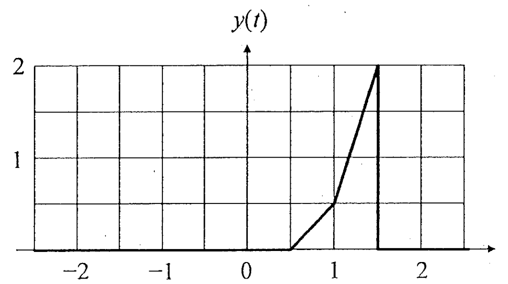
(a) (5 pts) Consider the system with input $x(t)$ and output $y(t)$ described by the following equation:

$y(t) = \frac{1}{x(t)}$. Circle "yes" or "no" in each row of the table below to indicate whether or not this system is linear, time-invariant, and/or causal.

Linear	yes	no
Time-Invariant	yes	no
Causal	yes	no

(b) (5 pts) The functions $x(t)$ and $y(t)$ are shown below where $y(t) = x(Bt + C)$. Find B and C and write them below the figure.





$y(t) = x(-2(t-1))$
$= x(-2t+2$

$B = -2$

$C = +2$

(c) (5 pts) Simplify the following expression: $\int_0^t \cos(\pi\tau)[\delta(\tau+2) + \delta(\tau-1)]d\tau =$

$t>0$ $\quad \int_0^t \cos(\pi)\,\delta(\tau-1)\,d\tau = \boxed{-u(t-1)}$

will accept this for full credit since problem wasn't clearly stated

$t<0$ $\quad -\int_t^0 \delta(\tau+2)\,d\tau = -u(\tau+2)\Big|_t^0$

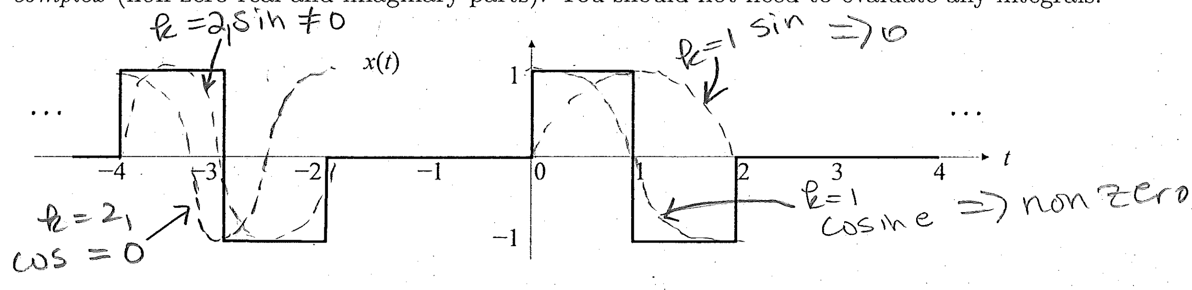
$= -u(2) + u(t+2)$

$= -1 + u(t+2)$

Will also accept this for full credit:

$\boxed{-1 + u(t-2) - u(t-1)}$

(d) (10 pts) Consider the Fourier series of the periodic signal $x(t)$ shown below with a period of $T_0 = 4$s. In the table below, circle *exactly one* attribute for each of the $k = 0$, $k = 1$, and $k = 2$ Fourier coefficients; that is, indicate if a_k is either *zero*, *non-zero real*, *imaginary* or *complex* (non-zero real and imaginary parts). You should not need to evaluate any integrals.



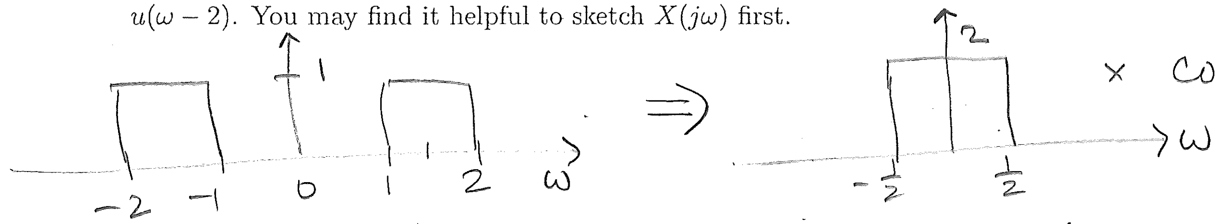
$k = 0$	zero	non-zero real	imaginary	complex	← DC $= 0$
$k = 1$	zero	non-zero real	imaginary	complex	← only cosine terms $\neq 0$
$k = 2$	zero	non-zero real	imaginary	complex	← only sine terms $\neq 0$

(e) (5 pts) Find $X(j\omega)$, the Fourier transform of the signal $x(t) = \sum_{n=0}^{\infty} e^{-at}\delta(t - nT)$ $= e^{-at}u(t) \cdot \sum_{n=-\infty}^{\infty}\delta(t-nT)$

$$X(j\omega) = \frac{1}{\cancel{2\pi}}\left(\frac{1}{a+j\omega}\right) * \left(\frac{\cancel{2\pi}}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{T}\right)\right)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty}\frac{1}{a+j\left(\omega - \frac{2\pi k}{T}\right)}$$

(f) (10 pts) Find $x(t)$, the inverse Fourier transform of $X(j\omega) = u(\omega+2) - u(\omega+1) + u(\omega-1) - u(\omega-2)$. You may find it helpful to sketch $X(j\omega)$ first.



$\times \cos(1.5t)$

$$x(t) = \frac{2\sin(\frac{1}{2}t)}{\pi t}\cdot\cos\left(\frac{3}{2}t\right)$$

these are the same

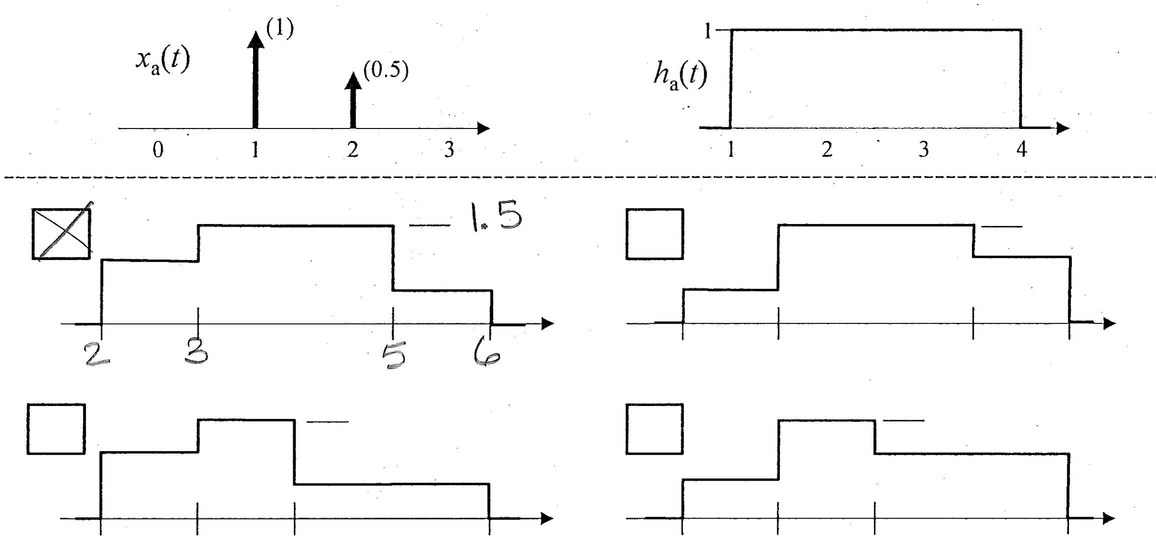
or: $X(j\omega) = [u(\omega+2) - u(\omega-2)] - [(u(\omega+1) - u(\omega-1)] \Rightarrow x(t) = \frac{\sin(2t)}{\pi t} - \frac{\sin(t)}{\pi t}$

Work either way for full credit.

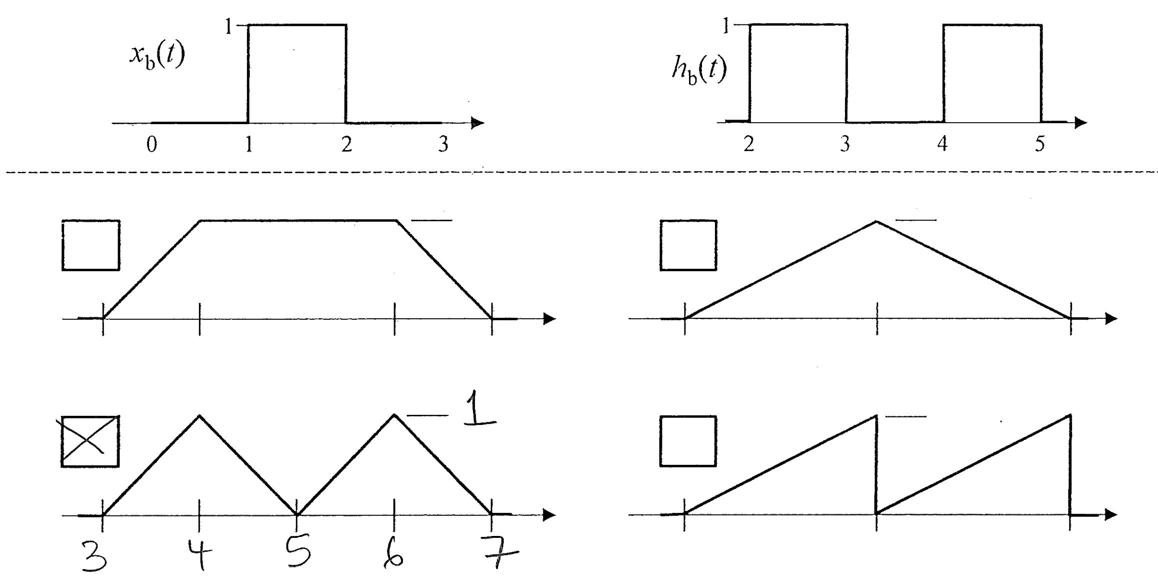
Problem F.2:

For each part of this problem, consider the convolution $y(t) = x(t) * h(t)$. For each part, $x(t)$ and $h(t)$ are different, and four choices for $y(t)$ are shown. Indicate your choice of $y(t)$ by putting an "X" inside the box adjacent to your selected plot. For each chosen $y(t)$, label its horizontal axis at all tic marks shown, and also label the vertical axis to show the maximum value.

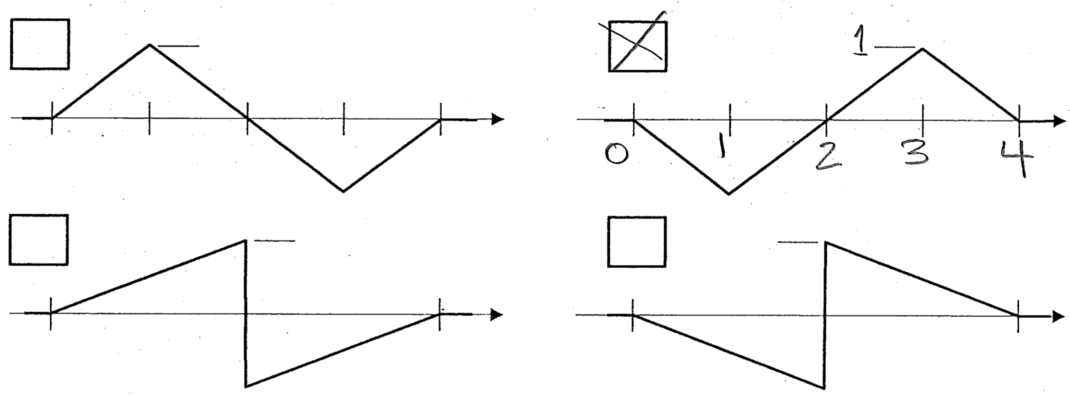
(a) (5 points)



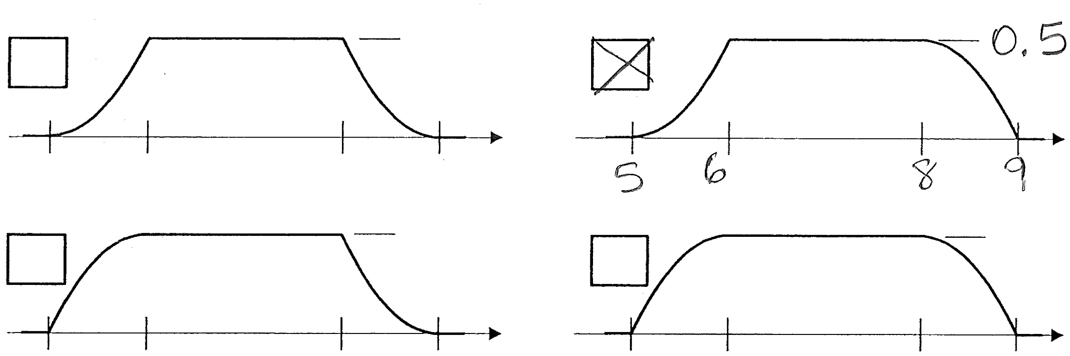
(b) (5 points)



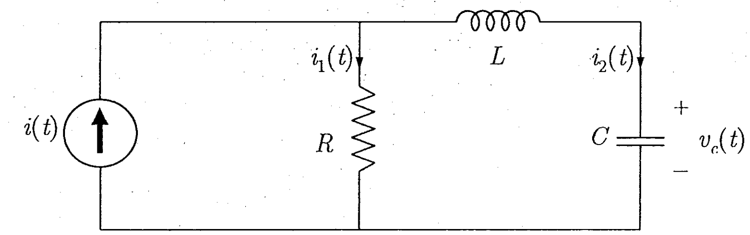
(c) (5 points)



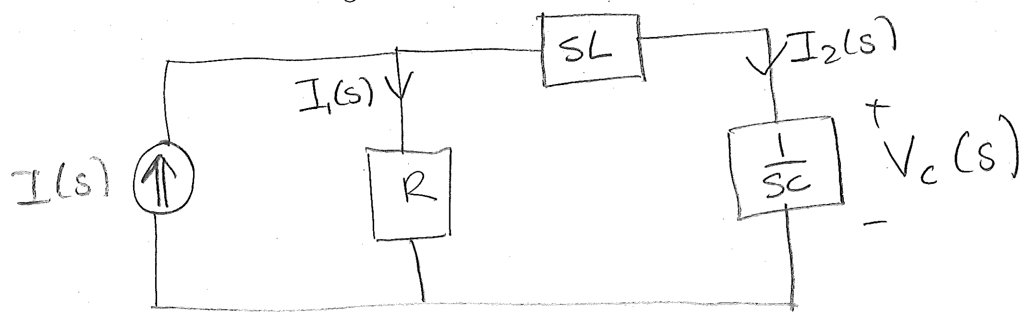
(d) (5 points)



Problem F.3:
Consider the circuit shown below with input current source $i(t)$:



(a) (10 pts) Draw this circuit in the s-domain assuming zero initial conditions. Properly label all quantities indicated on the original circuit for full credit.



(b) (10 pts) Determine the transfer function relating the input current source to the voltage across the capacitor, $H(s) = V_c(s)/I(s)$, by analyzing it in the s-domain. Simplify $H(s)$ so that it is expressed as a ratio of polynomials in s.

Current divider

$$I_2(s) = I(s) \cdot \frac{R}{R + SL + \frac{1}{SC}} = I(s) \frac{SCR}{s^2LC + SCR + 1}$$

$$V_c(s) = \frac{1}{SC} I_2(s) = I(s) \frac{R}{s^2LC + SCR + 1}$$

$$H(s) = \frac{V_c(s)}{I(s)} = \frac{R}{s^2LC + SCR + 1}$$

(c) (5 pts) Let $i(t) = I_0 u(t)$. Use the Final Value Theorem to find the steady state output voltage.

$$V_c(s) = H(s) \frac{I_0}{S} \qquad \lim_{t \to \infty} v_c(t) = I_0 H(0) = I_0 R$$

Problem F.4:
Consider a system with input $x(t)$ and output $y(t)$ whose impulse response is $h(t) = \delta(t) + \delta(t - t_d)$. This system is a reasonable model for a signal with an echo (often called "multipath") where t_d, the time delay of the echo, is unknown.

(a) (10 pts) Find $H(j\omega)$ in terms of t_d. Simplify it as much as possible, which will help you with part (c).

$$H(j\omega) = 1 + e^{-j\omega t_d}$$
$$= e^{-j\omega t_d/2}\left(e^{j\omega t_d/2} + e^{-j\omega t_d/2}\right)$$
$$= 2e^{-j\omega t_d/2} \cos\left(\omega t_d/2\right)$$

(b) (10 pts) Find $y(t)$ for $t_d = 4$ and $x(t) = 5 + u(t) - u(t-1) + 2\cos(\frac{\pi}{2}t)$.

Do each part separately, 1^{st} & last in the frequency domain

$$H(j\omega) = 2e^{-j2\omega} \cos(2\omega)$$

5: $H(j0) = 2$

$2\cos(\frac{\pi}{2}t)$: $H(j\frac{\pi}{2}) = 2e^{-j\pi} \cos(\pi) = +2$

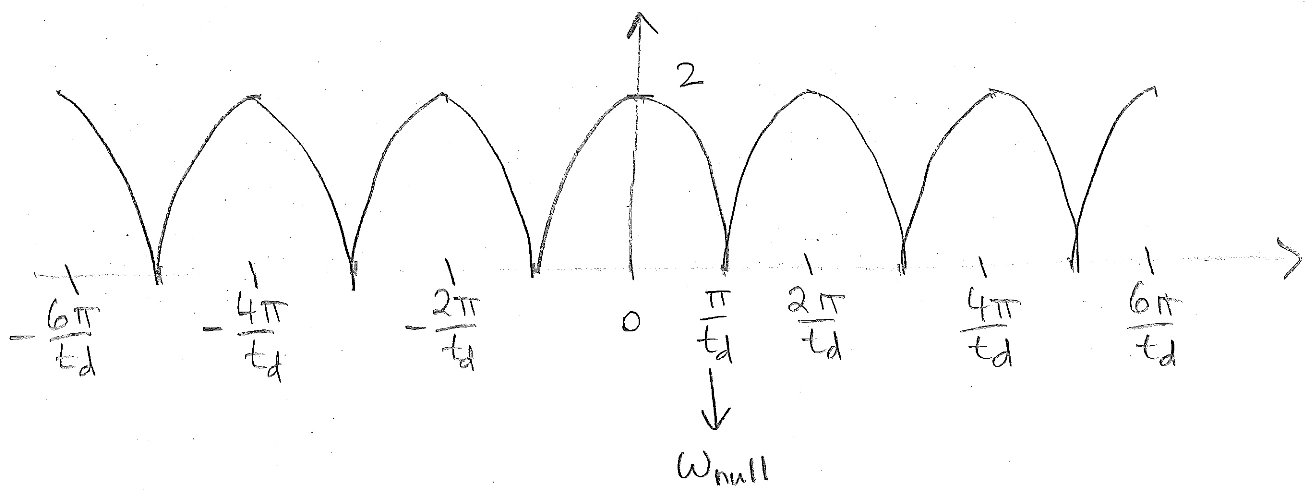
$$y(t) = 10 + \underbrace{u(t) - u(t-1) + u(t-4) - u(t-5)}_{\text{by convolution}} + 4\cos\left(\frac{\pi}{2}t\right)$$

for an arbitrary t_d peaks at $\frac{\omega t_d}{2} = n\pi$

(c) (10 pts) Accurately sketch $|H(j\omega)|$ for $|\omega| < 6\pi/t_d$.

$\omega = \frac{2n\pi}{t_d}$

$|H(j\omega)| = 2\cos\left(\frac{\omega t_d}{2}\right)$ from (a)



(d) (5 pts) One strategy for finding t_d is to excite the system with a delta function; i.e., $x(t) = \delta(t)$, and analyze the response $y(t)$ in the frequency domain by finding its nulls (frequencies where $Y(j\omega) = 0$). Express t_d in terms of ω_{null}, the first null of $Y(j\omega)$, for $x(t) = \delta(t)$. (This should be very straightforward if part (c) is correct).

$$\omega_{null} = \frac{\pi}{t_d}$$

$$\boxed{t_d = \frac{\pi}{\omega_{null}}}$$

Problem F.5:

(5 pts each) The table below gives transfer functions of six second order LTI systems. For each system, find the pole locations and decide whether the system is stable or unstable. If the system is stable, circle the attributes regarding filter type, damping and resonance that apply to that system. The attribute "Resonant" means that there is a peak in the frequency response for a low-pass or high-pass filter. If the system is unstable, give an example of one bounded input $x(t)$ starting at $t = 0$ that results in an unbounded output $y(t)$.

(a) $H(s) = \dfrac{2}{s^2 + 6s + 5}$, Poles: $P_1 = -1, \; P_2 = -5$

Circle One: Stable Unstable

If stable, circle the appropriate attributes associated with $H(s)$:

Low-pass Filter Band-pass Filter High-pass Filter

Underdamped Critically Damped Overdamped Resonant

If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output

$x(t) =$

(b) $H(s) = \dfrac{2s^2}{s^2 + 2s + 10}$, Poles: $P_{1,2} = -1 \pm j3$

$s^2 + 2s + 1 + 9$

$2\zeta\omega_n = 2$

$\zeta = \frac{2}{2 \cdot 3} = \frac{1}{3}$

Circle One: Stable Unstable

If stable, circle the appropriate attributes associated with $H(s)$:

Low-pass Filter Band-pass Filter High-pass Filter

Underdamped Critically Damped Overdamped Resonant

If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output

$x(t) =$

(c) $H(s) = \dfrac{36}{s^2 + 36}$, Poles: $P_{1,2} = \pm j6$

Circle One: Stable Unstable

If stable, circle the appropriate attributes associated with $H(s)$:

Low-pass Filter Band-pass Filter High-pass Filter

Underdamped Critically Damped Overdamped Resonant

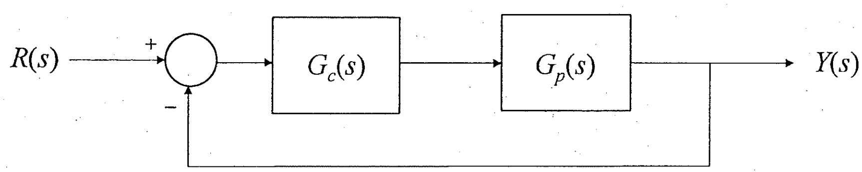
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output

$x(t) = \cos(6t)u(t)$ (must excite with $\omega = 6$)

(d) $H(s) = \dfrac{20s}{s^2 + 10s + 25}$, Poles: $p_{1,2} = -5$	
Circle One: Stable Unstable	
If stable, circle the appropriate attributes associated with $H(s)$:	
Low-pass Filter Band-pass Filter High-pass Filter	
Underdamped Critically Damped Overdamped Resonant	
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output	
$x(t) =$	
(e) $H(s) = \dfrac{s^2}{s^2 + 4s - 5}$, Poles: $p_1 = -5$, $p_2 = +1$	$(s+5)(s-1)$
Circle One: Stable Unstable	
If stable, circle the appropriate attributes associated with $H(s)$:	
Low-pass Filter Band-pass Filter High-pass Filter	
Underdamped Critically Damped Overdamped Resonant	
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output	
$x(t) = u(t)$ (or any other input)	
(f) $H(s) = \dfrac{100}{s^2 + 8s + 25}$, Poles: $p_{1,2} = -4 \pm j3$	$s^2+8s+16+9$
Circle One: Stable Unstable	$2\zeta\omega_n = 8$
If stable, circle the appropriate attributes associated with $H(s)$:	$\zeta = \dfrac{8}{2 \cdot 5} = 0.8$
Low-pass Filter Band-pass Filter High-pass Filter	
Underdamped Critically Damped Overdamped Resonant	
If unstable, specify a bounded input starting at $t = 0$ that results in an unbounded output	
$x(t) =$	

Problem F.6:

This problem considers velocity control of a plant $G_p(s)$ using a controller $G_c(s)$ where $G_c(s) = K_p$ (proportional control) and $G_p(s) = \dfrac{1}{s(s+a)}$.



(a) (5 pts) Find the transfer function $H(s)$ of the closed loop system in terms of a, the plant pole location, and K_p. Express your answer as a ratio of polynomials with all like terms combined.

$$H(s) = \frac{\dfrac{K_p}{s(s+a)}}{1 + \dfrac{K_p}{s(s+a)}} = \frac{K_p}{s^2 + as + K_p}$$

(b) (10 pts) For $K_p > 0$ and $a > 0$, indicate whether the following statements about the closed loop system are true or false. Select "Can't Tell" if there is insufficient information to determine if it is true or false.

The system is stable for all $K_p > 0$ and $a > 0$	True	False	Can't Tell
The system perfectly tracks a step function input	True	False	Can't Tell
The system is overdamped	True	False	Can't Tell
The natural frequency ω_n depends only on K_p	True	False	Can't Tell
The damping ratio ζ depends only on a	True	False	Can't Tell

(c) (10 pts) Consider the case where $a = 10$. Find K_p such that $\zeta = 5/6$. Where are the poles of the resulting closed loop system?

$$s^2 + 10s + K_p = s^2 + 2(5/6)\omega_n s + \omega_n^2$$

$$10 = \frac{10}{6}\omega_n \qquad \boxed{K_p = \omega_n^2 = 36}$$

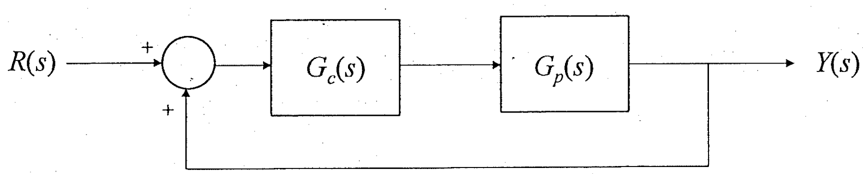
$$\omega_n = 6$$

$$s^2 + 10s + 36 = s^2 + 10s + 25 + 11 = (s+5)^2 + 11$$

$$P_{1,2} = -5 \pm \sqrt{11}$$

Problem F.7:

Consider the feedback system shown below, which differs from that of the previous problem by having *positive* feedback (see the positive sign where $y(t)$ feeds into the summing junction). The transfer function of a closed loop system with positive feedback is $H_{+}(s) = \dfrac{G_c(s)G_p(s)}{1 - G_c(s)G_p(s)}$. Positive feedback is usually undesirable, but it is often quite easy to accidentally hook up a system incorrectly such that the feedback is positive rather than negative (Prof. Michaels knows this from personal experience). As for the previous problem, consider the situation for which the plant output, $y(t)$, is the velocity of a mechanical system, and the goal is to control velocity.



(a) (5 pts) Find the transfer function $H_{+}(s)$ of the closed loop system with positive feedback for $G_c(s) = K_p$ (proportional control) and $G_p(s) = \dfrac{1}{s(s+a)}$. Express your answer as a ratio of polynomials with all like terms combined.

$$H(s) = \frac{\frac{K_p}{s(s+a)}}{1 - \frac{K_p}{s(s+a)}} = \frac{K_p}{s^2+as-K_p}$$

(b) (10 pts) For $a = 9$ and $K_p = 10$, find the complete step response; that is, find $y(t)$ for $r(t) = u(t)$.

$$Y(s) = \frac{H(s)}{s} = \frac{10}{s(s^2+9s-10)} = \frac{C_1}{s} + \frac{C_2}{(s+10)} + \frac{C_3}{(s-1)}$$

$$C_1 = \frac{10}{-10} = -1; \quad C_2 = \frac{10}{s(s-1)}\Big|_{s=-10} = \frac{10}{(-10)(-11)} = \frac{1}{11}$$

$$C_3 = \frac{10}{s(s+10)}\Big|_{s=1} = \frac{10}{1(11)} = \frac{10}{11}$$

$$\boxed{y(t) = -u(t) + \frac{1}{11}e^{-10t}u(t) + \frac{10}{11}e^{t}u(t)}$$

(c) (10 pts) Keeping in mind that $y(t)$ is the velocity of a mechanical system, consider the case whereby the systems engineer immediately observes the undesirable step response from part (b) and turns off the input as soon as possible; i.e., $r(t) = u(t) - u(t - t_d)$. What is $y(t)$ for $t >> t_d$? Does it return to zero (i.e., eventually stops moving), does it stabilize to a specific value (i.e., runs at a constant velocity), or does it keep increasing (until something limits the velocity)?

By linearity,

$$y(t) = -u(t) + \frac{1}{11}e^{-10t}u(t) + \frac{10}{11}e^{t}u(t)$$
$$+u(t-t_d) - \frac{1}{11}e^{-10(t-t_d)}u(t-t_d) - \frac{10}{11}e^{(t-t_d)}u(t-t_d)$$

For $t >> t_d$, all terms drop out except for the e^t terms

$$\left.\begin{aligned} y(t) &\cong \frac{10}{11}e^{t} - \frac{10}{11}e^{t}e^{-t_d} \\ &= \frac{10}{11}e^{t}(1-e^{-t_d}) \end{aligned}\right\} \text{For large } t >> t_d$$

The velocity keeps on increasing, but at a somewhat lower rate